# Fractional Quantum Hall State at Filling Factor $\nu = 1/4$ in Ultra-High-Quality GaAs Two-Dimensional Hole Systems

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Single-component fractional quantum Hall states (FQHSs) at even-denominator filling factors may host non-Abelian quasiparticles that are considered to be building blocks of topological quantum computers. Such states, however, are rarely observed in the lowest-energy Landau level, namely at filling factors $\nu < 1$. Here, we report evidence for an even-denominator FQHS at $\nu = 1/4$ in ultra-high-quality two-dimensional hole systems confined to modulation-doped GaAs quantum wells. We observe a deep minimum in the longitudinal resistance at $\nu = 1/4$, superimposed on a highly insulating background, suggesting a close competition between the $\nu = 1/4$ FQHS and the magnetic-field-induced, pinned Wigner solid states. Our experimental observations are consistent with the very recent theoretical calculations that predict that substantial Landau level mixing, caused by the large hole effective mass, can induce composite fermion pairing and lead to a non-Abelian FQHS at $\nu = 1/4$. Our results demonstrate that Landau level mixing can provide a very potent means for tuning the interaction between composite fermions and creating new non-Abelian FQHSs.

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Even-denominator fractional quantum Hall states (FQHSs) are fascinating condensed matter phases. The best-known example is the even-denominator FQHS at Landau level (LL) filling factor $\nu = 5/2$ observed in GaAs two-dimensional electron systems (2DESs) when a first excited ($N = 1$) spin LL is half-occupied [1,2]. It is generally believed to be a BCS-type, paired state of flux-particle composite fermions (CFs) [3–6]. This state may have non-Abelian quasiparticles as its excitations, and be of potential use in fault-tolerant, topological quantum computing [7–9].

The CF pairing that leads to the stability of the $\nu = 5/2$ FQHS is facilitated by the node in the in-plane wave function of electrons in the $N = 1$ LL as it allows them to come closer to each other. Such pairing is much harder to achieve in the ground state ($N = 0$) LL, consistent with the near absence of even-denominator FQHSs. Instead, the ground state at $\nu = 1/2$ (and $1/4$) is a compressible CF Fermi sea, flanked by a plethora of odd-denominator FQHSs at nearby fillings [10]. An exception is a 2DES with *bilayer* charge distribution. A FQHS at $\nu = 1/2$ was observed in 2DESs confined to wide GaAs quantum wells (QWs) [11–14] and double QWs [15]. These were originally interpreted as a two-component, Abelian FQHS described by the Halperin-Laughlin ($\psi_{331}$) wave function [16,17], with the layer or electric sub-band index playing the role of an extra degree of freedom. Although the two-component origin of the $\nu = 1/2$ FQHS is widely accepted for the double QWs where interlayer tunneling is negligible, recent experiments [18–20] and theory [21] suggest that in wide QWs where interlayer tunneling is significant, the $\nu = 1/2$ FQHS is likely a single-component, non-Abelian state. In addition, another even-denominator FQHS was reported in wide GaAs QWs at $\nu = 1/4$ [22,23], and theory suggests it is also likely a single-component, non-Abelian state, topologically equivalent to an $f$-wave paired state of CFs [24]. We emphasize that, for both $\nu = 1/2$ and $1/4$ FQHSs in wide QWs, the thick and bilayerlike charge distribution is crucial as it leads to a softening of the Coulomb repulsion and CF pairing.

Here, we report experimental evidence for a developing FQHS at $\nu = 1/4$ in ultra-high-quality 2D *hole* systems (2DHSs) confined to *narrow* GaAs QWs with *single-layer* charge distributions. We attribute this surprising observation to the much larger effective mass of GaAs 2D holes ($m^* \simeq 0.5$, in units of free electron mass) [25] compared to electrons ($m^* = 0.067$), and the ensuing severe LL mixing (LLM). LLM is often parametrized as the ratio of the Coulomb energy to cyclotron energy, $\kappa = (e^2/4\pi\epsilon l_B)/(\hbar eB/m^*)$, and is proportional to $m^* B^{1/2}$, where $l_B = (\hbar/eB)^{1/2}$ is the magnetic length. LLM can play a crucial role in determining the many-body ground states in different 2D material systems, including semiconductor heterostructures [26,27] and atomically thin 2D materials (e.g., monolayer graphene [28]). For example, it can affect the stabilization of possible non-Abelian FQHSs at $\nu = 5/2$ [26,27] and high-field Wigner crystal [29]. Most relevant to our Letter, very recent theoretical calculations suggest that

 

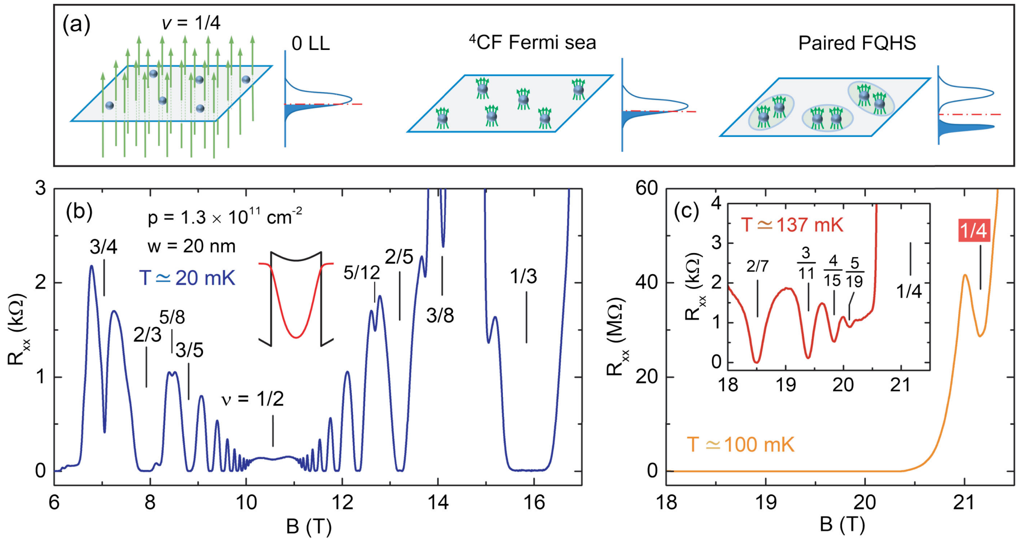


FIG. 1. (a) Schematics of the pairing mechanism for even-denominator FQHS at $\nu = 1/4$. The blue spheres represent holes, and the green vertical arrows the magnetic field flux quanta. The curved short arrows in the right two panels represent magnetic flux quanta attached to holes to form four-flux composite fermions ($^4$CFs). If LLM is strong, $^4$CFs pair and condense to form a FQHS at $\nu = 1/4$. (b) $R_{xx}$ vs $B$ trace near $\nu = 1/2$ for a 2DHS, with density $1.3 \times 10^{11}$ cm$^{-2}$ and QW width 20 nm, taken at $T \simeq 20$ mK with $I = 20$ nA. Inset shows the self-consistently calculated hole charge distribution (red) and potential (black). (c) $R_{xx}$ vs $B$ trace near $\nu = 1/4$, taken at $T \simeq 100$ mK with $I = 0.1$ nA. Inset shows $R_{xx}$ vs $B$ between $\nu = 2/7$ and $1/4$ taken at $T \simeq 137$ mK. A higher current ($I = 50$ nA) is used to reduce noise.

substantial LLM can destabilize the CF Fermi sea at even-denominator fillings in the lowest LL and lead to the emergence of a single-component, non-Abelian FQHS through CF pairing [Fig. 1(a)] [30]. Our results, together with the calculations of Ref. [30], establish ultra-high-quality GaAs 2DHSs as a platform for hosting and creating exotic, non-Abelian FQHSs through LLM.

We studied 2DHSs in GaAs QWs grown on GaAs (001) substrates using molecular beam epitaxy. The samples have $Al_xGa_{1-x}As$ barriers and are modulation doped with C. They were grown following the optimization of the growth chamber vacuum integrity and the purity of the source materials [31], and have extremely high mobilities, up to $\simeq 6 \times 10^6$ cm$^2$/Vs [32]. The widths ($w$) of the QWs range from 20 to 35 nm, and their 2D hole densities ($p$) from 0.41 to 1.3, in units of $10^{11}$ cm$^{-2}$, which we use throughout this Letter. We performed our experiments on $4 \times 4$ mm$^2$, Van der Pauw samples, with alloyed In:Zn contacts at the four corners and side midpoints. We cooled the samples in a dilution refrigerator, and measured the longitudinal resistance ($R_{xx}$) using the conventional lock-in amplifier techniques.

In Figs. 1(b) and 1(c) we present $R_{xx}$ vs magnetic field ($B$) traces for a 2DHS with $p = 1.3$ and $w = 20$ nm. Near $\nu = 1/2$, the sample exhibits a smooth and shallow minimum, flanked by numerous odd-denominator FQHSs following the Jain sequence $\nu = n/(2n \pm 1)$, where $n$ is an integer [3,10]. This is consistent with a compressible Fermi sea of two-flux CFs ($^2$CFs) being the ground state when the lowest spin LL is half-occupied [10]. We also observe emerging even-denominator FQHSs at $\nu = 3/4$, 3/8, 5/8, and 5/12 [33,34]. In Fig. 1(c) and its inset, we show $R_{xx}$ at higher $B$ at elevated temperatures ($\simeq$100 and 137 mK). For $18 < B < 20.5$ T, $R_{xx}$ remains in the kΩ range, and we observe odd-denominator FQHSs at $\nu = 2/7$, 3/11, 4/15, and 5/19. These are the Jain-sequence states of four-flux CFs ($^4$CFs) that follow $\nu = n/(4n - 1)$ [10]. Their presence, together with the higher-order FQHSs flanking $\nu = 1/2$, attest to the exceptionally high quality of the 2DHS.

In Fig. 1(c), when $B$ exceeds 20.5 T, $R_{xx}$ sharply increases and attains values $\simeq$40 MΩ, even at a relatively high temperature of $\simeq$100 mK. The 2DHS becomes highly insulating in this field range, as we demonstrate later in this Letter. Such $B$-induced insulating phases have been previously reported in GaAs 2DESs at $\nu \lesssim 1/5$ [35–37] and in GaAs 2DHSs at $\nu \lesssim 1/3$ [38–40]. They are generally believed to signal the formation of Wigner solids (WSs) pinned by the ubiquitous disorder [29,38–40].

The highlight of our Letter is the observation, for the first time, of a very deep and sharp $R_{xx}$ minimum at $\nu = 1/4$, signaling a developing FQHS at this filling. The fact that the $R_{xx}$ minimum at $\nu = 1/4$ appears on top of the insulating background suggests a close competition between the FQHS and WS states at $\nu = 1/4$. This is reminiscent of the recent observation of a developing FQHS at $\nu = 1/7$ in ultra-high-mobility 2DESs, also competing with surrounding WS states [41–43].

In order to confirm that the $R_{xx}$ minimum we observe at $\nu = 1/4$ is intrinsic to ultra-high-quality 2DHSs, we measured several samples from different wafers with

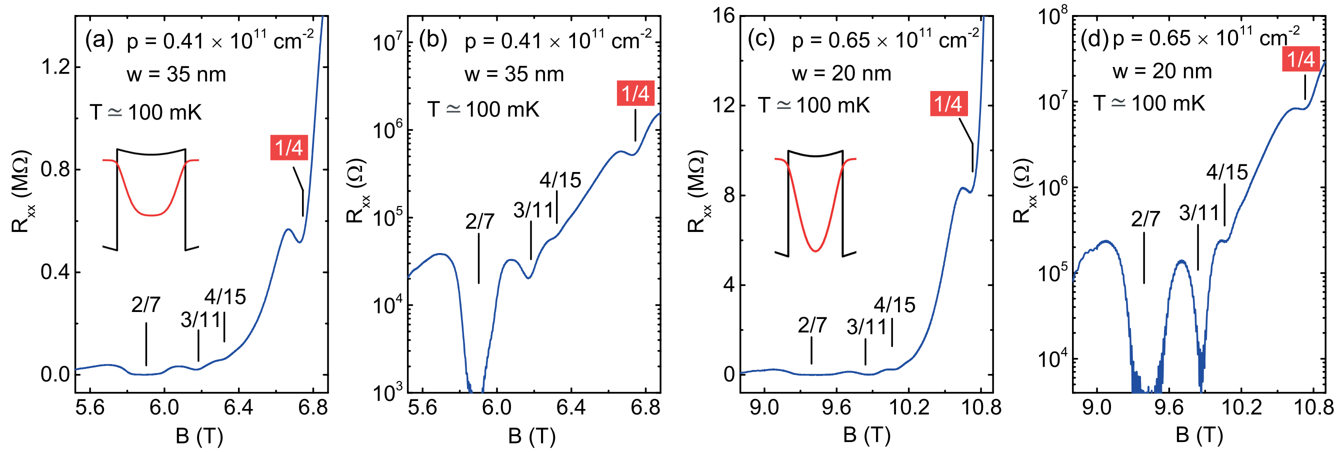


FIG. 2. $\nu = 1/4$ FQHSs in 2D hole samples with different densities: (a),(b) $p = 0.41 \times 10^{11}$ cm$^{-2}$, and (c),(d) $p = 0.65 \times 10^{11}$ cm$^{-2}$. (b) and (d) are replots of (a) and (c) in *log* scales for $R_{xx}$. Insets in (a) and (c): self-consistently calculated hole charge distribution (red) and potential (black).

various hole densities. In Figs. 2 and 3, we present data for three samples with $p = 0.41$, 0.65, and 1.0, and QW widths 35, 20, and 20 nm, respectively. Similar behavior is observed in all samples. On the low-field side of $\nu = 1/4$, we observe $R_{xx}$ minima at odd-denominator $\nu = 2/7$ and $3/11$, and a minimum or an inflection point at $4/15$, signaling developing FQHSs belonging to the $\nu = n/(4n-1)$ sequence; these are seen more clearly in *log* scale plots of Figs. 2(b), 2(d), and 3(a) inset. As we increase $B$ and approach $\nu = 1/4$, $R_{xx}$ grows very rapidly, and the 2DHSs enter the insulating phase. Remarkably, in all samples, a well-defined and sharp $R_{xx}$ minimum is seen at $\nu = 1/4$ superimposed on the insulating background.

We also investigated the temperature dependence of the $\nu = 1/4$ FQHS and its adjacent insulating phases. Figure 3(a) illustrates $R_{xx}$ traces at various temperatures for a sample with $p = 1.0$, and $w = 20$ nm. As $T$ increases, the $R_{xx}$ minimum at $\nu = 1/4$ gradually weakens and turns into an inflection point at 150 mK, while the background resistance decreases by more than 10 times. At $B \simeq 17.9$ T and intermediate temperatures, we observe a developing FQHS at $\nu = 3/13$, following the Jain-sequence states of $^4$CF $[\nu = n/(4n+1)]$.

To quantitatively analyze the insulating phases near $\nu = 1/4$, we deduce the activation energy $E_A$ from the relation $R_{xx} \propto e^{E_A/2kT}$. The energy scale $E_A$ is generally associated with the defect formation energy of the WS [41,45]. In Fig. 3(b), we show the Arrhenius plots of $R_{xx}$ vs $1/T$. The slopes of the linear fits yield $E_A$ at a given $B$. In Fig. 3(c), we present $E_A$ for $16 < B < 18$ T, where $R_{xx}$ exhibits insulating behavior. The data reveal a clear minimum in $E_A$ at $\nu = 1/4$ and a small dip at $\nu = 3/13$. Similar features were reported in GaAs 2DESs at very low fillings, e.g., at $\nu = 1/7$ and $2/13$, in recent measurements of ultra-high-mobility 2DES samples [41], and were interpreted as precursors to developing FQHSs [46–51]. We also note that the magnitude of $E_A$ in Fig. 3(c), ≃2–3 K, is comparable to $E_A$ measured at similar $B$ near $\nu = 1/7$ [41]. It is consistent with the theoretically calculated energies for the quantum bubble defect formation in WSs composed of CFs [45], and suggests that the insulating behavior adjacent $\nu = 1/4$ is intrinsic and originates from many-body phenomena rather than single-particle Anderson localization [52].

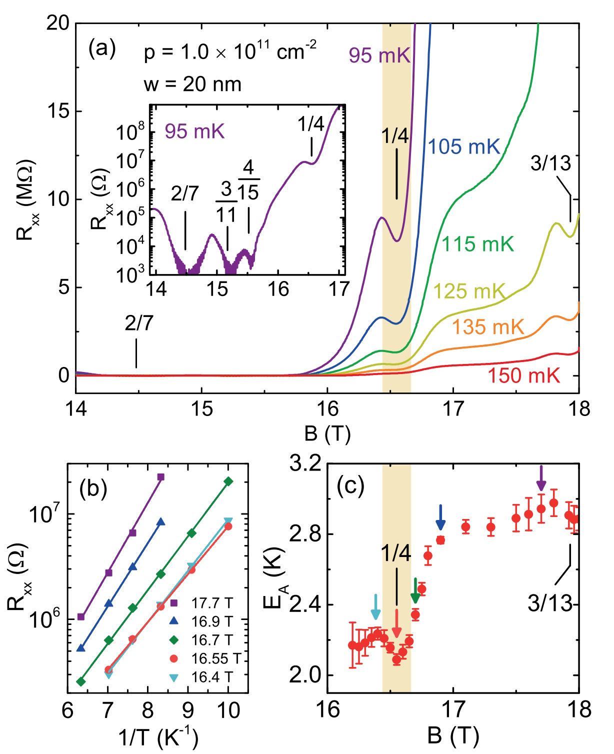


FIG. 3. (a) $R_{xx}$ vs $B$ traces taken at different temperatures for a 2DHS with $p = 1.0 \times 10^{11}$ cm$^{-2}$. Inset: $R_{xx}$ in *log* scale vs $B$ at $T \simeq 95$ mK. (b) Arrhenius plots of $R_{xx}$ vs $1/T$ at various magnetic fields, color-coded according to the fields marked by arrows in (c). The solid lines are linear fits to the data points according to $R_{xx} \propto e^{E_A/2kT}$, from which we obtain the activation energy $E_A$. (c) $E_A$ vs $B$.

An important requirement for a FQHS to be non-Abelian is that it is fully spin polarized [7,53]. Our observation of the $\nu = 1/4$ FQHS in a wide range of large perpendicular magnetic fields ($7 \lesssim B_\perp \lesssim 21$ T) in samples with different densities strongly suggests that it is fully spin polarized. This conclusion is supported by our data taken in tilted magnetic fields, which reveal that the $\nu = 1/4$ $R_{xx}$ minimum is always present, even when a large parallel magnetic field of $\simeq$12 T is applied (see Supplemental Material (SM) [43]).

To understand the origin of the observed $\nu = 1/4$ FQHS in our 2DHSs, it is helpful to compare our results to other 2D systems. In GaAs 2DESs confined to narrow QWs, the ground state at $\nu = 1/4$ is a Fermi sea of $^4$CFs. This is supported by transport experiments where $R_{xx}$ is featureless near $\nu = 1/4$, and odd-denominator Jain-sequence states of $^4$CF at $\nu = n/(4n \pm 1)$ are observed [41,54]. The Fermi wave vector of the $^4$CF Fermi sea has indeed been directly measured by geometrical resonance [55]. In 2DESs confined to wide GaAs QWs, however, a FQHS was observed at $\nu = 1/4$ [22,23]. This is explained by theory, suggesting that the large electron layer thickness in wide QWs can modify the interaction, making the CF Fermi sea at $\nu = 1/4$ unstable to $f$-wave pairing [24]. We can easily rule out the above mechanism for the $\nu = 1/4$ FQHS in our 2DHSs because our samples are all narrow QWs, and have single-layer charge distributions (see insets to Figs. 1 and 2).

A $\nu = 1/4$ FQHS has also been reported at an isospin transition in monolayer graphene [56]. This was interpreted as a multicomponent FQHS incorporating correlations between electrons on different carbon sublattices [56]. Similar physics was also observed in GaAs 2D holes: when the two lowest-energy LLs cross at $\nu = 1/2$, an even-denominator FQHS manifests itself as the ground state [57]. However, it is extremely unlikely that the $\nu = 1/4$ FQHSs in our 2D hole samples originate from a LL crossing. The LL crossing at $\nu = 1/2$ or other fillings in 2DHSs occurs for carefully tuned sample parameters (density, QW width, tilt angle, etc.) [57–60]. In contrast, we observe a $\nu = 1/4$ FQHS in several samples spanning a wide range of densities, QW widths, and tilt angles. It is hard to imagine that a crossing of two LLs occurs at $\nu = 1/4$ in all these samples.

Recent theoretical calculations by Zhao *et al.* [30] offer a potential explanation for our experimental findings. Using fixed-phase diffusion Monte Carlo calculations, the authors predict that significant LLM could give rise to the pairing of CFs at 1/4 filling, leading to the destabilization of CF Fermi sea and the emergence of non-Abelian FQHSs [Fig. 1(a)] [30]. Specifically, they propose a transition from a compressible CF Fermi sea to an incompressible, non-Abelian, paired FQHS [61] at $\nu = 1/4$ when the LLM parameter $\kappa$ reaches a value of $\simeq$6 to 7. This critical value is much higher than $\kappa$ for GaAs 2DESs, typically $\lesssim$1, consistent with the Fermi sea as the CF ground state at $\nu = 1/4$.

GaAs 2D holes, however, have a much larger $\kappa$ because of their larger effective mass. For the samples investigated in our Letter, if we use the calculated $B = 0$ effective mass and simply assume linear LLs as a function of $B$, we estimate that $\kappa$ at $\nu = 1/4$ is between 3 and 8, close to the critical $\kappa$ estimated by the calculations [30]. However, this agreement is likely fortuitous, because both the $\kappa$ we quote for our samples and the calculated critical $\kappa$ are rough estimates [30,43,62].

A natural question that arises is the relation between the $\nu = 1/4$ FQHS and the FQHS at $\nu = 3/4$ reported recently in 2DHSs with similar ultrahigh quality [33]. Assuming particle-hole symmetry in the lowest spin LL, these FQHSs may be viewed as a pair of particle-hole conjugate states. While severe LLM is evidently crucial in stabilizing both the 3/4 and 1/4 FQHSs, LLM is also known to break the $\nu \leftrightarrow (1 - \nu)$ particle-hole symmetry, suggesting that the two states likely have distinct origins. This conjecture is supported by the very different behaviors for GaAs 2DHSs near $\nu = 1/4$ and 3/4. On the flanks of $\nu = 1/4$, we observe numerous odd-denominator FQHSs at $\nu = n/(4n - 1)$. In contrast, no signs of odd-denominator FQHSs are observed at $\nu = 1 - n/(4n - 1)$ in the same samples (see Fig. S3 in SM [43]). Furthermore, the theory of Ref. [30] for $\nu = 1/4$ cannot explain other even-denominator FQHSs observed in the lowest LL of 2DHSs, e.g., at $\nu = 3/8$, 5/8, and 5/12. On the other hand, instead of being viewed as the particle-hole counterpart of the 1/4 FQHS, the FQHS at 3/4 filling of the 2DHS LL can be mapped to an *effective filling* $\nu^* = 3/2$ of $^2$CF Lambda levels [33]. In this picture, the excited CF Lambda level is half-occupied, reminiscent of the 5/2 FQHS in GaAs 2DESs where the excited electron spin LL is half-occupied. Similar explanations can be applied to the FQHSs at $\nu = 3/8$, 5/8, and 5/12 [34].

In closing, we emphasize that we observe signatures of the 1/4 FQHS in a highly resistive, insulating regime where there is a close competition between the FQHSs at fractional fillings and the (pinned) WS states at nearby fillings. The observation of insulating phases is consistent with previous experiments and theories that have shown that severe LLM in 2DHSs can favor WS states at $\nu \lesssim 1/3$ [29,38–40]. The 1/4 FQHS therefore manifests itself amidst a challenging landscape where a small but finite amount of disorder can disturb the many-body ground states. The significant improvement in the quality of GaAs 2DHSs [32] is evidently playing a crucial role in the unraveling of intriguing interaction phenomena previously concealed because of the presence of higher amounts of disorder. While our report of new correlated states in an ultrapure GaAs 2DHS advances the field of many-body condensed matter physics, it should also stimulate related future work in other 2D carrier systems, including atomically thin 2D materials such as transition-metal dichalcogenides (e.g., $WSe_2$) and bilayer graphene. These materials also experience severe LLM in the quantum Hall regime

[63,64] and, if their quality could be further improved, they should provide a fertile ground for the observation of exotic even-denominator FQHSs in the lowest LL.

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