Mass-Independent Scheme to Test the Quantumness of a Massive Object

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(Received 18 April 2023; accepted 30 November 2023; published 16 January 2024)

The search for empirical schemes to evidence the nonclassicality of large masses is a central quest of current research. However, practical schemes to witness the irreducible quantumness of an arbitrarily large mass are still lacking. To this end, we incorporate crucial modifications to the standard tools for probing the quantum violation of the pivotal classical notion of macrorealism (MR): while usual tests use the same measurement arrangement at successive times, here we use two different measurement arrangements. This yields a striking result: a *mass-independent* violation of MR is possible for harmonic oscillator systems. In fact, our adaptation enables probing quantum violations for literally any mass, momentum, and frequency. Moreover, coarse-grained position measurements at an accuracy much worse than the standard quantum limit, as well as knowing the relevant parameters only to this precision, without requiring them to be tuned, suffice for our proposal. These should drastically simplify the experimental effort in testing the nonclassicality of massive objects ranging from atomic ions to macroscopic mirrors in LIGO.

DOI: 10.1103/PhysRevLett.132.030202

Introduction and motivation.—A cutting-edge research enterprise in contemporary physics is to explore realizable schemes for checking the validity of the quantum mechanics (QM) in the macroscopic regime, together with demonstrating its incompatibility with the world view based on the classical notion of macrorealism (MR) [1]. The goal is to expand as much possible the domain of such testability. This also has potentiality in providing useful empirical constraints on suggested modifications of quantum dynamical evolution in the macroscopic limit (such as the models of spontaneous wave function collapse [2–4]). Nonclassical massive matter states are also a resource for nonclassical gravity [5–10]. Testing nonclassicality via MR can be, in principle, much easier than creating highly nonclassical states. However, in practice it imposes very high demands on the initial control of parameters, as well as precise measurements, with its scaling becoming prohibitively difficult for large masses [11,12]. Here we show that appropriately modifying the schemes for testing MR in the context of massive objects provides a threefold advantage: (i) We can obtain a *mass-independent* violation of MR, so that the applicability domain becomes essentially unlimited. (ii) It does not require any tuning of other parameters—e.g., frequency, momentum—either. (iii) It becomes highly robust to measurement imprecision (no need to surpass the standard quantum limit, for example), opening up the scope for practical realizations.

The key tools for probing MR are “temporal correlators” from which one constructs the Leggett-Garg inequality (LGI) [13] and the no-signaling-in-time (NSIT) conditions [14]. Such relations are derived from a conjunction of the following assumptions, as formulated by Leggett and Garg [13,15] for characterizing the notion of MR: (i) At any instant, even if unobserved, a system is definitely in one of its possible states with all its observable properties having definite values (realism *per se*). (ii) It is possible to determine which of the states the system is in by ensuring that the measurement-induced disturbance is arbitrarily small, and thus not affecting the subsequent time evolution of the measured state of the system (noninvasive measurability). (iii) The outcome of a measurement is not affected by what will be measured subsequently (induction). Since both the LGI and the NSIT relations are consequences of MR, an experimental refutation of either of them, in accordance with the quantum mechanical predictions, would constitute a decisive evidence of macroscopic nonclassicality, together with certifying the validity of the QM principle of superposition of states. While the logical connection between the LGI and NSIT has been analyzed in various ways [16–18], for the purpose of the present work it suffices to regard violation of either of them as a sufficient condition for evidencing nonclassicality or quantumness. From the operational point of view, using the NSIT relations is, in general, more advantageous compared to LGI because of the lower

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number of required outcome probabilities. Furthermore, since the NSIT condition is violated by the presence of any nonvanishing quantum interference term [14], it is usually violated for a much wider parameter regime than the LGI.

With growing interest in this fundamentally significant topic, particularly over the past two decades, a number of experimental studies seeking to test MR in the macroscopic regime have been reported (for a useful review of the earlier experiments, see Ref. [19]). For characterizing "macroscopicity," these studies have used different parameters, ranging from the length scale of neutrino oscillation [20] to the difference of the magnetic moments corresponding to the two superposing superconducting-current states [16] and the spatial separation between the two superposing single-photon states (corresponding to the two arms of an interferometer) [21]. However, while "mass" seems to be a quite relevant parameter for characterizing "macroscopicity," no experiment testing MR has yet been performed based on systems having significantly large mass—a few earlier LGI-based experiments employing atomic systems have been confined to using, for example, a single cesium atom [22] and spin-bearing phosphorus impurities in silicon [23]. On the other hand, the tests of quantumness *per se* of macromolecules (without seeking to test MR) have so far reached only up to masses of about 10^4 amu [24,25]. Our present work is motivated toward filling this important gap in the relevant literature by formulating a suitable scheme that can enable scaling up the test of MR *vis-à-vis* QM to arbitrarily large masses of harmonically oscillating objects. To this end, we introduce a hitherto unexplored suitable variation of the LGI and the NSIT relations such that a *mass-independent* QM violation of MR is possible. In fact, for literally *any* choice of parameters—mass, initial momentum, frequency—our procedure can certify macroscopic quantumness and show violation of MR. It is an added advantage that the scheme, by working for highly imprecise measurements, provides a great facilitation of practical realization (in comparison to existing literature on large masses [11,12]).

The basic ideas of our scheme.—Let us begin by noting that the various versions of macrorealist inequalities/conditions that have been applied in different contexts usually consider the same observable to be successively measured on a single particle evolving in time. In contrast here, for the example considered, the successive measurements are invoked in such a way that they pertain to different observables. Let us now explain how this is realized. For a one-dimensional system harmonically oscillating between $x = -\infty$ and $x = \infty$, dividing this domain of oscillation into two subdomains, ranging from $x = -\infty$ to $x = \beta_i$ and from $x = \beta_i$ to $x = \infty$, where β_i is any real number, we consider coarse-grained spatial measurement at an instant $t = t_i$ that determines which one of these two regions the oscillating system is in at the given instant. A key element of our scheme is that the location $x = \beta_i$ of the boundary between the two regions for the type of measurement considered is chosen according to the instant $t = t_i$ at which the measurement is made. For convenience, considering that the initial coherent-state Gaussian wave function at $t = 0$ is peaked at $x = 0$, we choose the instant of the first measurement to be the initial instant, i.e., $t_1 = 0$, and we determine the boundary for this measurement to be located at $x = \beta_1 = 0$.

Next, for the subsequent measurement at the instant $t = t_2$, the appropriate choice of the location $x = \beta_2$ of the boundary between the two measurement regions is critical in order to achieve the desired mass independence of the quantum violation of MR. Toward attaining this goal, our analysis reveals that by suitably fixing β_2 for given values of t_2, it is possible to ensure the magnitudes of the quantum violations of both the two-time LGI [26] and the two-time NSIT relation [14,27] to be independent of mass, as well as of the other relevant experimental parameters, such as the initial peak momentum and the angular frequency. Here it needs to be pointed out that if the observable quantity is taken to be such that at any instant, it takes a value $+1$ (-1) depending on whether the system is in one region or in the other, it is then evident that in this example, such an observable quantity changes according to the location of the boundary demarcating the two measurement regions. Hence, for the purpose of the following analysis, the two-time LGI and the two-time NSIT relation invoked are, crucially, in terms of different observable quantities being measured at two different instants. Here, note that the derivations of LGI and NSIT relations from MR do not depend upon the measured quantity necessarily being the same at the different instants of successive measurements.

An important point to note here is that the measurement envisaged in our example can be designed such that an outcome is inferred when the detector is not triggered (negative result measurement). This ensures that there is no classical interaction with the measured object during measurement, thereby satisfying the assumption of noninvasive measurability, if the measured object is classical [15]. Such a measurement can be implemented in our setup by using, say, a probe beam illuminating one of the two regions (either from $x = \beta_i$ to $x = \infty$, or from $x = \beta_i$ to $x = -\infty$). If no scattered light is observed, an outcome is registered by inferring the presence of the oscillating object within the unilluminated region [11]. Other relevant specifics will be discussed with respect to the analysis of our scheme presented as follows.

The analysis.—We begin by explicitly writing the modified forms of the two-time LGI and the two-time NSIT relation involving sequential measurements of the observables denoted by Q and R at two different instants, $t = t_1$ and $t = t_2$, respectively, where $t_1 < t_2$. The NSIT condition implies that the probability of obtaining a particular outcome for the measurement of R at $t = t_2$ should be independent of whether any prior measurement has been carried out. That is, it can be regarded as the statistical version of noninvasive measurability. In the present context, the two-time NSIT condition can be expressed as

$$\mathrm{N}_{\pm} = p(R\pm) - [p(Q+, R\pm) + p(Q-, R\pm)] = 0, \tag{1}$$

where $p(Q\pm, R\pm)$ is the joint probability of getting the outcomes ± 1 at instant $t = t_1$ and ± 1 at instant $t = t_2$. $p(R\pm)$ is the probability of getting the outcome ± 1 at instant $t = t_2$, when no measurement is done at $t = t_1$. The magnitude of the quantum violation of the two-time NSIT condition will be denoted by the nonzero value of $|\mathrm{N}_{\pm}|$.

In this scenario, the two-time LGI can be expressed as [28]

$$\mathrm{L}_{s_1,s_2} = 1 + s_1\langle Q\rangle + s_2\langle R\rangle + s_1 s_2 \langle QR\rangle \geq 0, \quad \text{with} \quad s_1, s_2 \in \{+1, -1\}, \tag{2}$$

where the correlation function is $\langle QR\rangle = p(Q+, R+) - p(Q+, R-) - p(Q-, R+) + p(Q-, R-)$, and the expectation values are $\langle Q\rangle = p(Q+) - p(Q-)$, $\langle R\rangle = p(R+) - p(R-)$. Here $\langle R\rangle$ is defined when no measurement at $t = t_1$ is performed. In this case, magnitude of quantum violation of the two-time LGI will be denoted by the positive value of $\max_{s_1=\pm 1, s_2=\pm 1}(-\mathrm{L}_{s_1,s_2})$.

Consider the following initial Gaussian wave function of the coherent state peaked at $x = 0$ at instant $t = 0$:

$$\psi(x, t = 0) = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_0}} \exp\left(-\frac{x^2}{4\sigma_0^2} + \frac{\mathbb{i} p_0 x}{\hbar}\right), \tag{3}$$

with the initial momentum expectation value p_0 and the width $\sigma_0 = \sqrt{\hbar/(2m\omega)}$, where ω is the angular frequency of oscillation and m is the mass. The time evolution of this state is evaluated in this example using linear harmonic oscillator potential.

We consider measurements of Q and R at the instants $t = t_1$ and $t = t_2$, respectively, where Q and R correspond to the earlier-mentioned coarse-grained measurements. To put it specifically, Q is an observable quantity such that it takes a value $+1$ (-1) depending on whether the system is in the region from $x = \beta_1$ to $x = \infty$ (from $x = -\infty$ to $x = \beta_1$). Similarly, R is another observable quantity such that it takes a value $+1$ (-1) if the particle is in the region from $x = \beta_2$ to $x = \infty$ (from $x = -\infty$ to $x = \beta_2$). Such a coarse-grained position measurement at instant $t = t_i$ $(i = 1,\ 2)$ can be represented by the operator $\hat{O}_i = \int_{\beta_i}^{\infty} |x\rangle\langle x| dx - \int_{-\infty}^{\beta_i} |x\rangle\langle x| dx$, which has two eigenvalues: $+1$ and -1.

As mentioned earlier, $\beta_1 = 0$. With this choice of β_1, it can be shown that the expressions for the joint probabilities are the following functions of β_2 and other relevant parameters:

$$p(Q\pm, R-) = \frac{1}{4\sqrt{\pi}} \int_{-\infty}^{\gamma} dx \, \exp[-x^2] f(x, \omega t_2),$$

$$p(Q\pm, R+) = \frac{1}{4\sqrt{\pi}} \int_{\gamma}^{\infty} dx \, \exp[-x^2] f(x, \omega t_2), \tag{4}$$

$$\text{with} \quad f(x, \omega t_2) = \left|1 \pm \mathrm{erf}\left[\frac{-\mathbb{i} x}{\sin(\omega t_2)\sqrt{2 - 2\mathbb{i}\cot(\omega t_2)}}\right]\right|^2, \tag{5}$$

$$\text{and} \quad \gamma = \beta_2 \sqrt{\frac{m\omega}{\hbar}} - \frac{p_0 \sin(\omega t_2)}{\sqrt{\hbar m \omega}}, \tag{6}$$

where the error function $\mathrm{erf}(z) = (2/\sqrt{\pi}) \int_0^z dt \, \exp(-t^2)$. Similarly, we have the following form of probabilities:

$$p(Q\pm) = \frac{1}{2}, \qquad p(R\pm) = \frac{1}{2}[1 \mp \mathrm{erf}(\gamma)]. \tag{7}$$

The details of these calculations are given in the Supplemental Material [28].

Next, we have to choose β_2 suitably for the measurement at $t = t_2$. Our heuristic arguments based on physical ground [28] suggest that the location of the peak at the instant $t = t_2$, given by $x_0^{(t_2)} = p_0 \sin(\omega t_2)/(m\omega)$, together with the standard deviation $\Delta^{(t_2)} = \sqrt{\hbar/(2m\omega)}$ of the probability density without any prior measurement at $t = t_1$, would play a critical role in fixing β_2 suitably for our purpose. Guided by this consideration, our in-depth investigation reveals that if the choice of β_2 is made at $x_0^{(t_2)} \pm c\Delta^{(t_2)}$ with c being positive and of the order of 10^{-1} or 1, the possibility indeed arises for obtaining quantum violations of both the two-time NSIT condition and the two-time LGI. Here, a key point is that this choice of $\beta_2 = x_0^{(t_2)} \pm c\Delta^{(t_2)}$ leads to γ given by Eq. (6) becoming independent of m, ω, and p_0—i.e., γ is then determined only by the chosen value of c, whence $\gamma = \pm c/\sqrt{2}$. Consequently, the probability distributions (4) and (7) become functions of (ωt_2) only. This, therefore, enables the quantum violations of the two-time NSIT condition and the two-time LGI to become independent of mass. That this is indeed the case is confirmed comprehensively by evaluating numerically the integrations appearing in (4). In Table I, some illustrative results are presented by choosing, for example, $c = \sqrt{2}$. To sum up, the upshot of this entire study is that it is always, in principle, possible to choose t_2 suitably depending on the time period of the oscillating particle to obtain quantum violation of the two-time NSIT condition or the two-time LGI for any m, p_0, and ω.

Practical challenges with large mass and measurement precision.—Ideally, in our scheme, one of the two regions (either from $x = -\infty$ to $x = \beta_i$, or from $x = \beta_i$ to $x = \infty$)

TABLE I. Quantum violations of the two-time NSIT condition and the two-time LGI when the boundary between the two regions in the case of the second measurement is chosen to be located at $x = \beta_2 = p_0 \sin(\omega t_2)/(m\omega) + \sqrt{\hbar/(m\omega)}$. Here, $T = 2\pi/\omega$ denotes the time period.

				Magnitude of quantum violation of two-time NSIT:	Magnitude of quantum violation of two-time LGI:
m	p_0	ω	t_2	$\lvert\mathrm{N}_\pm\rvert$	$\max_{s_1=\pm1,s_2=\pm1}(-\mathrm{L}_{s_1,s_2})$
Any	Any	Any	$T/14$	0.12	0.08
Any	Any	Any	$T/8$	0.15	0.04
Any	Any	Any	$T/4$	0.17	No violation
Any	Any	Any	$T/3$	0.16	No violation
Any	Any	Any	$3T/8$	0.15	0.04
Any	Any	Any	$2T/5$	0.14	0.07
Any	Any	Any	$3T/4$	0.17	No violation

should be illuminated at an instant $t = t_i$. In practice, however, it is almost impossible to keep the boundary between the two regions (illuminated and unilluminated) fixed in all experimental runs. Rather, we can expect that the aforementioned boundary at $t = t_i$ will be at $x = \beta_i + \epsilon_i$ (with ϵ_i being a small positive/negative number depending on the accuracy of the experimental setup), where ϵ_i will be different in different runs. In effect, the observed violation of the NSIT condition will be the statistical average over different values of N_+, N_- corresponding to different values of ϵ_1 and ϵ_2. Similar will be the case for the LGI. It can be shown that the permissible ranges of ϵ_1 and ϵ_2 to get significant violation of the NSIT or the LGI are proportional to $1/\sqrt{m\omega}$ [28]. Hence, the required precision in fixing the boundary between the two measurement regions at any instant is increased with increasing mass. Nonetheless, the effect of increasing mass in this context can be offset by decreasing the angular frequency ω. Here, it is relevant to note that the lowest angular frequency of a harmonic well achieved to date is $\omega \sim 100$ kHz in the case of an optical trap [29], $\omega \sim 100$ Hz in an ion trap [30,31], and 1–10 Hz in a diamagnetic trap [32–34]. For small ω, the violations as mentioned in Table I are observed for large t_2. For any ω, damping has to be controlled so that the decoherence rate due to all force noises can be given by $\gamma = S_{FF}(\omega)(\Delta x)^2/\hbar^2 \ll 1/t_2$, where $S_{FF}(\omega)$ is the noise power density of force and Δx is the width of the wave function at t_2. Since both *with and without* measurement at t_1, $\Delta x \sim \sigma_0 = \sqrt{\hbar/(2m\omega)}$ (see Figs. 1 and 2 in [28]), the above condition reduces to $\sqrt{S_{FF}(\omega)} \ll \omega\sqrt{\hbar m}/\sqrt{\pi}$ for $t_2 \sim T$ (for example, with $m \sim 10$ kg and $\omega \sim 100$ Hz [35], $\sqrt{S_{FF}} \sim 10^{-15}$ N/$\sqrt{\mathrm{Hz}}$ is required). Interestingly, large masses indeed help here, for the obvious reason that force noise induces less random acceleration. The above consideration of decoherence is quite generic: all recoil noises, as well as all trapping noises, can be encompassed under the above limit on force noise.

It is well known that the balance between unwanted measurement backaction and the precision of optical measurements imposes a standard quantum limit (SQL) on the position measurement of a harmonically trapped object [36]. For the system considered by us, this limit is given by $\delta x = \sqrt{\hbar/(2m\omega)}$, the minimum uncertainty in position measurement [36,37]. Recently, it has been shown that one can surpass SQL, but only with highly delicate technologies [38,39]. In our proposed setup, probing the quantum violation of MR for any m, ω, and p_0 using the NSIT condition is possible even when the accuracies of the coarse-grained position measurements are much worse than in the SQL [28]. However, this is not the case for the LGI. Hence, the NSIT condition should be preferred for implementing our proposal.

Possible experimental implementations.—We can envisage implementations with nano- and micro-objects (typically up to $\sim 10^{-14}$ kg) based on so-called levitated mechanics [40] in various low-noise traps such as optical dipole traps, ion traps, and magnetic and diamagnetic traps in vacuum and at low temperature, as well as using much larger masses (e.g., ∼mg [41,42] and ∼10 kg in the gravitational wave detectors [35]). The mass independency of this MR test can be judiciously made use of in choosing the experimental setups optimized to reduce relevant decoherence effects and noises, as well as for addressing at the same time the need for high spatial detection resolution of the center-of-mass motion of the trapped particle. Specifically, one can observe the quantum violations of MR as described in Table I for any given mass dependent only on t_2 even if the values of ω and p_0 are different in different experimental runs—i.e., ω and p_0 need not be tuned in each run. The only requirement is that the values of ω and p_0 in each run need to be known in order to fix β_2. In particular, for testing the NSIT condition, it is sufficient to know these parameters to the precision of the order of SQL. On the contrary, for LGI, these parameters should be known with much more precision, implying less difficulty with testing the NSIT condition.

The preparation of the initial state will be accomplished by cooling—for instance, by feedback [43]—to a motional state of low occupation number, for which the only requirement is that the rate of acquiring information about the object must be much faster than the rate of its heating from environmental noise [44–46]. This technique has already been used for cooling to the ground state for $\omega \sim 100$ kHz traps [47,48], as well as for a large-mass and low-frequency ($\omega \sim 100$ Hz) gravitational wave detector [35]. The conditions are well within ultrahigh vacuum at 10^{-10} mbar and can be fulfilled in low-temperature environments even below 10 mK, and with vibration isolation.

While keeping the trapping a low-noise mechanism (e.g., an ion trap, magnetic, or diamagnetic trap, etc.), a promising possibility for the realization of measurements in this experiment is optical detection. If we are using the detection of scattered light from the particle to detect its motion, for an assumed near-unity efficiency of collection of the scattered light, and for n photons, we obtain a resolution of $\lambda/\sqrt{n}$ with $\lambda \sim \mu\mathrm{m}$. This implies the collection of $n \sim 10^8$ photons for reaching a spatial resolution of the ground-state spread σ_0 of a $m \sim 10^{-14}$ kg particle in a $\omega \sim 1$ Hz trap. If the detected scattered power is 1 nW, this information is acquired in $\sim 10^{-2}$ s, implying that all other heating rates, such as undetected scattered photons from interactions with blackbody photons and gas collisions, have to be at $\Gamma \leq 10^2$ Hz.

For increasing the critical detection efficiency in light-scattering techniques, one could use collection optics with a high numerical aperture—for example, parabolic mirrors [49]. The high spatial resolution could be achieved by illuminating only either to the left or right of $x = \beta_i$ at instant $t = t_i$ with a sharp drop of profile at the point $x = \beta_i$. If no scattered light is obtained after 0.01 s with a nW laser illuminating the left half, it immediately implies a $+1$ outcome, with the location $x = \beta_i$ being at angstrom resolution. Such spatial resolution has been achieved in the optical imaging of single molecules [50] and in optomechanical experiments [46,51] by using optical interferometry.

Finally, we note that most of the magnitudes of violations of NSIT or LGI are $\sim 10^{-1}$. Since n number of runs can determine outcome probabilities with uncertainty $\sim 1/\sqrt{n}$, we require 10^4 experimental runs to ensure that the statistical error is 1 order of magnitude less than the mean value of the violations.

Conclusions.—We have suitably modified the procedure for testing LGI and NSIT in order to show the violation of the classical notion of MR in a manner which is *independent* of the parameters: mass, momentum, and frequency. Moreover, this modification offers a quantum jump in simplifying experimental efforts in terms of measurement precision (even a more coarse-grained measurement than the SQL is sufficient) and tuning of the parameters. Naturally, this enormously broadens the scope for evidencing nonclassicality for large masses. No nonclassical state, such as a quantum superposition of distinct states (e.g., a Schrödinger cat state) or even a squeezed state, needs to be prepared *a priori*. Moreover, this approach does not require coupling with any ancillary quantum system or using nonlinearity. Rather, the starting point of our scheme is the most "classical-like" of all quantum states—namely, the coherent state, which has been prepared by feedback cooling in several systems [43], including 10 kg LIGO masses [35], and is imminent in several other systems. In fact, this can be regarded as a *scale-invariant test of nonclassicality*: the experimental data curves of MR violations with t_2 at the same fractions of T with widely different masses can be made to coincide with each other by adjusting p_0 and ω.

Note added.—A paper has appeared in parallel [52] reporting QM violations of the different forms of LGI for coherent states of a harmonic oscillator, focusing on maximizing these violations. In particular, the observable measured at different instants is taken to be the same while testing LGI. Consequently, mass-independent quantum violation has not been achieved in that paper.

We acknowledge fruitful discussions with Jonathan Halliwell and Clement Mawby. D. D. acknowledges the Royal Society (United Kingdom) for support through the Newton International Fellowship (No. NIF\R1\212007). D. H. acknowledges support from the NASI Senior Scientist Fellowship and QuEST-DST Project No. Q-98 of the Government of India. H. U. would like to acknowledge support from EPSRC through Grants No. EP/W007444/1, No. EP/V035975/1, and No. EP/V000624/1; the Leverhulme Trust (No. RPG-2022-57); the EU Horizon 2020 FET-Open Project TeQ (No. 766900); and the EU EIC Pathfinder Project QuCoM (No. 10032223). S. B. would like to acknowledge EPSRC Grants No. EP/N031105/1 and No. EP/S000267/1, and STFC Grant No. ST/W006227/1. S. B. and H. U. would like to acknowledge EPSRC Grant No. EP/X009467/1.

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