# Pointer States in the Born-Markov Approximation

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Quantum states least affected by interactions with environment play a pivotal role in both foundations and applications of quantum mechanics. Known as pointer states, they surprisingly lacked a systematic description. Working within the Born-Markov approximation, we combine methods of group theory and open quantum systems and derive general conditions describing pointer states. Contrary to common expectations, they are in general different from coherent states. Thus the two notions of being "closest to the classical"—one defined by the uncertainty relations and the other by the interaction with the environment—are in general different. As an example, we study spin-spin and spin-boson models with an arbitrary central spin $J$.

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A comprehensive understanding of how the classical reality emerges from the underlying quantum theory is one of the most fascinating challenges of modern physics. Decoherence program [1–5] has been highly successful in explaining the loss of quantum properties through the interaction with uncontrolled degrees of freedom (environment); see, e.g., Refs. [6–12]. The inevitable, in realistic situations, interactions with the environment lead to delocalization and destruction of phase relations, making certain quantum superpositions unobservable. On the other hand, the same process distinguishes some preferred states [13,14], which are least affected, and thus the system is most likely to be found in one of them. In this sense, perceived classicality can be explained through properties of certain robust quantum states [2].

Determining the preferred states, known as pointer states, in the general case has been a difficult open task since their introduction in Ref. [13]. Several formal definitions were given with the most fruitful being the predictability sieve idea [15], defining pointer states as states producing least entropy (for others see, e.g., Ref. [16]). Various examples of pointer states have been found so far, with the best known [17] in the quantum Brownian motion (QBM) model [2,5,18,19], where they happen to be the Glauber-Sudarshan coherent states [20,21]. Minimum uncertainty states were also proven to be universal pointer states for a general, linearly coupled free open evolution and that decoherence to them is generic [22], which is an extension of an earlier result [16], obtained in a simpler Markovian model. A general question when generalized coherent states [23] are the preferred states was analyzed in [24], using the Gorini-Kossakowski-Lindblad-Sudarshan (GKLS) [25–28] master equation and group-theoretical methods, showing that it is not generically the case unless specific conditions are met. Other results include Ref. [29], where similar findings to Ref. [17] were derived in the GKLS formalism [30], where the problem of how to engineer the coupling to turn a given pure state into a pointer state was studied in a non-Markovian setup, and Ref. [31], where pointer states for antiferromagnetic systems were numerically analyzed.

In this Letter, we derive a general framework for finding pointer states in the Born-Markov regime. To our knowledge, it is the first systematic analysis of pointer states in the Born-Markov regime. We work under a broad assumption of an existence of some Lie group structure behind the dynamics, which covers, among others, the canonical models of decoherence [5]. We analyze in detail the case of a compact, semisimple group, but our method applies to other groups too as we show on the QBM example of [17]. The equations that we derive define a new class of optimization problems, different from the minimum uncertainty problem, confirming that open dynamics selects its own robust states, different from the generalized coherent states (cf. [24]). We exemplify our methods with arbitrary spins coupled to bosonic or spin environments. Pointer states in these models were unknown before.

*General method.*—We consider an open system model, where a system of interest $S$, governed by the free Hamiltonian $H_0$, is coupled to the environment $E$ via a bilinear interaction term $H_I = A \otimes \mathcal{E}$. The most general coupling is a sum of such terms [5], and a generalization of our method to such couplings is straightforward. We will assume the weak-coupling limit and that the conditions of the Born-Markov approximation hold. Then system's reduced density matrix $\rho(t)$ satisfies then the Born-Markov master equation [5,32,33]:

$$\dot{\rho}(t) = -\mathrm{i}[H_0, \rho(t)] - \int_0^\infty \mathrm{d}\tau \nu(\tau)[A, [A(-\tau), \rho(t)]] + \mathrm{i}\int_0^\infty \mathrm{d}\tau \eta(\tau)[A, \{A(-\tau), \rho(t)\}], \quad (1)$$

 

where $\{A, B\} = AB + BA$,

$$A(-\tau) = e^{-\mathrm{i}H_0\tau} A e^{\mathrm{i}H_0\tau}, \tag{2}$$

and $\nu(\tau)$, $\eta(\tau)$ are, respectively, the noise and dissipation kernels defined via the environment correlation function:

$$\mathrm{Tr}_E[\rho_E(0)\mathcal{E}(\tau)\mathcal{E}] \equiv \nu(\tau) - \mathrm{i}\eta(\tau), \tag{3}$$

where $\mathcal{E}(\tau) = e^{\mathrm{i}H_E\tau}\mathcal{E}e^{-\mathrm{i}H_E\tau}$. The environment state $\rho_E$ is arbitrary here as long as it is stationary $[H_E, \rho_E] = 0$, which is a standard assumption [2,4,5].

The predictability sieve looks for those states, which generate the least entropy during the evolution (1). A convenient measure is the linear entropy (cf. [3,16]):

$$s(\rho) = 1 - \mathrm{Tr}[\rho^2], \tag{4}$$

related to purity. We assume a pure initial state,

$$\rho(0) = |\psi\rangle\langle\psi|, \tag{5}$$

and ask how much purity is lost during the evolution [17]. Let us assume that there exists a Lie group $G$ of dimension $N$, such that $H_0$ and $A$ are from its Lie algebra and can be represented using the generators $\{X_i\}_{i=1}^N$:

$$H_0 \equiv X_N, \qquad A \equiv \sum_{j=1}^N a_j X_j \tag{6}$$

(we choose $H_0$ as one of the generators and leave $A$ arbitrary only for the sake of definiteness). This is somewhat similar to the approach of [24]; however, instead of imposing the secular approximation and studying the algebra generated by the GKLS jump operators and the associated generalized coherent states, we assume the existence of a dynamical group already at the microscopic level. Both $H_0$ and $A$ must be Hermitian for physical reasons, which motivates the assumption that the whole Lie algebra is real and spanned by Hermitian operators, $X_j^\dagger = X_j$. Further assuming they can be represented in finite dimension, $G$ becomes a subgroup of sufficiently large unitary group and thus compact. In light of these identifications, the free evolution (2) is the adjoint action of $G$ in its Lie algebra:

$$X_j(-\tau) = e^{-\mathrm{i}X_N\tau} X_j e^{\mathrm{i}X_N\tau} = \sum_k R^N_{jk}(-\tau) X_k, \tag{7}$$

where $R^N_{jk}(0) = \delta_{jk}$. The indices $i, j, k, l \ldots \in 1 \ldots N$ are the Lie algebra indices of $G$. Matrix $R^N$ is nothing but the exponent of the structure constants $f_{ijk}$ of $G$, arranged into $N \times N$ matrix (the matrix of the $\mathrm{ad}_{X_N}$ action) and is given by

$$R^N_{jk}(t) = \left[e^{\mathrm{i}t\,\mathrm{ad}_{X_N}}\right]_{jk}, \qquad [\mathrm{ad}_{X_N}]_{jk} = \mathrm{i}f_{Njk}, \tag{8}$$

where $f_{ijk}$ are defined via $[X_i, X_j] = \mathrm{i}\sum_k f_{ijk} X_k$. In what follows we will omit the index $N$ for simplicity, writing $R_{jk}(\tau)$. Using this we can express the change of the entropy (4) as

$$\frac{1}{2}\dot{s} = \sum_{jkl} a_j a_l D_{jk} (\mathrm{Tr}[\rho^2\{X_l, X_k\}] - 2\mathrm{Tr}[\rho X_l \rho X_k]) + \sum_{jklm} a_j a_l \gamma_{jk} f_{lkm} \mathrm{Tr}[\rho^2 X_m], \tag{9}$$

where we have introduced constants [assuming the function (3) is regular enough for the integrals to exist]:

$$D_{jk} = \int_0^\infty \mathrm{d}\tau \nu(\tau) R_{jk}(-\tau), \qquad \gamma_{jk} = \int_0^\infty \mathrm{d}\tau \eta(\tau) R_{jk}(-\tau). \tag{10}$$

To make further analysis feasible, we assume as a first approximation [17] that the state $\rho(t)$ on the right-hand side above is approximately pure and evolves according to the free evolution; i.e.,

$$\rho(t) \approx e^{-\mathrm{i}X_N t} |\psi\rangle\langle\psi| e^{\mathrm{i}X_N t}. \tag{11}$$

Using Eq. (7) we obtain

$$\frac{1}{2}\dot{s} \approx \sum_{jklmn} a_j a_l D_{jk} R_{lm}(t) R_{kn}(t) C_{mn} \tag{12}$$

$$+ \sum_{jklmn} a_j a_l \gamma_{jk} f_{lkm} R_{mn}(t) \langle X_n \rangle, \tag{13}$$

where

$$C_{mn} = \langle\{X_m, X_n\}\rangle - 2\langle X_m\rangle\langle X_n\rangle \tag{14}$$

is the covariance matrix calculated in the initial state and $\langle X_m\rangle = \langle\psi|X_m|\psi\rangle$ is the average in the initial state.

Next we analyze matrices $R_{jk}(t)$. For a compact $G$, there exists a Killing-Cartan form, $h_{jk} = \mathrm{Tr}[\mathrm{ad}_{X_j} \mathrm{ad}_{X_k}]$ on the Lie algebra, invariant under (7) and serving as a metric (see, e.g., Ref. [34]). For simplicity, we will also assume that this metric is nondegenerate (so $G$ is semisimple, which is not crucial) so that $h_{jk} \propto \delta_{jk}$ and the adjoint matrices become orthogonal. Then from (7), $R_{jk}(t)$ are rotations around the $X_N$ axis and have a block form $R(t) = \mathrm{diag}[\mathrm{SO}(N-1), 1]$. The $\mathrm{SO}(N-1)$ part can be further decomposed into a direct sum of two-dimensional rotations:

$$R(t) = O^T[\oplus_\alpha R_\alpha(t)]O, \qquad R_\alpha(t) = \begin{bmatrix} \cos t\Omega_\alpha & \sin t\Omega_\alpha \\ -\sin t\Omega_\alpha & \cos t\Omega_\alpha \end{bmatrix}, \tag{15}$$

where $O \in \mathrm{SO}(N)$, $1 \le \alpha \le \alpha_N = \lfloor (N-1)/2 \rfloor$, the last block $R_{\alpha_N}(t) = 1$, and for even $N$ the one before the last block is also trivial; i.e.,

$$OR(t)O^T = \begin{bmatrix} R_1(t) & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & 1 \end{bmatrix}, \begin{bmatrix} R_1(t) & & & \\ & \cdot & & \\ & & \cdot & \\ & & & 1 \\ & & & & 1 \end{bmatrix} \tag{16}$$

for odd and even $N$, respectively. The form of the time dependence in (15) comes from the antisymmetry (the $G$ invariance of the Cartan-Killing form):

$$f_{ijk} = -f_{ikj}, \tag{17}$$

making $f_{ijk}$ totaly antisymmetric as $f_{ijk} = -f_{jik}$.

Thus for each particular model, the lhs of (12) is a linear combination of trigonometric functions and their squares, which, in principle, can be integrated (cf. [17]). However, the instantaneous entropy production $s(t)$ may not be the most indicative quantity due to its time fluctuations. Here we choose its longtime average as more representative:

$$\bar{s} = \lim_{\tau\to\infty} \frac{1}{\tau}[s(\tau) - s(0)] = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau dt\dot{s}(t); \tag{18}$$

meaning we average the entropy production over times much longer than any other timescales. In what follows, we apply the above time averaging to Eq. (12), using Eq. (15). The first term contains quadratic factors:

$$\overline{R_{lm}(t)R_{kn}(t)} = \sum_{k'l'm'n'} O_{l'l}O_{k'k}\sum_{\alpha\beta}\overline{R^{(\alpha)}_{l'm'}(t)R^{(\beta)}_{k'n'}(t)}\,O_{m'm}O_{n'n}, \tag{19}$$

where $R^{(\alpha)}_{jm}$ are the matrices of $R_\alpha$ embedded naturally into the whole space by adding rows and columns of zeros; i.e., $R^{(\alpha)}_{jm} = 0$ when $jm$ are outside of the $\alpha$ subspace. Let us define rotated generators and coefficients from (6):

$$\tilde{X}_{m'} \equiv \sum_m O_{m'm}X_m, \qquad \tilde{a}_{j'} \equiv \sum_j O_{j'j}a_j \tag{20}$$

$[\tilde{X}_N = X_N$ by the construction; cf. Eq. (16)], the corresponding covariance matrix $\tilde{C}_{mn}$. We show that [35]

$$\begin{aligned}&\sum_{jklmn} a_j a_l D_{jk}\overline{R_{lm}(t)R_{kn}(t)}C_{mn} \\ &\quad = \sum_{\alpha\le\alpha_N}\|\tilde{\mathbf{a}}_\alpha\|^2 D_\alpha\left[\Delta\tilde{X}^2_{\alpha0} + \Delta\tilde{X}^2_{\alpha1}\right] + a_N^2 D_0\Delta X_N^2,\end{aligned} \tag{21}$$

and for even $N$ there is an additional term $\tilde{a}^2_{N-1}D_0\Delta\tilde{X}^2_{N-1}$. Here $\tilde{\mathbf{X}}_\alpha = (\tilde{X}_{\alpha0}, \tilde{X}_{\alpha1})$ is the projection of $\tilde{\mathbf{X}}$ on the $\alpha$ subspace, similarly $\tilde{\mathbf{a}}_\alpha$, and $\Delta\tilde{X}^2_{\alpha i} = \langle\tilde{X}^2_{\alpha i}\rangle - \langle\tilde{X}_{\alpha i}\rangle^2$ are variances in the initial state $|\psi\rangle$. We decomposed $D_{jk}$ by inserting (15) into (10):

$$D^{(\alpha)}_{jk} \equiv \int_0^\infty \mathrm{d}\tau\nu(\tau)R^{(\alpha)}_{jk}(-\tau) = \begin{bmatrix} D_\alpha & f_\alpha\Omega_\alpha \\ -f_\alpha\Omega_\alpha & D_\alpha \end{bmatrix}_{jk}, \tag{22}$$

where $D_\alpha$ and $f_\alpha$ are generalized normal and anomalous diffusion coefficients, corresponding to $\Omega_\alpha$:

$$D_\alpha = \int_0^\infty \mathrm{d}\tau\nu(\tau)\cos\tau\Omega_\alpha, \qquad f_\alpha = \frac{-1}{\Omega_\alpha}\int_0^\infty \mathrm{d}\tau\nu(\tau)\sin\tau\Omega_\alpha. \tag{23}$$

Finally, we introduced $D_0 = \int \mathrm{d}\tau\nu(\tau)$, provided it exists (e.g., for Ohmic and super-Ohmic bosonic environments).

Similar analysis can be applied to the term linear in $R_{jk}(t)$ in (13). We first assume the odd group dimension $N$. From (15) and (16) the only nonzero term is $\overline{R_{NN}(t)} = R_{NN}(t) = 1$, so that

$$\sum_{jklmn} a_j a_l\gamma_{jk}f_{lkm}\overline{R_{mn}(t)}\langle X_n\rangle = -\sum_{jkl} a_j a_l\gamma_{jk}f_{Nkl}\langle X_N\rangle, \tag{24}$$

where we have used the total antisymmetry of $f_{ijk}$. We then decompose $\gamma_{jk}$ using (15):

$$\gamma = \sum_\alpha O^T\gamma_\alpha O, \qquad \gamma_\alpha = \begin{bmatrix} -\tilde{\Omega}^2_\alpha & -\gamma_\alpha\Omega_\alpha \\ \gamma_\alpha\Omega_\alpha & -\tilde{\Omega}^2_\alpha \end{bmatrix}, \tag{25}$$

where

$$\tilde{\Omega}^2_\alpha = -\int_0^\infty \mathrm{d}\tau\eta(\tau)\cos\tau\Omega_\alpha, \qquad \gamma_\alpha = \frac{1}{\Omega_\alpha}\int_0^\infty \mathrm{d}\tau\eta(\tau)\sin\tau\Omega_\alpha \tag{26}$$

are generalized frequency shift and the momentum damping coefficients. Similarly, (15) implies via differentiation of (8) a block-diagonal form of $\mathrm{ad}_{X_N}$:

$$\mathrm{ad}_{X_N} = \sum_{\alpha\le\alpha_N} O^T c_\alpha O, \qquad c_\alpha = \begin{bmatrix} 0 & -\mathfrak{i}\Omega_\alpha \\ \mathfrak{i}\Omega_\alpha & 0 \end{bmatrix}, \tag{27}$$

where the last block is zero due to (16). Recalling that $[\mathrm{ad}_{X_N}]_{jk} = \mathfrak{i}f_{Njk}$, we obtain [35]

$$\sum_{jkl} a_j a_l \gamma_{jk} f_{Nkl} = -\sum_{\alpha<\alpha_N} \|\tilde{\mathbf{a}}_\alpha\|^2 \Omega_\alpha^2 \gamma_\alpha. \tag{28}$$

For even dimension $N$, there is one more nonzero element in $\overline{R_{mn}(t)}$, corresponding to the $(N-1, N-1)$ element of the canonical form of $R(t)$: $[OR(t)O^T]_{N-1,N-1} = 1$; cf. Eq. (16). It leads to an additional term in Eq. (24):

$$\sum_{jklmn} a_j a_l \gamma_{jk} f_{lkm} O_{N-1,m} O_{N-1,n} \langle X_n \rangle = \mathbf{i} \sum_\alpha \langle \tilde{\mathbf{a}} | \boldsymbol{\gamma}_\alpha \cdot \widetilde{\mathrm{ad}}_{\tilde{X}_{N-1}} \tilde{\mathbf{a}} \rangle \langle \tilde{X}_{N-1} \rangle, \tag{29}$$

where $\langle | \rangle$ is a scalar product, $\mathrm{ad}_{\tilde{X}_m}$ is the ad operator of the transformed generator $\tilde{X}_m$, and $\widetilde{\mathrm{ad}}_{\tilde{X}_{N-1}} = O \mathrm{ad}_{\tilde{X}_{N-1}} O^T$ is the matrix of $\mathrm{ad}_{\tilde{X}_{N-1}}$ transformed to the new basis so that $[\tilde{X}_i, \tilde{X}_j] = \mathbf{i} \sum_k [\widetilde{\mathrm{ad}}_{\tilde{X}_i}]_{jk} \tilde{X}_k$.

*Main results.*—We obtain the following expressions for the asymptotic Born-Markov entropy production in the studied framework:

$$\frac{1}{2}\bar{s} \approx \sum_{\alpha\leq\alpha_N} \|\tilde{\mathbf{a}}_\alpha\|^2 D_\alpha \left[\Delta\tilde{X}_{\alpha 0}^2 + \Delta\tilde{X}_{\alpha 1}^2 + \frac{\Omega_\alpha^2 \gamma_\alpha}{D_\alpha} \langle X_N \rangle\right] \tag{30}$$

$$+ a_N^2 D_0 \Delta X_N^2, \tag{31}$$

for odd $N$, and

$$\frac{1}{2}\bar{s} \approx \sum_{\alpha\leq\alpha_N - 1} \|\tilde{\mathbf{a}}_\alpha\|^2 D_\alpha \left[\Delta\tilde{X}_{\alpha 0}^2 + \Delta\tilde{X}_{\alpha 1}^2 + \frac{\Omega_\alpha^2 \gamma_\alpha}{D_\alpha} \langle X_N \rangle\right]$$
$$+ \tilde{a}_{N-1}^2 D_0 \Delta\tilde{X}_{N-1}^2 + a_N^2 D_0 \Delta X_N^2 \tag{32}$$

$$+ \mathbf{i} \sum_\alpha \langle \tilde{\mathbf{a}} | \boldsymbol{\gamma}_\alpha \cdot \widetilde{\mathrm{ad}}_{\tilde{X}_{N-1}} \tilde{\mathbf{a}} \rangle \langle \tilde{X}_{N-1} \rangle, \tag{33}$$

for even $N$. The predictability sieve then defines the pointer states as

$$\mathrm{argmin}_\psi \bar{s}(\psi). \tag{34}$$

This is a complicated optimization problem. It can be somewhat simplified in the low-damping limit $\gamma_{jk}/D_\alpha \approx 0$, which holds, e.g., in high-temperature environments [4,5]. Dropping the nondynamical term $a_N X_N$ from (6) as it is usually the case, we obtain in this limit

$$\frac{1}{2}\bar{s} \approx \sum_{jk} g_{jk} \left[\langle \tilde{X}_j \tilde{X}_k \rangle - \langle \tilde{X}_j \rangle \langle \tilde{X}_k \rangle\right], \tag{35}$$

where we introduced the environment-dependent metric

$$g_{jk} = \bigoplus_{\alpha\leq\alpha_N} \|\tilde{\mathbf{a}}_\alpha\|^2 D_\alpha \mathbf{1}_2, \tag{36}$$

where $\mathbf{1}_2$ is the 2D unit matrix and for even $N$ the last nonzero block is equal to $\tilde{a}_{N-1}^2 D_0$. There is a similarity between (35) and the generalized coherent states defined via minimization of the $G$-invariant dispersion $\langle \Delta C \rangle = \sum h_{jk} [\langle \tilde{X}_j \tilde{X}_k \rangle - \langle \tilde{X}_j \rangle \langle \tilde{X}_k \rangle]$ [23]. However, the metric $g_{jk}$ is in general different from the Killing-Cartan form $h_{jk}$, which here is $\propto \mathbf{1}$. For example, in thermal models $g_{jk}$ can nontrivially depend on the temperature via the diffusion coefficients $D_\alpha$. We thus obtain (cf. [24]) the following.

*Corollary 1.*—Pointer states are in general different from coherent states for $G$ unless $\bar{s} \propto \langle \Delta C \rangle$.

*QBM example.*—It is interesting to revisit the seminal result of [17], which shows that for the QBM model the pointer states are the coherent states. The model is described by $H_0 = P^2/2M + M\Omega^2 Q^2/2$ and $A = Q$. The group $G$ is generated by the operators $\boldsymbol{X} = \{Q, P, \mathbf{1}, H_0\}$ and is known as the oscillator group [36]. It is noncompact [it is a projective representation of $Sp(2, \boldsymbol{R})$], but after the rescaling $q = \sqrt{M}\Omega Q$, $p = (1/\sqrt{M})P$, the matrix of the adjoint action (7) generated by $h_0 = (q^2 + p^2)/2$ becomes orthogonal (in the Lie algebra basis $\{q, p, \mathbf{1}, h_0\}$ and $[q, p] = \mathbf{i}\Omega$),

$$R(t) = \begin{bmatrix} \cos\Omega t & \sin\Omega t & & \\ -\sin\Omega t & \cos\Omega t & & \\ & & 1 & \\ & & & 1 \end{bmatrix}, \tag{37}$$

and is already in the canonical form (16). We can thus use our procedure and obtain the high-temperature entropy production equation (35):

$$\bar{s} \approx 2D[\Delta q^2 + \Delta p^2] = 2DM\Omega^2 \left[\Delta Q^2 + \frac{\Delta P^2}{M^2 \Omega^2}\right]. \tag{38}$$

Modulo an unimportant prefactor, the above equation is the same as obtained in [17] via a direct calculation; $D$ is given by (23) with $\Omega_\alpha = \Omega$. It now so happens that the rhs of Eq. (38) corresponds to an invariant dispersion for a subgroup $H$ of $G$, the Heisenberg-Weyl group generated by $\{Q, P, \mathbf{1}\}$. The minimization of (38) leads to the coherent states of $H$ [23], which are the celebrated Glauber-Sudarshan coherent states $|\alpha\rangle$ [20,21] and which are also coherent states for $G$ [23]. This situation is rather exceptional as, e.g., it is well known that higher order polynomials in $Q$, $P$ will not lead to any group structures, which in turn is connected to the known problems of the canonical quantization [37].

*Spin-J systems.*—As a further illustration, we consider a class of models where a central spin $J$ interacts with a thermal environment. As our general method is quite insensitive to the type of the environment, as long as the autocorrelation function is sufficiently regular, it will not matter below if the environment is bosonic,

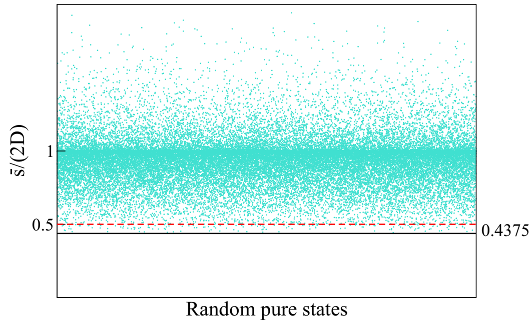


FIG. 1. Values of the rescaled entropy production $\bar{s}/2D$ for random pure states in the high-temperature limit for spin-1 system. Each dot corresponds to a single random pure state. The dashed horizontal line corresponds to the minimum value of $\bar{s}/2D$ for spin-coherent states. The black horizontal line indicates the true minimum value, 0.4375.

$$H = \Omega J_z + \sum_i \omega_i a_i^\dagger a_i - J_x \sum_i (g_i a_i^\dagger + g_i^* a_i), \tag{39}$$

or spin,

$$H = \Omega J_z + \sum_i \frac{\omega_i}{2} \sigma_z^{(i)} - J_x \sum_i g_i \sigma_x^{(i)}. \tag{40}$$

Above $J_i$ are the spin operators of the central system, $\sigma_k^{(i)}$ are the Pauli matrices for the $i$th spin, $a_i, a_i^\dagger$ are the annihilation and creation operators, respectively, of the environment, which is assumed to be thermal with the inverse temperature $\beta$. Both spin-spin and spin-boson models have become of significant importance recently due to their roles in such fields as, e.g., matter-wave interferometry [9,12], quantum dots [38], nitrogen-vacancy centers [39], and applied quantum information [40].

The models fall within our framework with $G = \mathrm{SU}(2)$, $H_0 = \Omega J_z$, $A = -J_x$. The matrix (7) is already in the canonical form and reads (after rescaling $X_N$):

$$R(t) = \begin{bmatrix} \cos \Omega t & -\sin \Omega t & \\ \sin \Omega t & \cos \Omega t & \\ & & 1 \end{bmatrix}. \tag{41}$$

From Eqs. (30) and (31), we immediately obtain the entropy production (with $a_x = -1$, the rest zero):

$$\bar{s} \approx 2D \left[ \Delta J_x^2 + \Delta J_y^2 + \frac{\gamma}{D} \langle J_z \rangle \right], \tag{42}$$

where $D$, $\gamma$ are from (23) and (26) and are given by the well-known expressions [5] with $\gamma/D = \tanh(\beta\Omega/2)$. Equation (42) is the same, up to an irrelevant positive prefactor, for both models, which can be seen, e.g., using the standard spin-oscillator mapping [5]. It is then easy to minimize (42) for spin-coherent states $|\boldsymbol{n}\rangle = e^{i\theta \boldsymbol{mJ}}|j, -j\rangle$, where $\boldsymbol{n}$ is a unit vector, $\boldsymbol{m} = (\sin\phi, -\cos\phi, 0)$, and $J_z|j, -j\rangle = -j|j, -j\rangle$. We obtain $\bar{s}/2D \approx j(1 - 1/2\sin^2\theta - \gamma/D \cos\theta)$, with the minimum at $\cos\theta = \gamma/D$ leading to

$$\frac{\bar{s}_{\min}}{2D} \approx \frac{j}{2} \left( 1 - \frac{\gamma^2}{D^2} \right). \tag{43}$$

For $\beta \to 0$, this implies $\bar{s}_{\min}/2D = j/2$, which is satisfied for spin-1/2 systems (e.g., two-level atoms), since then all pure states are coherent states by definition, but this is not so already for spin-1. The minimum for high temperature is achieved for the following $U(1)$ family of states [35], which are the true pointer states:

$$|\psi\rangle = \sqrt{\frac{5}{16}} (e^{i\psi}|1, 1\rangle + e^{-i\psi}|1, -1\rangle) + \sqrt{\frac{3}{8}} |1, 0\rangle, \tag{44}$$

and the minimum value is $\bar{s}_{\min}/2D = (7/16) \approx 0.4375 < 0.5$. Figure 1 is a numerical illustration of this fact. Although the overlap of (44) with spin-coherent states reaches $1/2 + \sqrt{15}/8 \approx 0.98$, the states $|\psi\rangle$ are superpositions of two orthogonal coherent states [35] showing their "nonclassicality".

*Concluding remarks.*—The use of the Born-Markov, rather than the GKLS, approach can be criticized due to the lack of complete positivity in some situations (see, however, Refs. [41,42]). This has been a matter of an ongoing debate, see, e.g., Refs. [43,44], and the references therein, despite the immense predictive power of the Born-Markov approach [5]. Let us state clearly that it is not the goal of our work to advocate for one approach or the other, but rather to explore how pointer states appear in the Born-Markov regime, which remains one of the most powerful approximation schemes in the open quantum systems theory.

Above, we studied compact, semisimple groups, but it is clear that our method is more universal. It is enough that the free evolution generates a group with some known canonical decomposition, like in the example of QBM and the noncompact oscillator group.

The pointer defining equations present a difficult optimization problem even in the low-damping regime and new mathematical tools will most probably be needed to tackle it. The physical characteristics of the states found here will require further studies (we know they are not minimum uncertainty states). So will the problem of how a general initial state evolves toward their mixture (cf. [22]) and if advanced decoherence forms such as quantum Darwinism and spectrum broadcast structures [45–49] can be defined around the pointer states found here; cf., e.g., [50] for an alternative classicization mechanism. We hope our work will stimulate further research into those and related topics and contribute to the debate on the dynamical emergence of classical properties, as well as have practical implications, e.g., for quantum state engineering.

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[1] H. D. Zeh, On the interpretation of measurement in quantum theory, Found. Phys. **1**, 69 (1970).
[2] E. Joos, H. D. Zeh, C. Kiefer, D. Giulini, J. Kupsch, and I.-O. Stamatescu, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin, 2003).
[3] W. H. Zurek, Decoherence, einselection, and the quantum origins of the classical, Rev. Mod. Phys. **75**, 715 (2003).
[4] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, New York, 2002).
[5] M. Schlosshauer, *Decoherence and the Quantum-to-Classical Transition* (Springer, Berlin, 2007).
[6] I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, Coherent quantum dynamics of a superconducting flux qubit, Science **21**, 1869 (2003).
[7] S. Deléglise, I. Dotsenko, C. Sayrin, J. Bernu, M. Brune, J.-M. Raimond, and S. Haroche, Reconstruction of non-classical cavity field states with snapshots of their decoherence, Nature (London) **455**, 510 (2008).
[8] H. Häffner, C. Roos, and R. Blatt, Quantum computing with trapped ions, Phys. Rep. **469**, 155 (2008).
[9] K. Hornberger, S. Gerlich, P. Haslinger, S. Nimmrichter, and M. Arndt, Colloquium: Quantum interference of clusters and molecules, Rev. Mod. Phys. **84**, 157 (2012).
[10] J. Moser, A. Eichler, J. Güttinger, M. I. Dykman, and A. Bachtold, Nanotube mechanical resonators with quality factors of up to 5 million, Nat. Nanotechnol. **9**, 1007 (2014).
[11] P. Tighineanu, C. L. Dreeßen, C. Flindt, P. Lodahl, and A. S. Sørensen, Phonon decoherence of quantum dots in photonic structures: Broadening of the zero-phonon line and the role of dimensionality, Phys. Rev. Lett. **120**, 257401 (2018).
[12] Y. Y. Fein, P. Geyer, P. Zwick, F. Kiałka, S. Pedalino, M. Mayor, S. Gerlich, and M. Arndt, Quantum superposition of molecules beyond 25 kDa, Nat. Phys. **15**, 1242 (2019).
[13] W. H. Zurek, Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?, Phys. Rev. D **24**, 1516 (1981).
[14] W. H. Zurek, Environment-induced superselection rules, Phys. Rev. D **26**, 1862 (1982).
[15] W. H. Zurek, Preferred states, predictability, classicality and the environment-induced decoherence, Prog. Theor. Phys. **89**, 281 (1993).
[16] L. Diósi and C. Kiefer, Robustness and diffusion of pointer states, Phys. Rev. Lett. **85**, 3552 (2000).
[17] W. H. Zurek, S. Habib, and J. P. Paz, Coherent states via decoherence, Phys. Rev. Lett. **70**, 1187 (1993).
[18] R. Feynman and F. L. Vernon, The theory of a general quantum system interacting with a linear dissipative system, Ann. Phys. (N.Y.) **24**, 118 (1963).
[19] P. Ullersma, An exactly solvable model for Brownian motion: I. Derivation of the Langevin equation, Physica **32**, 27 (1966).
[20] R. J. Glauber, Coherent and incoherent states of the radiation field, Phys. Rev. **131**, 2766 (1963).
[21] E. C. G. Sudarshan, Equivalence of semiclassical and quantum mechanical descriptions of statistical light beams, Phys. Rev. Lett. **10**, 277 (1963).
[22] J. Eisert, Exact decoherence to pointer states in free open quantum systems is universal, Phys. Rev. Lett. **92**, 210401 (2004).
[23] A. Perelomov, *Generalized Coherent States and Their Applications* (Springer, Berlin, 1986).
[24] S. Boixo, L. Viola, and G. Ortiz, Generalized coherent states as preferred states of open quantum systems, Europhys. Lett. **79**, 40003 (2007).
[25] A. Kossakowski, On quantum statistical mechanics of non-Hamiltonian systems, Rep. Math. Phys. **3**, 247 (1972).
[26] E. B. Davies, Markovian master equations, Commun. Math. Phys. **39**, 91 (1974).
[27] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Completely positive dynamical semigroups of $N$-level systems, J. Math. Phys. (N.Y.) **17**, 821 (1976).
[28] G. Lindblad, On the generators of quantum dynamical semigroups, Commun. Math. Phys. **48**, 119 (1976).
[29] G.-S. Paraoanu and H. Scutaru, Classical states via decoherence, Phys. Lett. A **238**, 219 (1998).
[30] K. Khodjasteh, V. V. Dobrovitski, and L. Viola, Pointer states via engineered dissipation, Phys. Rev. A **84**, 022336 (2011).
[31] H. C. Donker, H. De Raedt, and M. I. Katsnelson, Decoherence and pointer states in small antiferromagnets: A benchmark test, SciPost Phys. **2**, 010 (2017).
[32] A. G. Redfield, On the theory of relaxation processes, IBM J. Res. Dev. **1**, 19 (1957).
[33] F. Bloch, Generalized theory of relaxation, Phys. Rev. **105**, 1206 (1957).
[34] A. Kirillov, Jr., *An Introduction to Lie Groups and Lie Algebras*, in Cambridge Studies in Advanced Mathematics (Cambridge University Press, Cambridge, England, 2008).
[35] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.132.030203 for some definitions and proofs appearing in the main text.
[36] R. F. Streater, The representations of the oscillator group, Commun. Math. Phys. **4**, 217 (1967).
[37] H. Groenewold, On the principles of elementary quantum mechanics, Physica (Amsterdam) **12**, 405 (1946).
[38] B. Urbaszek, X. Marie, T. Amand, O. Krebs, P. Voisin, P. Malentinsky, A. Högele, and A. Imamoglu, Nuclear spin physics in quantum dots: An optical investigation, Rev. Mod. Phys. **85**, 79 (2013).
[39] M. W. Doherty, N. B. Manson, P. Delaney, F. Jelezko, J. Wrachtrup, and L. C. Hollenberg, The nitrogen-vacancy colour centre in diamond, Phys. Rep. **528**, 1 (2013).
[40] J. A. Bergou, M. Hillery, and M. Saffman, *Quantum Information Processing: Theory and Implementation* (Springer International Publishing, Cham, 2021).

[41] J. Jeske, D. J. Ing, M. B. Plenio, S. F. Huelga, and J. H. Cole, Bloch-Redfield equations for modeling light-harvesting complexes, J. Chem. Phys. **142**, 064104 (2015).

[42] G. McCauley, B. Cruikshank, D. I. Bondar, and K. Jacobs, Accurate Lindblad-form master equation for weakly damped quantum systems across all regimes, npj Quantum Inf. **6**, 74 (2020).

[43] M. Winczewski, A. Mandarino, M. Horodecki, and R. Alicki, Bypassing the intermediate times dilemma for open quantum system, arXiv:2106.05776.

[44] D. Tupkary, A. Dhar, M. Kulkarni, and A. Purkayastha, Fundamental limitations in Lindblad descriptions of systems weakly coupled to baths, Phys. Rev. A **105**, 032208 (2022).

[45] W. H. Zurek, Quantum Darwinism, Nat. Phys. **5**, 181 (2009).

[46] J. K. Korbicz, P. Horodecki, and R. Horodecki, Objectivity in a noisy photonic environment through quantum state information broadcasting, Phys. Rev. Lett. **112**, 120402 (2014).

[47] T. P. Le and A. Olaya-Castro, Strong quantum Darwinism and strong independence are equivalent to spectrum broadcast structure, Phys. Rev. Lett. **122**, 010403 (2019).

[48] T. K. Unden, D. Louzon, M. Zwolak, W. H. Zurek, and F. Jelezko, Revealing the emergence of classicality using nitrogen-vacancy centers, Phys. Rev. Lett. **123**, 140402 (2019).

[49] J. K. Korbicz, Roads to objectivity: Quantum Darwinism, spectrum broadcast structures, and strong quantum Darwinism—A review, Quantum **5**, 571 (2021).

[50] T. Bhattacharya, S. Habib, and K. Jacobs, Continuous quantum measurement and the emergence of classical chaos, Phys. Rev. Lett. **85**, 4852 (2000).