Measurement-Induced Quantum Synchronization and Multiplexing

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Measurements are able to fundamentally affect quantum dynamics. We here show that a continuously measured quantum many-body system can undergo a spontaneous transition from asynchronous stochastic dynamics to noise-free stable synchronization at the level of single trajectories. We formulate general criteria for this quantum phenomenon to occur and demonstrate that the number of synchronized realizations can be controlled from none to all. We additionally find that ergodicity is typically broken, since time and ensemble averages may exhibit radically different synchronization behavior. We further introduce a quantum type of multiplexing that involves individual trajectories with distinct synchronization frequencies. Measurement-induced synchronization appears as a genuine nonclassical form of synchrony that exploits quantum superpositions.

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Synchronization is a universal concept in science and technology. Synchronous motion typically arises in coupled nonlinear oscillators when they collectively adjust their individual frequencies [1–8]. Such phase-locking processes are omnipresent in physical, chemical, biological, and engineering systems. Synchronization occurs in classical [1–8] as well as in quantum [9–23] systems. In both cases, three general synchronization mechanisms are commonly distinguished: Synchronized behavior may (i) spontaneously appear in interacting systems owing to their mutual coupling, (ii) be forced by an external periodic drive, or (iii) be induced by noise [1–23]. However, classical and quantum theories are fundamentally different. An essential question is, therefore, to identify purely nonclassical forms of synchrony, determine their distinct quantum properties, and explore their potential applications.

An intrinsic feature of quantum mechanics is measurement backaction that randomly perturbs the state of a measured system [24–26]. As a consequence, observing a quantum object may dramatically affect its dynamics in a nonclassical manner. A case in point is the quantum Zeno effect, where frequent measurements slow down time evolution [27]. While detection backaction is often detrimental, limiting measurement accuracy and causing decoherence [24–26], it has recently been realized that it may also be used to control complex many-body systems. Quantum measurements may indeed trigger phase transitions, such as entanglement phase transitions [28–38] and topological phase transitions [39–41], when the measurement rate, or strength, is varied. These measurement-induced phase transitions originate from the nontrivial interplay between unitary dynamics and the monitoring action of a detector, underscoring the potential constructive role of quantum measurements [28–41].

We here demonstrate measurement-induced synchronization in a continuously monitored quantum system. We concretely show that an otherwise closed many-body system may undergo a spontaneous transition from stochastic asynchronous dynamics to noise-free stable (anti)synchronization at the level of individual trajectories, when subjected to standard homodyne detection [24–26]. We formulate general criteria for measurement-induced synchronization in generic quantum systems based on the existence of decoherence-free subspaces [42–46]. Decoherence-free subspaces are special parts of a system's Hilbert space that play an important role in quantum information science, since information encoded in them is protected from the environment [42–46]. Whereas (anti)synchronization appears along all trajectories in classical systems, we show that the number of synchronized quantum trajectories is controlled by the overlap between the initial state and the decoherence-free subspaces. We reveal that synchronization may appear at the trajectory level while being absent at the ensemble level—and vice versa. In general, knowledge of the ensemble average is, hence, not sufficient to provide information about the synchronized behavior of single realizations. We characterize this breaking of ergodicity by evaluating the fidelity between ensemble and time-averaged states [47]. We further introduce a quantum form of multiplexing [48], where individual synchronized trajectories at multiple frequencies are possible by engineering coherent superpositions of decoherence-free subspaces with distinct frequencies. Linear superposition, an essential resource of quantum theory [49], thus appears as a useful feature for quantum synchronization. Finally, we illustrate our results by analyzing measurement-induced synchronization in a quantum spin chain [50].

Measurement-induced synchronization.—We consider a closed quantum system with Hamiltonian H that is continuously monitored using a standard homodyne detection scheme [24–26]. Its stochastic density operator ρ_W evolves according to the Itô stochastic master equation

$$\mathrm{d}\rho_W = -i[H,\rho_W]\mathrm{d}t + \left(L\rho_W L^\dagger - \frac{1}{2}\{L^\dagger L,\rho_W\}\right)\mathrm{d}t + (L\rho_W + \rho_W L^\dagger - \langle L + L^\dagger\rangle\rho_W)\mathrm{d}W(t), \tag{1}$$

where L denotes the measurement operator and $\mathrm{d}W$ is a Wiener noise increment satisfying $\mathrm{d}W(t)^2 = \mathrm{d}t$ [24–26]. We focus on a single measurement operator, but the analysis may easily be extended to an arbitrary number of them. The density operator ρ_W describes a particular realization of the quantum process. Taking the ensemble average over all trajectories, Eq. (1) reduces to the usual Lindblad master equation $\dot{\rho} = -i[H,\rho] + \mathcal{D}[L]\rho$, where $\rho = \mathbb{E}[\rho_W]$ is the averaged density operator and $\mathcal{D}[L]\bullet = L\bullet L^\dagger - \{L^\dagger L,\bullet\}/2$ denotes the dissipator [24–26]. We note that quantum synchronization has been mostly investigated at the ensemble level so far [9–23]. The fluctuating statistics of synchronization have been examined for coupled mechanical oscillators (including nonlinear Van der Pol oscillators) by unraveling their dynamics in Refs. [51,52]; in both cases, the evolution is ergodic (since there is a unique steady state), and the corresponding trajectories remain stochastic.

We begin by seeking conditions for stable measurement-induced (anti)synchronization, corresponding to local system observables that oscillate at the same frequency (and with stable amplitude), to occur along an individual quantum trajectory (we will see below that different trajectories may possibly exhibit different synchronization frequencies and amplitudes). This dynamical synchronization criterion has been widely used at the ensemble level [53–59]. However, the averaged dynamics does not allow one to draw conclusions about individual realizations, which is why one needs to go beyond existing conditions for quantum synchronization [59] that are no longer applicable. In particular, the occurrence of synchronization in the mean does generally not imply synchronization along single trajectories, and vice versa. Typically, the temporal behavior of a continuously monitored system remains stochastic throughout the whole evolution. Thus, to ensure stable synchronization, we require the onset of unperturbed oscillations with constant amplitude. A sufficient condition is the existence of a decoherence-free subspace (DFS), such that $L|q_k\rangle = c_k|q_k\rangle$, where $|q_k\rangle$ are eigenstates of the Hamiltonian and c_k are complex numbers [42–46] (different constants c_k generally correspond to distinct decoherence-free subspaces). This follows from the observation that a decoherence-free subspace remains decoherence-free along a trajectory (1), since $\mathrm{d}\rho_W^{\mathrm{DFS}} = -i[H,\rho_W^{\mathrm{DFS}}]\mathrm{d}t$. Contrary to (averaged) Lindblad dynamics, where the state space needs to be considered as a whole, here, each (decoherence-free) subspace must be treated independently to be able to properly account for synchronization along an individual trajectory. A sufficient condition for stable (anti)synchronization is, thus, that the decoherence-free subspace contains only a single eigenmode, in which case (anti)synchronization appears at the frequency of that eigenmode [60].

We next examine the dynamics of the synchronization process by first assuming the presence of a single decoherence-free subspace. To that end, we show that, when the measurement operator is Hermitian, the probability that individual realizations get spontaneously trapped in the decoherence-free subspace is equal to the initial support on that subspace. We start by writing the solution of the stochastic master equation (1) as $\rho_W(t) = \sum_m u_m|\Psi_m(t)\rangle\langle\Psi_m(t)|$, where u_m is the probability to prepare the pure state $|\Psi_m(0)\rangle$. We further partition the system Hilbert space into the decoherence-free subspace (with basis states $\{|q_k\rangle\}$) and its orthogonal complement (with basis states $\{|p_l\rangle\}$). Any pure state $|\Psi(t)\rangle$ may then be expanded as $|\Psi(t)\rangle = \sum_k q_k(t)|q_k\rangle + \sum_l p_l(t)|p_l\rangle$, with $q_k(t) = \langle q_k|\Psi(t)\rangle$ and $p_l(t) = \langle p_l|\Psi(t)\rangle$ (we omit the state index m for convenience). The probabilities to find the system at time t in the decoherence-free subspace or its complement are, thus, respectively, $|q(t)|^2 = \sum_k |q_k(t)|^2$ and $|p(t)|^2 = \sum_l |p_l(t)|^2 = 1 - |q(t)|^2$, since total probability is conserved. By additionally assuming that $L = L^\dagger$, we obtain using Eq. (1) the differential for the probability $|q(t)|^2$ (Supplemental Material [61]):

$$\mathrm{d}(|q|^2) = 2|q|^2\left(c(t)(1-|q|^2) - \sum_{m,n} p_m^* p_n L_{mn}\right)\mathrm{d}W, \tag{2}$$

where we have defined $\sum_k |q_k(t)|^2 c_k \equiv |q(t)|^2 c(t)$ and $L_{mn} = \langle p_m|L|p_n\rangle$. Equation (2) describes the temporal evolution of the overlap with the decoherence-free subspace for individual realizations. It has the form of a free Brownian motion with state-dependent diffusion [66]. The corresponding Fokker-Planck equation for the probability density $P(|q|^2,t)$ is accordingly [66]

$$\frac{\partial P}{\partial t} = 2\frac{\partial^2}{\partial(|q|^2)^2}\left[|q|^4\left(c(t)(1-|q|^2) - \sum_{m,n} p_m^* p_n L_{mn}\right)^2 P\right] \tag{3}$$

with steady-state solution (Supplemental Material [61])

$$P^{\mathrm{s}}(|q|^2) = \left(1-|q(0)|^2\right)\delta(|q|^2) + |q(0)|^2\delta(|q|^2-1). \tag{4}$$

A single stochastic trajectory will, therefore, asymptotically select the decoherence-free subspace $|q|^2 = 1$ (and become unitary) with probability $|q(0)|^2$ or the orthogonal complement $|q|^2 = 0$ (and remain stochastic) with probability $1 - |q(0)|^2$. When a trajectory gets trapped indefinitely in a decoherence-free subspace, the quantum system undergoes a spontaneous transition from random to noiseless evolution, which may support (anti)synchronization. These results can be easily extended to arbitrary mixed states and multiple decoherence-free subspaces (Supplemental Material [61]). The above scenario bears similarities with dissipative freezing, which has recently been investigated for quantum jump processes [67–69]. However, dissipative freezing entails freezing into arbitrary symmetry sectors where the evolution is generally stochastic, and stable synchronization is, therefore, absent.

Example of a quantum spin chain.—The above discussion is valid for generic quantum many-body systems. We now apply our findings to an XY chain of N spins in a transverse field [50]. The corresponding Hamiltonian is

$$H = \frac{J}{2}\sum_{j=1}^{N-1}(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y) + h\sum_{j=1}^{N}\sigma_j^z, \tag{5}$$

where $\sigma_j^{x,y,z}$ are the usual Pauli operators, J is the interaction constant, and $h = 1$ is the field strength. The quantum XY chain is an important system in statistical physics and quantum information science [50]. For concreteness, we consider a chain of $N = 8$ spins and perform a homodyne measurement of the polarization of the third spin by setting $L = \sqrt{\Gamma}\sigma_3^z$, where Γ is the measurement strength. This model admits two two-dimensional decoherence-free subspaces with $c_1 = -1$ and $c_2 = 1$ (Supplemental Material [61]). Both subspaces support antisynchronization, at the same frequency, of the average polarizations, $\langle\sigma_1^z\rangle_W$ and $\langle\sigma_8^z\rangle_W$, of the two edge spins, with $\langle\sigma_j^z\rangle_W = \mathrm{Tr}[\sigma_j^z\rho_W]$ (the dynamics of the remaining magnetizations are presented in Supplemental Material [61]). To illustrate the nonintuitive behavior of the system and highlight its quantum properties, we compare two scenarios: (i) In the first case, the system is initially in a statistical mixture of the first decoherence-free subspace and the orthogonal complement, $|q_1(0)|^2 = 2/5$, $|q_2(0)|^2 = 0$, $|p(0)|^2 = 3/5$ [Figs. 1(a)–1(c)], whereas (ii) in the second case, the initial state is chosen such that the chain is in a linear superposition of the first and second decoherence-free subspaces, $|q_1(0)|^2 = |q_2(0)|^2 = 1/2$, $|p(0)|^2 = 0$ [Figs. 1(d)–1(f)]. Figure 1(a) demonstrates measurement-induced stable quantum antisynchronization, where the first

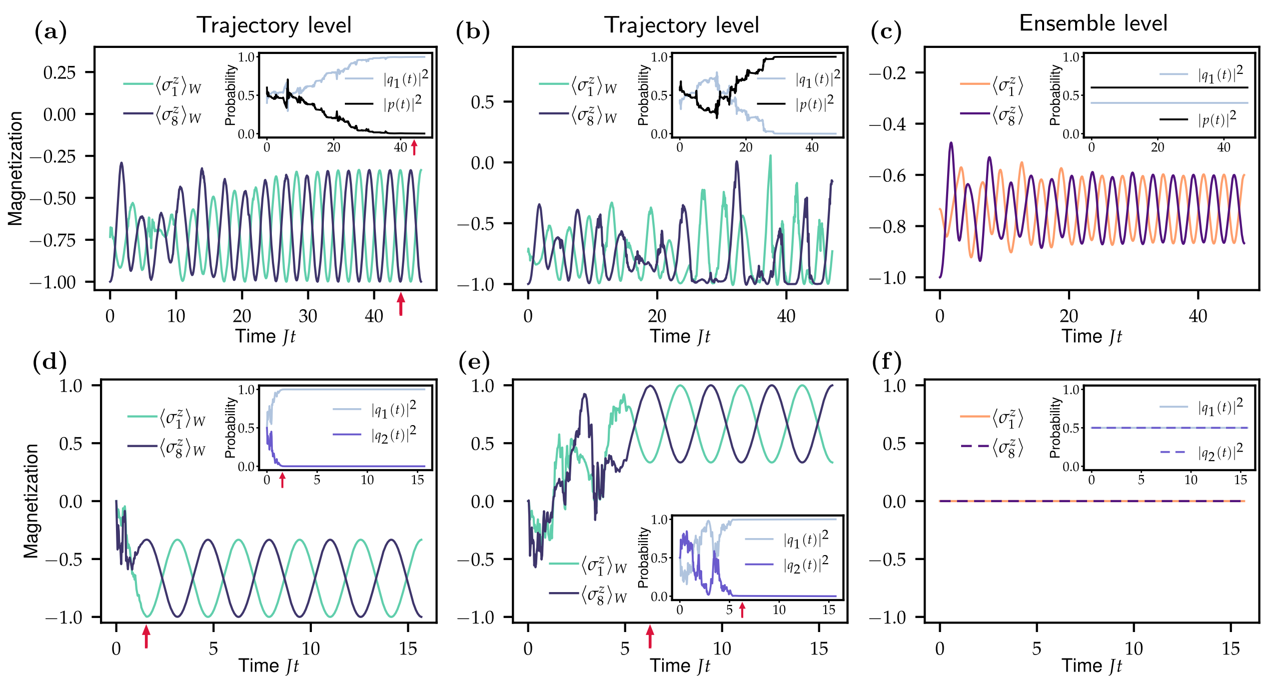

FIG. 1. Stable measurement-induced antisychronization along quantum trajectories. Evolution of the end magnetizations, $\langle\sigma_1^z\rangle_W$ and $\langle\sigma_8^z\rangle_W$, of a XY spin chain [Eq. (5)] with $N = 8$ spins subjected to homodyne measurement of the third z polarization with measurement operator $L = \sqrt{\Gamma}\sigma_3^z$. (a) Spontaneous transition (red arrow) from noisy asynchronous to noiseless antisynchronization, when the system gets trapped in the first decoherence-free subspace with probability $|q_1|^2 = 1$. The system is initially prepared in a mixture of the first decoherence-free subspace and the orthogonal complement, $|q_1(0)|^2 = 2/5$ and $|p(0)|^2 = 3/5$. (b) No synchronization occurs when the system gets trapped in the orthogonal complement with probability $|p|^2 = 1$. (c) Antisynchronization also appears at the ensemble level for this configuration. (d),(e) When the state is initially prepared in a superposition of the two decoherence-free subspaces, $|q_1(0)|^2 = 1/2$ and $|q_2(0)|^2 = 1/2$, stable antisychronization occurs at the trajectory level, when the system gets spontaneously trapped in either one of the decoherence-free subspaces. (f) However, in this case, there is no synchronization at the ensemble level. The reduced measurement strength is $\Gamma/J = 0.7/\pi$.

and the last spins oscillate with identical frequencies. As predicted by Eqs. (3) and (4), two-fifths of the trajectories undergo a spontaneous transition (inset) from random asynchronous to noiseless antisynchronized evolution, as they get trapped in the first decoherence-free subspace. The remaining trajectories, by contrast, end up in the orthogonal complement, stay noisy, and do not synchronize [Fig. 1(b)]. The two end spins also exhibit antisynchronization at the ensemble level, with average polarizations $\langle\sigma_j^z\rangle = \mathrm{Tr}[\sigma_j^z\rho]$ [Fig. 1(c)]. On the other hand, in case (ii), half of the trajectories get localized in each decoherence-free subspace, implying that antisynchronization occurs along all realizations [Figs. 1(d) and 1(e)]. However, here antisynchronization is completely absent at the ensemble level [Fig. 1(f)], indicating that individual realizations can strongly deviate from the mean. This genuinely nonclassical phenomenon is a consequence of quantum superposition. It may be experimentally observed in superconducting circuits that have been used to realize spin chains [70,71] and monitor individual trajectories of continuously measured qubits [72,73].

Ergodicity breaking and classical noise.—The identified difference between trajectory and ensemble properties is related to ergodicity breaking [74]. We quantify nonergodic behavior with the mean fidelity, $\mathbb{E}[F(\bar{\rho}_W,\rho^{\mathrm{s}})] = \mathbb{E}\big[\mathrm{Tr}\big[\sqrt{\sqrt{\bar{\rho}_W}\rho^{\mathrm{s}}\sqrt{\bar{\rho}_W}}\big]^2\big]$, between the time-averaged state, $\bar{\rho}_W = \lim_{T\to\infty}\int_0^T \mathrm{d}t\,\rho_W/T$, and the ensemble-averaged state ρ^{s} [47], for a given initial condition. When one decoherence-free subspace exists and the operator L is Hermitian, the system Hilbert space may be decomposed into two mutually orthogonal subspaces that can be associated to orthogonal blocks of the density matrix [75]. When more decoherence-free subspaces are present, additional orthogonal subspaces may be identified. We assume that every block j has a unique steady state ρ_j^{s}. These asymptotic states are orthogonal [75,76]. The ensemble-averaged density matrix can then be written as $\rho^{\mathrm{s}} = \sum_j w_j\rho_j^{\mathrm{s}}$, where w_j is the probability that the initial state is prepared in block j. Since each block contains exactly one stationary state ρ_j^{s}, every trajectory localizes into one of them with probability w_j, where the evolution is ergodic [77]. As a result, $\bar{\rho}_{W,j} = \rho_j^{\mathrm{s}}$, and we obtain [78]

$$\mathbb{E}[F(\bar{\rho}_W,\rho^{\mathrm{s}})] = \sum_k w_k F\left(\bar{\rho}_{W,k}, \sum_j w_j\rho_j^{\mathrm{s}}\right) = \sum_k w_k^2. \qquad (6)$$

The mean fidelity is, thus, given by the inverse participation ratio, a prominent measure of localization [80,81], of the initial state over the subspaces. Equation (6) is lower bounded by $1/N$, where N is the number of blocks (Supplemental Material [61]). The dynamics is accordingly ergodic only when the system starts in one of the subspaces. Figure 2(a) displays the mean fidelity (6) as a function of the overlap with the first decoherence-free subspace, $|q_1(0)|^2$, for the example of the quantum spin chain (5). Taking the initial state $\rho(0) = w_{q_1}|q_1\rangle\langle q_1| + w_p|p\rangle\langle p|$, with $w_{q_1} = |q_1(0)|^2$ and $w_p = |p(0)|^2$, the mean fidelity is simply given by $\mathbb{E}[F(\bar{\rho}_W,\rho^{\mathrm{s}})] = w_{q_1}^2 + (1-w_{q_1})^2$ (purple line), in perfect agreement with numerical simulations of the quantum trajectories (purple dots).

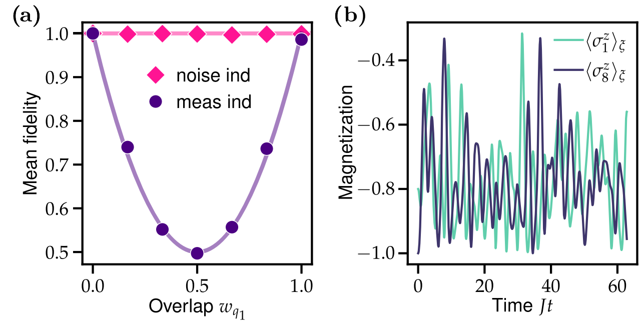

FIG. 2. Ergodicity breaking. (a) Mean fidelity between time and ensemble-averaged states [Eq. (6)] as a function of the overlap, $w_{q_1} = |q_1(0)|^2$, with the first decoherence-free subspace, for quantum (measurement) noise (purple line). Dynamics is nonergodic unless the chain starts in one of the subspaces (dots show simulations with 100 trajectories). Dynamics is always ergodic for classical noise (pink). (b) Corresponding trajectories are not synchronized (shown for $w_{q_1} = 0.3$).

It is instructive to further compare synchronization induced by quantum (measurement) noise and by classical noise [23]. Setting $L = -i\sqrt{\Gamma}\sigma_3^z$, the randomness no longer depends on the state of the system, leading to stochastic unitary dynamics, $\dot{\rho}_\xi = -i[H + \sqrt{\Gamma}\xi(t)\sigma_3^z, \rho_\xi]$ (in Stratonovich convention), with classical white noise with zero mean and unit variance $\langle\xi(t)\xi(t')\rangle = \delta(t-t')$ [25]. Figure 2(a) shows that, for classical noise, the evolution is always ergodic (pink line and diamonds) for any initial overlap $|q_1(0)|^2$ with the first decoherence-free subspace. In this situation, there is no (anti)synchronization along single trajectories [Fig. 2(b)], although for finite $|q_1(0)|^2$, synchronous behavior appears at the ensemble level [23]. The effects of classical and quantum noises on the system, hence, differ significantly.

Application to quantum multiplexing.—Multiplexing is a standard technique in classical communication by which signals with different frequencies are simultaneously transmitted through the same medium [48]. Measurement-induced synchronization allows for a quantum form of multiplexing by preparing the system in a superposition of multiple decoherence-free subspaces with distinct frequencies. According to Eq. (4), the initial overlap with each subspace then controls the amount of trajectories that are (anti)synchronized at the respective frequencies. Figure 3 presents an example of quantum multiplexing using the quantum XY chain [Eq. (5)] with $N = 9$ spins and the polarization of the fifth spin, $L = \sqrt{\Gamma}\sigma_5^z$, being measured. Contrary to the previous example, this system supports

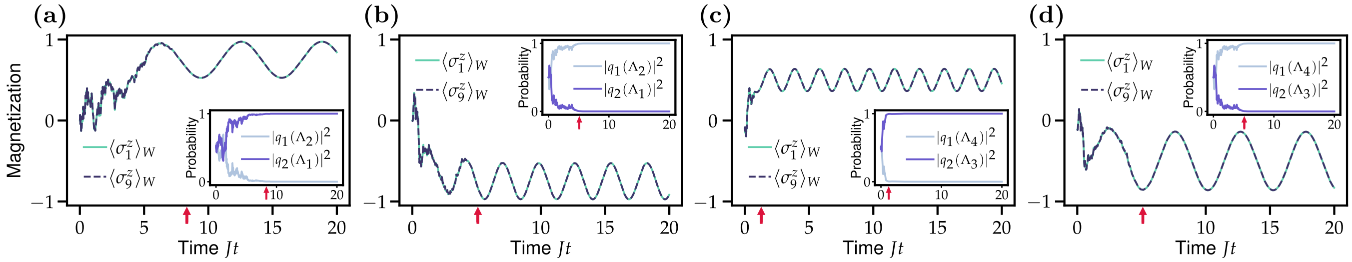

FIG. 3. Quantum multiplexing and stable measurement-induced synchronization. Evolution of the end spin magnetizations, $\langle\sigma_1^z\rangle_W$ and $\langle\sigma_9^z\rangle_W$, of a XY spin chain [Eq. (5)] with $N = 9$ spins, subjected to homodyne measurement of the fifth z polarization with measurement operator $L = \sqrt{\Gamma}\sigma_5^z$. The chain is prepared in a linear superposition of two decoherence-free subspaces that support the same four eigenfrequencies: $\Lambda_1 = J$, $\Lambda_2 = \sqrt{5}J$, $\Lambda_3 = [\sqrt{5}+1]J$, and $\Lambda_4 = [\sqrt{5}-1]J$. (a),(b) When the system is initialized in $|\Psi(0)\rangle = [|q_1(\Lambda_2)\rangle + |q_2(\Lambda_1)\rangle]/\sqrt{2}$, stable synchronization occurs with eigenfrequencies $\Lambda_{2(1)}$ in the subspace with $c_{1(2)}$. (c),(d) When the system is initialized in $|\Psi(0)\rangle = [|q_1(\Lambda_4)\rangle + |q_2(\Lambda_3)\rangle]/\sqrt{2}$, stable synchronization occurs with eigenfrequencies $\Lambda_{4(3)}$ in the subspace with $c_{1(2)}$. The reduced measurement strength is $\Gamma/J = 0.1$.

stable synchronized trajectories at various frequencies. It indeed possesses two eight-dimensional decoherence-free subspaces with the same eigenmodes and eigenfrequencies given by $\Lambda_1 = J$, $\Lambda_2 = \sqrt{5}J$, $\Lambda_3 = [\sqrt{5}+1]J$, and $\Lambda_4 = [\sqrt{5}-1]J$. To implement multiplexing, we prepare a superposition of the two subspaces, $|\Psi(0)\rangle = [|q_1(\Lambda_j)\rangle + |q_2(\Lambda_k)\rangle]/\sqrt{2}$, where $|q_i(\Lambda_j)\rangle$ indicates a state with eigenfrequency Λ_j in the subspace belonging to c_i. Depending on the choice of the eigenmodes, one half of the trajectories can be synchronized at frequency Λ_j and the other half at frequency Λ_k [Figs. 3(a)–3(d)]. More frequencies could be superposed in configurations with more decoherence-free subspaces.

Conclusions.—Classical synchronization plays an important role for classical communication systems [82,83]. The study of nonclassical types of synchrony and the exploration of their potential applications for quantum communication purposes are, hence, of great interest. We have here analyzed quantum synchronization induced by continuous (homodyne) measurements. In particular, we have shown that a many-body system may undergo a spontaneous transition from random asynchronous dynamics to stable noise-free (anti)synchronization at the level of individual trajectories. The number of (nonergodic) synchronized realizations is given by the initial overlap with a decoherence-free subspace and can, thus, be controlled by preparing a linear superposition of states living in different subspaces. A quantum form of frequency multiplexing is consequently possible when these decoherence-free subspaces are associated with different eigenfrequencies. These results highlight the significance of coherent superpositions for quantum synchronization.

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