Spin-Triplet Superconductivity from Quantum-Geometry-Induced Ferromagnetic Fluctuation

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We show that quantum geometry induces ferromagnetic fluctuation resulting in spin-triplet superconductivity. The criterion for ferromagnetic fluctuation is clarified by analyzing contributions from the effective mass and quantum geometry. When the non-Kramers band degeneracy is present near the Fermi surface, the Fubini-Study quantum metric strongly favors ferromagnetic fluctuation. Solving the linearized gap equation with the effective interaction obtained by the random phase approximation, we show that the spin-triplet superconductivity is mediated by quantum-geometry-induced ferromagnetic fluctuation.

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Introduction.—Unconventional superconductivity beyond the canonical Bardeen-Cooper-Schrieffer theory shows rich physical phenomena including high-temperature superconductivity and topological superconductivity. Various fluctuations arising from many-body interactions play the main role in the Cooper pairing for unconventional superconductivity, and low-dimensional fluctuations are particularly favorable. For example, it is argued that high-temperature superconductivity in cuprates is mediated by two-dimensional antiferromagnetic fluctuation [1–3]. Also, in iron-based high-temperature superconductors the extended s-wave pairing is mediated by orbital [4–6] or antiferromagnetic [7,8] fluctuation [9–11].

However, searching for topological superconductivity [12–15] with Majorana fermion [16–18] is an unresolved problem of modern condensed matter physics, which is attributed to the fact that the platform for topological superconductivity is rare in nature. Spin-triplet superconductors are canonical candidates, and it is expected that ferromagnetic fluctuation mediates the spin-triplet Cooper pairing. However, candidate materials are restricted to a few heavy-fermion systems with three-dimensional multiple bands [19–26].

In the two-dimensional isotropic continuum models, ferromagnetic fluctuation is not favored because of the constant density of states (DOS), which may imply the absence of two-dimensional spin-triplet superconductivity. Even for the anisotropic lattice systems, most quasi-two-dimensional superconductors do not show ferromagnetic fluctuation and antiferromagnetic fluctuations are rather ubiquitous, as we mentioned above for cuprates and iron-based compounds. Thus, spin-triplet superconductivity from ferromagnetic fluctuation is expected to require peculiar band structures, and the search for such systems is challenging for both materials and theoretical models. In this Letter, nevertheless, we propose a guiding principle for realizing ferromagnetic fluctuation in two-dimensional systems by referring to the quantum geometry of Bloch electrons, which is recently attracting much attention in various fields [27–46].

The importance of quantum geometry in superconductors has recently been recognized as it gives correction to the superfluid weight [34–37]. In the flatband systems [36,37,47–51] the superfluid weight from Fermi-liquid theory vanishes, and the quantum geometric contribution determines the superfluid weight. The quantum geometry also plays essential roles in the monolayer FeSe [52] and some finite-momentum Cooper pairing states [53–57]. However, how quantum geometry affects the pairing mechanism of superconductivity has not been revealed. This Letter elucidates a way to create a pairing glue of unconventional superconductivity via quantum geometry.

To show that the quantum geometry enables strong ferromagnetic fluctuation in two-dimensional systems, resulting in spin-triplet superconductivity, we elucidate the criterion for ferromagnetic fluctuation in the multiband system with SU(2) symmetry. We find that the criterion is given by the generalized electric susceptibility (GES) which is defined as a natural extension of the electric susceptibility to metals. The GES contains the terms obtained by the effective mass and the quantum geometry.

The key physics of quantum-geometry-induced ferromagnetic fluctuation, which is shown below, is nontrivial quantum geometry, especially Fubini-Study quantum metric [28,58], from non-Kramers band degeneracy. As shown in this Letter, the dispersive Lieb lattice model with non-Kramers band degeneracy shows strong ferromagnetic fluctuation by this mechanism. Solving the linearized gap equation with the effective interaction calculated by the random phase approximation (RPA), spin-triplet superconductivity is demonstrated.

Criterion for ferromagnetic fluctuation in multiband Hubbard models.—We consider the multiband Hubbard model with SU(2) symmetry, which contains multiple degrees of freedom such as orbitals and sublattices, as shown in the Supplemental Material [59]. The SU(2) symmetry means that the spin-orbit coupling and the magnetic field are absent. For the interacting Hamiltonian, we consider the on-site Coulomb interaction U strong enough for the superconducting transition, by assuming strongly correlated materials. We then focus on the momentum dependence of the fluctuation, which mainly determines the superconducting symmetry [3]. While we consider two-dimensional systems, the following discussions apply to three-dimensional systems.

Throughout this Letter, U is treated in the RPA scheme. When the system has only one band, the spin (charge) susceptibility $\chi_{\mathrm{s(c)}}(\boldsymbol{q}, i\Omega_n)$ can be obtained as $\chi_{\mathrm{s(c)}}(\boldsymbol{q}, i\Omega_n) = \chi^0_{\mathrm{s(c)}}(\boldsymbol{q}, i\Omega_n)/[1 \mp (U/2)\chi^0_{\mathrm{s(c)}}(\boldsymbol{q}, i\Omega_n)]$ by using the bare spin (charge) susceptibility of noninteracting systems, $\chi^0_{\mathrm{s(c)}}(\boldsymbol{q}, i\Omega_n)$. The interaction does not change the position of peaks in the momentum $\boldsymbol{q}$ space. Therefore, also for most multiband systems, it is expected that the momentum dependence of fluctuations arises from the bare susceptibility. Because the low-frequency spin (charge) fluctuation plays the dominant role in mediating superconductivity, hereafter we focus on the static fluctuations at $\Omega_n = 0$.

In multiband systems with SU(2) symmetry, the bare spin and charge susceptibilities hold the relationship $\chi^0_{\mathrm{s}}(\boldsymbol{q}) = \chi^0_{\mathrm{c}}(\boldsymbol{q}) = 2\chi^0(\boldsymbol{q})$ with the bare susceptibility $\chi^0(\boldsymbol{q})$. Thus, our main concern is the presence and absence of the peak of $\chi^0(\boldsymbol{q})$ at $\boldsymbol{q} = 0$, corresponding to the presence and absence of ferromagnetic fluctuation. The structure of susceptibility $\chi^0(\boldsymbol{q})$ around $\boldsymbol{q} = 0$ is determined by the curvature $\lim_{\boldsymbol{q}\to 0}\partial_{q_\mu}\partial_{q_\nu}\chi^0(\boldsymbol{q})$ with $\mu, \nu = x, y$ [63]. As a result, the criterion for the ferromagnetic fluctuation is given by the sign of the curvature (see Fig. 1). Ferromagnetic fluctuation may be present when $\lim_{\boldsymbol{q}\to 0}\partial_{q_\mu}\partial_{q_\nu}\chi^0(\boldsymbol{q})$ is negative. Otherwise, ferromagnetic fluctuation is prohibited.

The curvature $\lim_{\boldsymbol{q}\to 0}\partial_{q_\mu}\partial_{q_\nu}\chi^0(\boldsymbol{q})$ itself has a physical meaning. For the discussion, it is useful to consider the charge susceptibility in insulators at zero temperature, instead of the spin susceptibility. Based on the Kubo formula, the curvature expresses the correction to the charge density, $\delta\langle\hat{n}(\boldsymbol{r})\rangle$, by the external electric field $E_\nu(\boldsymbol{r})$ as, $\delta\langle\hat{n}(\boldsymbol{r})\rangle = -\sum_{\mu\nu}\partial_{r_\nu}[\lim_{\boldsymbol{q}\to 0}\frac{1}{2}\partial_{q_\mu}\partial_{q_\nu}\chi^0_{\mathrm{c}}(\boldsymbol{q})E_\nu(\boldsymbol{r})]$ [59]. This means that the curvature $\lim_{\boldsymbol{q}=0}\partial_{q_\mu}\partial_{q_\nu}\chi^0(\boldsymbol{q})$ is the electric susceptibility. Thus, by generalizing the concept of the electric susceptibility to metals, we define the GES as $\chi^{0:\mu\nu}_{\mathrm{e}} \equiv \lim_{\boldsymbol{q}=0}\partial_{q_\mu}\partial_{q_\nu}\chi^0(\boldsymbol{q})$ [59].

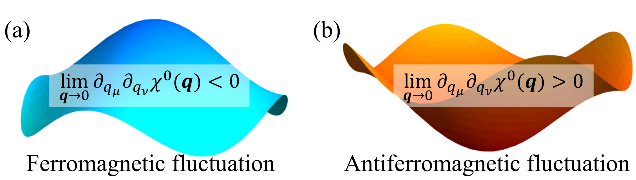

FIG. 1. Schematic figures for (a) ferromagnetic and (b) antiferromagnetic fluctuation. We illustrate the $\boldsymbol{q}$-dependence of $\chi^0(\boldsymbol{q})$.

Formula of GES.—Here, we derive the formula of GES [59], $\chi^{0:\mu\nu}_{\mathrm{e}} = \chi^{0:\mu\nu}_{\mathrm{e:geom}} + \chi^{0:\mu\nu}_{\mathrm{e:mass}}$,

$$\chi^{0:\mu\nu}_{\mathrm{e:geom}} = 2\sum_n \int \frac{d\boldsymbol{k}}{(2\pi)^2}\left(\frac{f'[\epsilon_n(\boldsymbol{k})]}{2}g^{\mu\nu}_n(\boldsymbol{k}) + f[\epsilon_n(\boldsymbol{k})]X^{\mu\nu}_n(\boldsymbol{k})\right), \tag{1}$$

$$\chi^{0:\mu\nu}_{\mathrm{e:mass}} = -2\sum_n \int \frac{d\boldsymbol{k}}{(2\pi)^2}\frac{f^{(2)}[\epsilon_n(\boldsymbol{k})]}{12}[m^{\mu\nu}_n(\boldsymbol{k})]^{-1}, \tag{2}$$

where $\epsilon_n(\boldsymbol{k})$ is the energy of the noninteracting Hamiltonian $\sigma_0 \otimes H_0(\boldsymbol{k})$, which follows $H_0(\boldsymbol{k})|u_n(\boldsymbol{k})\rangle = \epsilon_n(\boldsymbol{k})|u_n(\boldsymbol{k})\rangle$ with the Bloch wave function $|u_n(\boldsymbol{k})\rangle$. Note that σ_0 is the unit matrix of spin space and n is the band index. Thus, GES is given by the two terms, $\chi^{0:\mu\nu}_{\mathrm{e:geom}}$ and $\chi^{0:\mu\nu}_{\mathrm{e:mass}}$.

The first term $\chi^{0:\mu\nu}_{\mathrm{e:geom}}$ named the quantum geometric term is determined by the geometric quantities, namely, the Fubini-Study quantum metric $g^{\mu\nu}_n(\boldsymbol{k}) = \sum_{m(\neq n)}A^\mu_{nm}(\boldsymbol{k})A^\nu_{mn}(\boldsymbol{k}) + \mathrm{c.c.}$ and the positional shift $X^{\mu\nu}_n(\boldsymbol{k}) = \sum_{m(\neq n)}[A^\mu_{nm}(\boldsymbol{k})A^\nu_{mn}(\boldsymbol{k}) + \mathrm{c.c.}]/[\epsilon_m(\boldsymbol{k}) - \epsilon_n(\boldsymbol{k})]$ with the Berry connection $A^\mu_{nm}(\boldsymbol{k}) = i\langle\partial_{k_\mu}u_n(\boldsymbol{k})|u_m(\boldsymbol{k})\rangle$. This term arises from purely interband effects and is absent in single-band systems. In this term, the contributions from the quantum metric and the positional shift are competitive. First, the quantum metric [28,58], which is the counterpart of the Berry curvature [64], represents the distance between two adjacent states and is a positive definite tensor. Therefore, combined with negative $f'[\epsilon_n(\boldsymbol{k})]$, the contribution from the quantum metric is always negative, favoring ferromagnetic fluctuation. Second, the positional shift [29] means the shift of electrons by the external electric field. In insulators at zero temperature, the contribution from the positional shift corresponds to the well-known formula of electric susceptibility [65]. This term can be rewritten as it is proportional to $F_{nm}(\boldsymbol{k})[A^\mu_{nm}(\boldsymbol{k})A^\nu_{mn}(\boldsymbol{k}) + \mathrm{c.c.}]$ with the integrand of the Lindhard function, $F_{nm}(\boldsymbol{k},\boldsymbol{q}) = \{f[\epsilon_m(\boldsymbol{k})] - f[\epsilon_n(\boldsymbol{k}+\boldsymbol{q})]\}/[\epsilon_n(\boldsymbol{k}+\boldsymbol{q}) - \epsilon_m(\boldsymbol{k})] \xrightarrow{\boldsymbol{q}\to 0} F_{nm}(\boldsymbol{k})$. Therefore, this contribution is always positive, which favors antiferromagnetic fluctuation.

Importantly, both quantum metric and positional shift diverge at the non-Kramers band-degenerate point. Therefore, quantum geometry plays an essential role when non-Kramers band degeneracy exists. However, the total geometric term does not diverge because of the cancellation of two contributions, as shown in the Supplemental Material [59].

The effective-mass term $\chi^{0:\mu\nu}_{\mathrm{e:mass}}$ of GES is the purely intraband effect and is determined by the band dispersion through the effective mass $[m^{\mu\nu}_n(\boldsymbol{k})]^{-1} = \partial_{k_\mu}\partial_{k_\nu}\epsilon_n(\boldsymbol{k})$. In single-band systems, only this term is finite. This term can be positive or negative. For the hyperbolic dispersion $\epsilon_n(\boldsymbol{k}) = k^2/2m$, the effective-mass term is zero because the DOS and effective mass are constants, which means the absence of ferromagnetic fluctuation, as shown in the Supplemental Material [59].

GES with non-Kramers band degeneracy.—Because the non-Kramers band degeneracy enhances the quantum geometry, we focus on the Lieb lattice, which has been realized in ultracold atoms allowing us to tune the strength of U [66,67], with the experimental test in mind. The Lieb lattice hosts the flat band with threefold band degeneracy, and the ground state shows the flatband ferromagnetism [68]. To distinguish the quantum-geometry-induced ferromagnetic fluctuation from the flatband ferromagnetism, we study the dispersive Lieb lattice model in which the second and third-nearest-neighbor hoppings are finite. Unlike the usual Lieb lattice with only the nearest-neighbor hopping, the flat band becomes dispersive, and the threefold band degeneracy at the M point $[\boldsymbol{k} = (\pi, \pi)]$ is partially lifted, while the twofold degeneracy remains protected by the C_4 rotation symmetry, as shown in the Supplemental Material [59].

The dispersive Lieb lattice model is illustrated in Fig. 2(a). The Fermi surfaces for the chemical potential $\mu_c = 0.5$, 0.7, and 0.9 are shown in Fig. 2(b), and the band dispersion is in Fig. 2(c). The band-degenerate point lies on the Fermi surface, when $\mu_c = 0.7$. As shown in Fig. 2(d), the maximum of DOS corresponds to $\mu_c = 0.7$.

In Fig. 3(a), we show the chemical-potential dependence of GES $\chi_e^{0:xx}$. In some regions near the Lifshitz transitions ($\mu_c \simeq -0.1$ and 0.9), the GES shows the dip structure. This structure is induced by the effective-mass term. The effective-mass contribution from each band is proportional to an odd function $f^{(2)}[\epsilon_n(\boldsymbol{k})]$, and therefore, the effective-mass term tends to cancel out between the states below and above the Fermi energy. However, the cancellation is incomplete for μ_c near the Lifshitz transition point, and thus, the effective-mass term gives a negative GES. This is an understanding of why ferromagnetic fluctuation appears at finite temperatures when the Fermi surface is small, from the viewpoint of the GES.

In contrast, accompanying the band degeneracy on the Fermi surface, we obtain the maximally negative value of GES $\chi_e^{0:xx}$ at $\mu_c = 0.7$, which is dominated by the quantum geometric contribution. As expected from the band degeneracy at the M point, the quantum geometric term of the GES mainly comes from the region near the M point. This is verified by the $\boldsymbol{k}$-resolved quantum geometric contribution shown in Fig. 3(b). We find a large negative contribution to the GES from the vicinity of the M point, which in turn induces ferromagnetic fluctuation.

As we have mentioned, the quantum metric gives a negative contribution to the GES, while the positional shift positively contributes. Our results imply that the quantum metric overcomes the positional shift when the band-degenerate point lies on the Fermi surface. This can be intuitively understood from the formula of the quantum geometric term. The quantum metric contributes to the GES with $f'[\epsilon_n(\boldsymbol{k})]$, which is divergent on the Fermi surface at low temperatures, $[f'(0) \propto 1/T]$. On the other hand, $F_{nm}(\boldsymbol{k})$ in the positional shift contribution is a regular function. Therefore, the quantum metric becomes

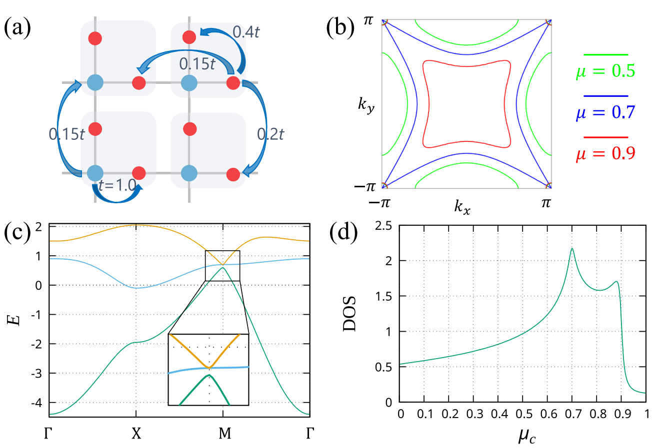

FIG. 2. (a) The dispersive Lieb lattice model, unit cell (gray box), and hopping integrals (blue arrows). The first-nearest-neighbor hopping is taken as the unit of energy, $t = 1$. (b) The Fermi surface for $\mu_c = 0.5$ (green line), 0.7 (blue line), and 0.9 (red line). (c) The band dispersion. (d) The DOS.

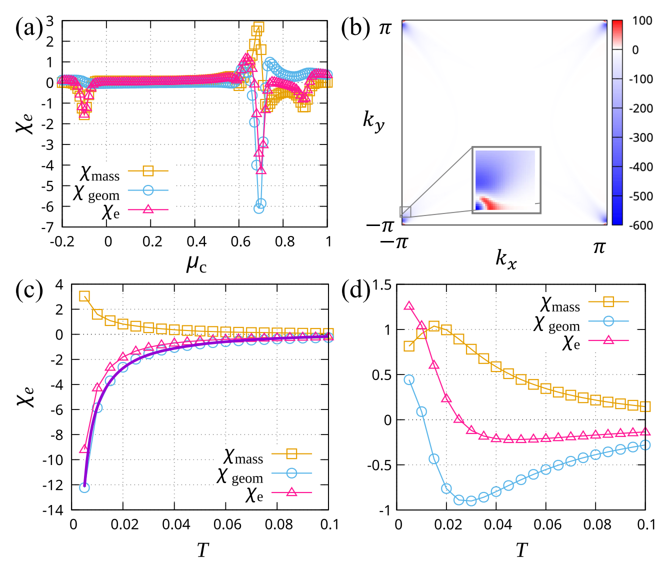

FIG. 3. GES $\chi_e^{0:xx}$ of the dispersive Lieb lattice model. In (a), (c), and (d), the triangles, circles, and squares show $\chi_e^{0:xx}$, $\chi_{e:\mathrm{geom}}^{0:xx}$, and $\chi_{e:\mathrm{mass}}^{0:xx}$, respectively. (a) The μ_c dependence for $T = 0.01$. (b) The quantum geometric contribution to the GES from each $\boldsymbol{k}$ point for $(\mu_c, T) = (0.7, 0.02)$. Inset: the contribution near the M point with band degeneracy. (c), (d) The temperature dependence for $\mu_c = 0.7$ and $\mu_c = 0.65$, respectively. The purple line in (c) is a fitting curve $\chi_{e:\mathrm{geom}}^{0:xx} \simeq -0.0631779/T + 0.462022$.

significant in the presence of band degeneracy at low energies. Consistent with the intuitive explanation, the geometric term is negatively enhanced at low temperatures owing to the contribution of quantum metric, as shown in Fig. 3(c). The geometric term is well fitted by the scaling $\chi^{0:\mu\nu}_{\mathrm{e:geom}} = a/T + b$ with constants a, b. Thus, we conclude that the quantum metric on the Fermi surface induces ferromagnetic fluctuation when the non-Kramers band degeneracy lies on the Fermi surface.

However, when the band-degenerate point is slightly off the Fermi surface and temperature decreases so that $T \ll |\mu_{\mathrm{c}} - 0.7|$, the negative geometric term is suppressed as shown in Fig. 3(d). This is consistent with the fact that the quantum metric contribution is a Fermi-surface term. As the band-degenerate point moves away from the Fermi surface by much more than T, $f'[\epsilon_n(\boldsymbol{k})]$ near the M point decays, and the quantum metric contribution is suppressed. At low temperatures, the positional shift contribution overcomes the quantum metric contribution, and the quantum geometric term is positive. Thus, in this case, the ferromagnetic-antiferromagnetic crossover of fluctuation occurs as the temperature decreases.

Quantum-geometry-induced ferromagnetic fluctuation.—Then, to justify the above discussion, we show the bare spin susceptibility defined by $\chi^0_{\mathrm{s}}(\boldsymbol{q}) = 2\sum_{nm}\int[d\boldsymbol{k}/(2\pi)^2]\times F_{nm}(\boldsymbol{k},\boldsymbol{q})[1 - D_{nm}(\boldsymbol{k},\boldsymbol{q})]$, where the quantum distance $D_{nm}(\boldsymbol{k},\boldsymbol{q}) \equiv 1 - |\langle u_n(\boldsymbol{k}+\boldsymbol{q})|u_m(\boldsymbol{k})\rangle|^2$ is closely related to the quantum geometry. Quantum geometry suppresses $\chi^0_{\mathrm{s}}(\boldsymbol{q})$ at $\boldsymbol{q} \neq 0$ via nonzero quantum distance $D_{nn}(\boldsymbol{k},\boldsymbol{q})$, which is expanded as $\sim\sum_{\mu\nu} g^{\mu\nu}_n(\boldsymbol{k}) q_\mu q_\nu + \cdots$ with the quantum metric. However, $\chi^0_{\mathrm{s}}(\boldsymbol{0})$ is not suppressed, and ferromagnetic fluctuation is relatively enhanced. The Van Vleck susceptibility arising from $D_{nm}(\boldsymbol{k},\boldsymbol{q})$ for $n \neq m$ corresponds to the positional-shift contribution to the GES. For comparison, we also define the bare spin susceptibility without quantum geometry, $\chi^0_{\mathrm{s:band}}(\boldsymbol{q}) = 2\sum_n \int[d\boldsymbol{k}/(2\pi)^2] \times F_{nn}(\boldsymbol{k},\boldsymbol{q})$, in which magnetic fluctuation is determined by only the effective-mass term. By comparing these two quantities, we can elucidate the effects of quantum geometry.

In Figs. 4(a) and 4(b), we show $\chi^0_{\mathrm{s}}(\boldsymbol{q})$ and $\chi^0_{\mathrm{s:band}}(\boldsymbol{q})$ in the dispersive Lieb lattice model for $\mu_{\mathrm{c}} = 0.7$. As expected by Fig. 3(a) showing the negative GES, $\chi^{0:\mu\nu}_{\mathrm{e}} = \lim_{\boldsymbol{q}=0}\partial_{q_\mu}\partial_{q_\nu}\chi^0(\boldsymbol{q})$, the bare spin susceptibility shows ferromagnetic fluctuation [Fig. 4(a)]. However, antiferromagnetic fluctuation is obtained when we neglect the quantum geometry [Fig. 4(b)]. Thus, we conclude that the quantum geometry induces the ferromagnetic fluctuation. It is emphasized that the maximum of DOS at $\mu_{\mathrm{c}} = 0.7$ is not sufficient for the ferromagnetic fluctuation; the relative enhancement of $\chi^0_{\mathrm{s}}(0)$ compared to $\chi^0_{\mathrm{s}}(\boldsymbol{q} \neq 0)$ by the quantum distance-quantum geometry is essential. Note that the momentum dependence of spin susceptibility plays an essential role in unconventional superconductivity [1,3]. We also show $\chi^0_{\mathrm{s}}(\boldsymbol{q})$ for $(\mu_{\mathrm{c}}, T) = (0.65, 0.05)$ and $(\mu_{\mathrm{c}}, T) = (0.65, 0.01)$ in Figs. 4(c) and 4(d), respectively. Consistent with Fig. 3(d), we confirm the crossover from ferromagnetic to antiferromagnetic fluctuation as the temperature decreases.

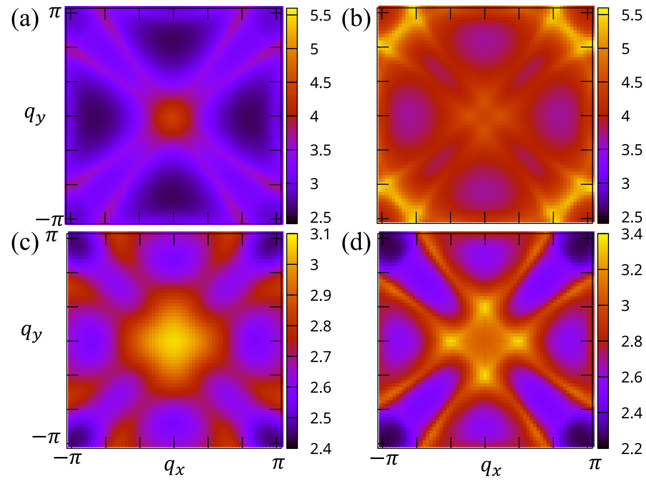

FIG. 4. The bare spin susceptibility in the dispersive Lieb lattice model. (a) $\chi^0_{\mathrm{s}}(\boldsymbol{q})$ and (b) $\chi^0_{\mathrm{s:band}}(\boldsymbol{q})$ for $(\mu_{\mathrm{c}}, T) = (0.7, 0.01)$ with the same color bar. (c),(d) $\chi^0_{\mathrm{s}}(\boldsymbol{q})$ for $(\mu_{\mathrm{c}}, T) = (0.65, 0.05)$ and (0.65,0.01), respectively.

Spin-triplet superconductivity.—Finally, we show that quantum-geometry-induced ferromagnetic fluctuation mediates spin-triplet superconductivity. To see this, we set the on-site interaction as $U = 0.86$ and solve the linearized gap equation, $\lambda^{\mathrm{t(s)}}\Delta_{ll'}(\boldsymbol{k}) = -(1/N\beta)\sum_{\boldsymbol{k}'\omega_n}\sum_{\{l_i\}} V^{\mathrm{t(s)}}_{ll_1,l_2l'}(\boldsymbol{k}-\boldsymbol{k}')\mathcal{G}_{l_1l_3}(\boldsymbol{k}', i\omega_n)\Delta_{l_3l_4}(\boldsymbol{k}')\mathcal{G}_{l_2l_4}(-\boldsymbol{k}', -i\omega_n)$, using the effective interaction obtained by RPA $V^{\mathrm{t(s)}}(\boldsymbol{q})$, which is $\simeq[-1(3)/4]U\chi_{\mathrm{s}}(\boldsymbol{q})U$ in single-band systems [1,3,69–71] but here extended to multiband systems, as shown in the Supplemental Material [59]. Here, $\mathcal{G}(\boldsymbol{k}, i\omega_n)$ is the Green function with the Matsubara frequency, $i\omega_n$. The instability of spin-triplet (singlet) superconductivity with the form factor $\Delta(\boldsymbol{k})$ is determined by the maximum eigenvalue $\lambda^{\mathrm{t(s)}}$. While the mean-field formalism overestimates the transition temperature, the dynamical effect of effective interaction is expected not to alter the superconducting symmetry, as in the cases of ^{3}He [69] and cuprates [72].

Figure 5(a) shows the spin susceptibility at $\mu_{\mathrm{c}} = 0.7$ obtained by RPA. Ferromagnetic fluctuation is enhanced by the Coulomb interaction, as we see from the comparison to Fig. 4(a). Eigenvalues of the linearized gap equation are shown in Fig. 5(b) for spin-singlet extended-s-wave (orange line) and spin-triplet p-wave (blue line) superconductivity, as shown in the Supplemental Material [59]. It is revealed that the spin-triplet superconductivity is stabilized around $\mu_{\mathrm{c}} \simeq 0.7$ and 0.9 corresponding to the negative peak of GES in Fig. 3(a).

Especially, we obtain the largest eigenvalue at $\mu_{\mathrm{c}} = 0.7$ where quantum geometry induces ferromagnetic

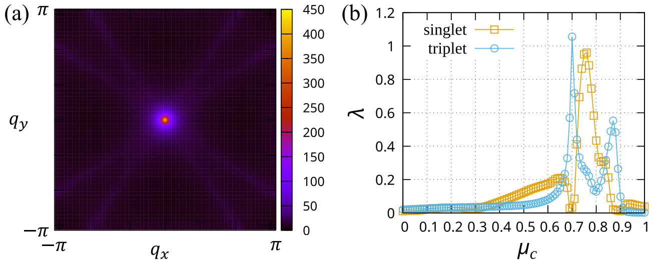

FIG. 5. (a) The spin susceptibility obtained by RPA for $(\mu_c, T) = (0.7, 0.01)$. (b) The eigenvalues of the linearized gap equation at $T = 0.01$. The blue and orange lines show the maximum eigenvalue for spin-triplet and spin-singlet superconductivity, respectively. Eigenvalues for all the irreducible representations are shown in the Supplemental Material [59].

fluctuation. Combined with the large DOS, the strong ferromagnetic fluctuation enhanced by interaction gives a large eigenvalue for spin-triplet superconductivity. Thus, we conclude spin-triplet superconductivity from quantum-geometry-induced ferromagnetic fluctuation.

Discussion.—In this Letter, we show that quantum geometry induces ferromagnetic fluctuation and results in spin-triplet superconductivity. The Fubini-Study quantum metric on the Fermi surface is an essential quantity for this mechanism of magnetism and superconductivity. Using the dispersive Lieb lattice model, we demonstrated that the non-Kramers band degeneracy on the Fermi surface plays the central role in enhancing the quantum-geometry-induced phenomena. In the diverse studies on unconventional superconductivity, the quantum geometry of electrons coupled to many-body effects has not been focused on. Stimulated by recent developments in the topology and geometry of quantum materials, we shed light on a route to spin-triplet superconductivity and, thereby, topological superconductivity.

A question of interest is whether our theory can be applied to other systems as well. To answer this, we have calculated the GES of Raghu's model [73] for iron-based superconductors, as shown in the Supplemental Material [59]. This model has the non-Kramers band degeneracy at the Γ point. Also in this model, the quantum geometry induces ferromagnetic fluctuation due to the non-Kramers band degeneracy. In addition, we confirmed the quantum-geometry-induced ferromagnetic fluctuation in other models with the flat band and various band touching including the usual Lieb lattice model [74]. Thus, a wide range of materials with non-Kramers band degeneracy [75,76] are candidates for quantum-geometry-induced ferromagnetism and superconductivity. We expect that future material-specific studies will be stimulated by our Letter. The exploration of two-dimensional materials with high tunability, e.g., by band engineering through heterostructures, gate voltage, strain, and twist angle is also expected.

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