

All Gauged Curvature-Squared Supergravities in Five Dimensions

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We present a complete basis to study gauged curvature-squared supergravity in five dimensions. We replace the conventional ungauged Riemann-squared action with a new log invariant, offering a comprehensive framework for all gauged curvature-squared supergravities. Our findings address long-standing challenges and have implications for precision tests in the AdS/CFT correspondence.

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Introduction.—Twenty-six years after its discovery, the AdS/CFT correspondence has entered a new era in which precision tests beyond the leading order have become increasingly important, owing to developments in both field theory and gravity. On the one hand, integrability and localization techniques allow one to compute the observables in superconformal field theories (SCFTs) exactly at finite couplings. On the other hand, the development of superconformal tensor calculus and superspace techniques—see the reviews [1–4]—in conjunction with the computational capabilities offered by computer algebra programs has significantly advanced the construction of exact, off-shell higher-derivative supergravity models.

In this Letter, we present all gauged curvature-squared supergravity invariants in five dimensions based on the off-shell dilaton Weyl multiplet. After going on shell, our invariants describe the most general four-derivative corrections to the five-dimensional minimal gauged supergravity, which is a universal sector to all string compactifications preserving at least eight supercharges. The gauged aspect is necessary to accommodate a supersymmetric anti-de Sitter (AdS) solution and, thus, is of broad interest in holography. In particular, due to recent advancements in AdS black hole microstate counting [5–12] using the dual CFT, a precise matching between the gravity and CFT results at the next-to-leading order clearly requires the knowledge of the complete curvature-squared supergravity actions. Previous works have made attempts to compute four-derivative corrections based on partial results in the

literature, and certain assumptions were made. Using our full results, it can be shown that, in fact, some of the assumptions are invalid, thus finally furnishing the stage for new, next-to-leading-order analyses on the gravity side of the AdS/CFT correspondence.

The construction of gauged curvature-squared invariants is notoriously hard, as opposed to their ungauged counterparts, which have been fully known for more than a decade [13–15]. The primary difficulty stems from the absence of a straightforward transition from ungauged to gauged theories. In fact, the complete basis of invariants must be constructed from completely different starting points. For instance, certain ungauged curvature-squared models are attainable through the application of superconformal tensor calculus, utilizing the dilaton Weyl multiplet. In contrast, their gauged counterparts necessitate the use of a modified version of the same multiplet [16], which has an entirely different field content and transformation rules. Furthermore, the deformation necessary for the construction of gauged supergravity models renders certain established higher-derivative supergravity building techniques impractical, thus further complicating the task. In fact, it takes an interplay between superconformal tensor calculus [16] and superspace techniques [17], together with a series of new, daunting computations finalized only in the results presented here, to yield the complete set of gauged curvature-squared invariants.

This Letter aims to explicitly show that the past challenges can be overcome by changing the basis of curvature-squared supergravities, which previously employed the Weyl tensor squared, Riemann tensor squared, and Ricci scalar squared as fundamental building blocks. We demonstrate that by replacing the Riemann-squared action with the log invariant, in which the leading term comes with the Ricci tensor squared, it is possible to explicitly establish all gauged curvature-squared supergravities in five dimensions.

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The outcomes presented in our Letter mark a significant advancement, paving the way to a complete study of physical results beyond the leading supergravity approximation in five dimensions. This development holds particular promise for precision tests of the AdS₅/CFT₄ correspondence. In this context, we derive the anomaly coefficients in the dual SCFT₄, which apparently depend on all curvature-squared couplings.

Construction of the invariants.—We start by introducing the field content of the standard Weyl multiplet of conformal supergravity in five dimensions [18]. Our notation and conventions correspond to that of [18]. We denote the spacetime indices by μ, ν, \dots , Lorentz indices by a, b, \dots , SU(2) indices by i, j, \dots , and spinor indices by α, β, \dots . The multiplet is described by a set of independent gauge fields: the vielbein e_μ^a , the gravitino ψ_μ^i , the SU(2) gauge fields V_μ^{ij} , and a dilatation gauge field b_μ . The other gauge fields associated with the remaining symmetries, including the spin connection ω_μ^{ab} , the S -supersymmetry connection ϕ_μ^i , and the special conformal connection f_μ^a , are composite fields; i.e., they are determined in terms of the other fields by imposing certain curvature constraints. The standard Weyl multiplet also contains a set of matter fields: a real antisymmetric tensor T_{ab} , a fermion χ_α^i , and a real scalar D . A more detailed discussion of the superconformal transformations of the various fields can be found, e.g., in [16,18].

Below, we will make use of a variant multiplet of conformal supergravity, known as the gauged dilaton Weyl multiplet [16,19]. For this multiplet, the independent gauge fields remain the same as the standard Weyl multiplet, but the matter content is replaced with $\{\sigma, C_\mu, B_{\mu\nu}, L_{ij}, E_{\mu\nu\rho}, N, \psi^i, \phi^i\}$. This is obtained by coupling the standard Weyl multiplet to on-shell vector and linear multiplets. The vector multiplet consists of a scalar field σ , the gaugino ψ_α^i , an Abelian gauge vector C_μ with field strength $G_{\mu\nu} = 2\partial_{[\mu}C_{\nu]}$, and an SU(2) triplet of auxiliary fields $Y^{ij} = Y^{(ij)}$. The linear multiplet contains an SU(2) triplet of scalars $L^{ij} = L^{(ij)}$, a gauge three-form $E_{\mu\nu\rho}$, a scalar N , and an SU(2) doublet ϕ_α^i . The bosonic matter fields of the vector and the standard Weyl multiplet are then expressed as follows [16]:

$$\begin{aligned} Y^{ij} &= -\frac{g}{2}\sigma^{-1}L^{ij} + \text{ft}, \\ T_{ab} &= \frac{1}{8}\sigma^{-1}G_{ab} + \frac{1}{48}\sigma^{-2}\epsilon_{abcde}H^{cde} + \text{ft}, \\ D &= \frac{1}{4}\sigma^{-1}\nabla^a\nabla_a\sigma + \frac{1}{8}\sigma^{-2}(\nabla^a\sigma)\nabla_a\sigma - \frac{1}{32}R \\ &\quad - \frac{1}{16}\sigma^{-2}G^{ab}G_{ab} - \left(\frac{26}{3}T^{ab} - 2\sigma^{-1}G^{ab}\right)T_{ab} \\ &\quad + \frac{g}{4}\sigma^{-2}N + \frac{g^2}{16}\sigma^{-4}L^2 + \text{ft}, \end{aligned} \quad (1)$$

where “ft” stands for omitted fermionic terms and $H_{abc} = e_a^\mu e_b^\nu e_c^\rho H_{\mu\nu\rho}$ denotes the three-form field strength $H_{\mu\nu\rho} := 3\partial_{[\mu}B_{\nu\rho]} + \frac{3}{2}C_{[\mu}G_{\nu\rho]} + \frac{1}{2}gE_{\mu\nu\rho}$. In the above, the covariant derivative is denoted by

$$\nabla_a = e_a^\mu(\partial_\mu - \omega_\mu^{bc}M_{bc} - b_\mu\mathbb{D} - V_\mu^{ij}U_{ij}), \quad (2)$$

with M_{ab} , \mathbb{D} , and U_{ij} being the Lorentz, dilatation, and SU(2) generators, respectively. The dilatation connection b_μ is pure gauge and will be set to zero throughout. The mapping (1) allows us to easily convert every invariant involving a coupling to the standard Weyl multiplet to that written in terms of the gauged dilaton Weyl multiplet. The ungauged map and the models can simply be obtained by setting $g = 0$ in (1). In this case, the fields of the linear multiplet decouple from the map (1), and the multiplet reduces to the ungauged dilaton Weyl multiplet with $32 + 32$ off-shell degrees of freedom [18,21].

In the superconformal tensor calculus, the so-called BF action principle plays a fundamental role in the construction of general supergravity-matter couplings; see Refs. [17,18,21–25] for the 5D case. It is based on an appropriate product of a linear multiplet with an Abelian vector multiplet:

$$e^{-1}\mathcal{L}_{\text{BF}} = A_a E^a + \rho N + \mathcal{Y}_{ij}L^{ij} + \text{ft}. \quad (3)$$

Here, we use $\{\rho, A_\mu, \mathcal{Y}_{ij}, \lambda_\alpha^i\}$ to denote the field content in an arbitrary vector multiplet, and the bosonic part of the constrained vector E_a is related to the three-form gauge field E_{abc} via $E_a = -(1/12)\epsilon_{abcde}\nabla^b E^{cde}$. In any construction that involves composite expressions for the fields of the linear multiplet in terms of the vector multiplet, the BF action yields a vector-coupled action in the (gauged) dilaton Weyl background. Using the off-shell map given in [15], a vector multiplet can be identified with fields in the gauged dilaton Weyl multiplet as

$$\begin{aligned} \mathcal{Y}^{ij} &\rightarrow \frac{1}{4}i\sigma^{-1}\bar{\psi}^i\psi^j - \frac{g}{2}\sigma^{-1}L^{ij}, & \rho &\rightarrow \sigma, \\ A_\mu &\rightarrow C_\mu, & \lambda^i &\rightarrow \psi^i, \end{aligned} \quad (4)$$

which gives rise to off-shell models that are purely expressed in terms of the fields of the (gauged) dilaton Weyl multiplet. By appropriately choosing primary composite linear multiplets, Eq. (3) becomes the building block for constructing various curvature-squared invariants.

In the superconformal approach, the off-shell formulation of minimal 5D supergravity can be achieved by coupling the standard Weyl multiplet to two off-shell conformal compensators: a vector multiplet and a linear multiplet. Within this setup, supersymmetric completions of the Weyl tensor squared and Ricci scalar squared were constructed in [26] and [14], respectively. The

Weyl tensor-squared invariant is based on a composite linear multiplet comprised solely of standard Weyl multiplet fields. To construct the Ricci scalar-squared invariant, one starts by defining a composite vector multiplet in terms of a linear multiplet. This composite vector multiplet is then substituted into the vector multiplet action obtained using (3).

While the Weyl tensor-squared and Ricci scalar-squared actions based on the dilaton Weyl multiplet were presented as ungauged models [14], they can simply be gauged by using the maps (1) and (4). At this point, it is worthwhile to mention that the on-shell results for these gauged actions differ from those presented by [27]. The reason is that Ref. [27] assumes that the map between the standard Weyl and the dilaton Weyl multiplet is not modified in the gauged case. However, as shown in [16] and presented as in Eq. (1), these expressions are indeed deformed. For completeness, we present the off-shell gauged results in Supplemental Material [28].

The third invariant necessary to obtain all the curvature-squared models in five dimensions was constructed as the Riemann-squared invariant in the ungauged dilaton Weyl basis in [13]. However, this model does not refer to the standard Weyl multiplet; hence, the prescription to obtain gauged models cannot be applied. Furthermore, the construction methodology cannot be extended to the gauged dilaton Weyl multiplet.

Alternatively, a third independent, locally superconformal invariant containing the Ricci tensor-squared term can

be constructed [17,29], which provides the correct basis to study gauged curvature-squared supergravity, as we shall discuss momentarily. In this case, the lowest component of the composite linear multiplet is given by the field L_{Log}^{ij} . This is obtained by making use of the standard Weyl multiplet and by acting with six Q -supersymmetry transformations on the field $\log \rho$, with ρ being the lowest component of a compensating vector multiplet [30]. The rest of the composite “log multiplet” is then obtained by acting with up to two more Q -supersymmetry transformations on L_{Log}^{ij} . Because of the complexity of computing up to eight supersymmetry transformations, the explicit form of the log multiplet, including all fermionic terms, has been obtained only recently with the aid of the CADABRA software [31,32]. These lengthy results will be published elsewhere [33]; see also [29,34] for the complete analysis of the gauged supergravity case. Inserting the resulting composite multiplet into (3) yields the explicit form of a new “log invariant” which will be presented in [33] in the standard Weyl basis. Then, the log invariant in the gauged dilaton Weyl background can be obtained by employing the map (1) and (4). For the purpose of this Letter, it suffices to present its bosonic sector in the gauge

$$\sigma = 1, \quad b_\mu = 0, \quad \psi^i = 0. \quad (5)$$

The gauged log invariant in the dilaton Weyl background, which includes a Ricci-squared term, reads

$$\begin{aligned} e^{-1} \mathcal{L}_{\text{Log}} = & -\frac{1}{6} R_{ab} R^{ab} + \frac{1}{24} R^2 + \frac{1}{6} R^{ab} G_{ab}^2 + \frac{1}{3} R H_{ab} G^{ab} - \frac{4}{3} R_{ab} H^{ac} G_c^b - \frac{1}{3} R H^2 - \frac{1}{12} \epsilon^{abcde} C_a V_{bc}^{ij} V_{deij} \\ & + \frac{1}{6} V^{abij} V_{abij} - 2(H^2)^2 + \frac{16}{3} H_{ab}^2 H^{ac} G_c^b - \frac{4}{3} H^2 H_{ab} G^{ab} + \frac{2}{3} H_{ab} H_{cd} (G^{ab} G^{cd} - 2G^{ac} G^{bd}) \\ & + \frac{2}{3} H^2 G^2 - \frac{4}{3} H^{2ab} G_{ab}^2 - \frac{1}{3} H_{ab} G^{ab} G^2 + G_{ab}^2 H^{ac} G_c^b - \frac{1}{48} (G^2)^2 - \frac{1}{24} G^4 - \frac{1}{6} \nabla_c G^{ac} \nabla^b G_{ab} \\ & + 2\nabla_a H_{bc} \nabla^{[a} H^{bc]} + \frac{1}{48} \epsilon^{abcde} \nabla^f G_{ef} (4H_{ab} - G_{ab})(4H_{cd} - G_{cd}) \\ & + \frac{g}{6} \left(RN - 4NH_{ab} G^{ab} - 2NG^2 + V_{ab}^{ij} L_{ij} (G^{ab} + 4H^{ab}) + 12NH^2 - 6\nabla^a \nabla_a N \right) \\ & - \frac{g^2}{24} \left(2RL^2 - L^2(G^2 - 4G^{ab} H_{ab} - 24H^2) + 4N^2 + 6\nabla^a L^{ij} \nabla_a L_{ij} \right) + \frac{2}{3} NL^2 g^3 + \frac{5}{24} L^4 g^4, \end{aligned} \quad (6)$$

where $V_{ab}^{ij} = 2\partial_{[a} V_{b]}^{ij} - 2V_{[a}^{k(i} V_{b]k}^{j)}$. Furthermore, we have used the following notations: $H^{ab} = -(1/12)\epsilon^{abcde} H_{cde}$, $H^2 = H^{ab} H_{ab}$, $G^2 = G^{ab} G_{ab}$, $H_{ab}^2 := H_a^c H_{bc}$, $G_{ab}^2 := G_a^c G_{bc}$, $G^4 = G^{2ab} G_{ab}^2$, and $H^4 = H^{2ab} H_{ab}^2$.

Note that the supersymmetric Riemann-squared invariant can be obtained by taking the following linear combination of the ungauged Weyl-squared invariant presented in [13] and setting $g = 0$ in the log invariant (6):

$$\mathcal{L}_{\text{Riem}}^2 = \mathcal{L}_{\text{Weyl}}^2 + 2\mathcal{L}_{\text{Log}}|_{g=0}. \quad (7)$$

The resulting action is identical to the one presented in [13] up to total derivatives.

Going on shell and dual CFT.—Now let us study a certain linear combination of the Einstein-Hilbert and all three curvature-squared invariants

$$(16\pi G)\mathcal{L}_{2\partial+4\partial} = \mathcal{L}_{\text{EH}} + \lambda_1 \mathcal{L}_{\text{Weyl}^2} + \lambda_2 \mathcal{L}_{\text{Log}} + \lambda_3 \mathcal{L}_{R^2}, \quad (8)$$

where G is Newton's constant and all the invariants are given in the gauged dilaton Weyl multiplet background. $\mathcal{L}_{\text{Weyl}^2}$ and \mathcal{L}_{R^2} , respectively, denote the Weyl tensor-squared and Ricci scalar-squared actions which are obtained by employing the maps (1) and (4) in the standard Weyl multiplet results of [14]. Their explicit form is not crucial here, but they are given in Supplemental Material [28] for the reader's convenience. The two-derivative Lagrangian \mathcal{L}_{EH} is obtained by using the linear multiplet action in the standard Weyl multiplet basis [15,16] and the sequential use of the maps (1) and (4). Note that, in this section, we rescaled the Lagrangians such that the coefficient of their leading curvature-squared term is normalized to unity. To go on shell, we fix the gauge according to (5) and break SU(2) down to U(1) by choosing

$$L_{ij} = \frac{1}{\sqrt{2}}\delta_{ij}L, \quad V_a^{ij} = V_a'^{ij} + \frac{1}{2}\delta^{ij}V_a. \quad (9)$$

Consequently, the two-derivative Lagrangian becomes

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{EH}} = & L \left(R - \frac{1}{2}G_{ab}G^{ab} + 4H_{ab}H^{ab} + 2V_a'^{ij}V_{ij}^a \right) \\ & + L^{-1}\partial_a L \partial^a L - 2L^{-1}E_a E^a - 2\sqrt{2}E_a V^a - 2N^2 L^{-1} \\ & - 4gC_a E^a - 2gNL - 4gN - \frac{1}{2}g^2 L^3 + 2g^2 L^2. \end{aligned} \quad (10)$$

From the total Lagrangian (8), several auxiliary fields can be solved from their field equations up to $\mathcal{O}(\lambda_i)$:

$$\begin{aligned} N &= -\frac{1}{2}gL(2+L) + \mathcal{O}(\lambda_i), \\ E_a &= \mathcal{O}(\lambda_i), \quad V_a'^{ij} = \mathcal{O}(\lambda_i). \end{aligned} \quad (11)$$

To arrive at the five-dimensional gauged minimal supergravity, we first dualize $B_{\mu\nu}$ to a new 1-form gauge field \tilde{C}_μ following the procedure in [15,16]. We then truncate the model consistently by imposing

$$L = 1 + \mathcal{O}(\lambda_i), \quad \tilde{C}_a = C_a + \mathcal{O}(\lambda_i). \quad (12)$$

Following (12), the field equation of E_{abc} now implies

$$V_a = -\frac{3}{\sqrt{2}}gC_a + \mathcal{O}(\lambda_i). \quad (13)$$

Plugging (11)–(13) back to the total Lagrangian (8), one obtains the on-shell theory up to first order in λ_i . It is

important to note that, in the procedure outlined above, the $\mathcal{O}(\lambda_i)$ terms arising from substituting (11)–(13) to the two-derivative action either vanish (proportional to the leading-order equations of motion of auxiliary fields) or can be removed by field redefinitions [35]. To recover the standard convention of minimal supergravity, we rescale the graviphoton and the U(1) coupling according to $C_a \rightarrow (1/\sqrt{3})C_a, g \rightarrow \sqrt{2}g$. To conclude, following [36], the resulting Lagrangian can be further simplified by redefining the metric and the U(1) gauge field. Eventually, the on-shell model is recast in the form below:

$$\begin{aligned} (16\pi G)e^{-1}\mathcal{L}_{2\partial+4\partial} = & c_0 R + 12c_1 g^2 - \frac{1}{4}c_2 G_{ab}G^{ab} \\ & + \frac{1}{12\sqrt{3}}c_3 \epsilon^{abcde} C_a G_{bc} G_{de} \\ & + \lambda_1 \mathcal{L}_{\text{GB}}|_{\text{onshell}}, \end{aligned} \quad (14)$$

where the various coefficients are

$$\begin{aligned} c_0 &= 1 + \left(\frac{28}{3}\lambda_1 - 20\lambda_2 - 4\lambda_3 \right) g^2, \\ c_1 &= 1 + \left(\frac{50}{9}\lambda_1 - \frac{28}{3}\lambda_2 + \frac{52}{3}\lambda_3 \right) g^2, \\ c_2 &= 1 + \left(\frac{64}{9}\lambda_1 - \frac{92}{3}\lambda_2 - \frac{76}{3}\lambda_3 \right) g^2, \\ c_3 &= 1 - 12(\lambda_1 + 3\lambda_2 + 3\lambda_3)g^2, \end{aligned} \quad (15)$$

and the on-shell Gauss-Bonnet invariant is given by

$$\begin{aligned} \mathcal{L}_{\text{GB}}|_{\text{onshell}} = & R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2 + \frac{1}{8}G^4 \\ & - \frac{1}{2}W_{abcd}G^{ab}G^{cd} + \frac{1}{2\sqrt{3}}\epsilon^{abcde}C_a R_{bc}{}^{fg}R_{defg}, \end{aligned} \quad (16)$$

where W_{abcd} is the Weyl tensor. This on-shell action is consistent with the generic result presented in [37] for the proper choice of parameters. Based on the on-shell model (14) we find that the AdS₅ radius receives corrections from the higher-derivative terms and is given by

$$\ell = g^{-1} \left(1 + \frac{8}{9}g^2\lambda_1 - \frac{16}{3}g^2\lambda_2 - \frac{32}{3}g^2\lambda_3 \right). \quad (17)$$

The effective Newton's constant from Eq. (14) is then

$$G_{\text{eff}} = G + G \left(-\frac{28}{3}\lambda_1 + 20\lambda_2 + 4\lambda_3 \right) g^2. \quad (18)$$

The AdS₅ vacuum preserves maximal eight supercharges [38,39], and the dual field theory should be a $D = 4, \mathcal{N} = 1$

CFT. Utilizing (17) and (18), the a and c Weyl anomaly coefficients of the dual CFT can be obtained via the standard holographic renormalization procedure [40,41]:

$$\begin{aligned} a &= \frac{\pi}{8g^3G} - \frac{9\pi(\lambda_2 + \lambda_3)}{2gG}, \\ c &= \frac{\pi}{8g^3G} + \frac{\pi(2\lambda_1 - 9\lambda_2 - 9\lambda_3)}{2gG}, \end{aligned} \quad (19)$$

using which one finds the results above are consistent with R -symmetry anomaly whose coefficients are related to those of the Weyl anomaly via [42,43]

$$5a - 3c = \frac{\pi c_3}{4g^3G}, \quad a - c = -\frac{\pi\lambda_1}{gG}. \quad (20)$$

Conclusions and outlook.—In this Letter, we provide the correct and complete basis to study curvature-squared gauged supergravity in five dimensions. Our new results show explicitly that in the ungauged case, only the Weyl-squared invariant is relevant in computing physical quantities as the rest can be redefined away, generalizing the nonrenormalization theorems for $D = 4$, $\mathcal{N} = 2$ [44]. Based on the new results, we successfully computed the anomaly coefficients governing dual four-dimensional SCFTs. As four-dimensional SCFTs are characterized by two anomaly coefficients, one would naturally anticipate the emergence of only two independent linear combinations among the three four-derivative couplings. Indeed, our analysis confirms this expectation, with λ_2 and λ_3 consistently appearing together in the anomaly coefficients as the combination $\lambda_2 + \lambda_3$. However, when examining the on-shell action (14), it becomes evident that λ_2 and λ_3 do not share this combination. Indeed, it can be shown that the log invariant and the Ricci scalar invariant both contribute to the thermodynamics of general non-Bogomol'nyi-Prasad-Sommerfield (BPS) AdS black holes, and the λ_2 , λ_3 dependence cannot be arranged into the combination $\lambda_2 + \lambda_3$ [45] as in some (BPS) quantities [37,46–48].

We can also generalize the results by coupling multiple vector multiplets which also enjoy an off-shell formulation. Given the simple form the 6D ungauged Gauss-Bonnet invariant [49–51] and the relation between dilaton Weyl multiplets in these two dimensions [13], it may be feasible to reformulate the 5D Gauss-Bonnet invariant into a more elegant expression that facilitates the construction of intriguing solutions. For instance, the nonexistence of supersymmetric AdS₅ black ring solutions in the two-derivative theory [52] raises the intriguing question of whether this situation changes in the presence of higher-derivative interactions. Our new invariants enable the computation of corrections to the entropy of (1/16)-BPS black holes [38,39,53–55], thereby extending the precision test of black hole microstate counting to the next-to-leading order. It is also interesting to extend the recently proposed

equivariant localization [56,57] beyond the leading two-derivative cases.

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Weyl multiplet based on an on-shell hypermultiplet in the background of the standard Weyl multiplet was recently constructed [20]. This might be used for alternative gaugings. In this Letter, we focus on the gauged (vector) dilaton Weyl multiplet of [16]; see also [17] for its construction in superspace.

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- with \mathcal{W} being the superfield describing the primary field strength of a compensating vector multiplet. In (21), $\nabla^{ij} = \nabla^{\alpha(i} \nabla_{\alpha}^{j)}$, and ∇_{α}^i is the conformal superspace spinor derivative. The primary component fields are then obtained by projection to their lowest $\theta = 0$ components, $\rho = \mathcal{W}|_{\theta=0}$ and $L_{\text{Log}}^{ij} = \mathcal{L}_{\text{Log}}^{ij}|_{\theta=0}$ —see Refs. [17,29] for detail.
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