

Single Transverse Spin Asymmetry as a New Probe of Standard-Model-Effective-Field-Theory Dipole Operators

Xin-Kai Wen,^{1,†} Bin Yan^{2,*}, Zhite Yu^{3,‡} and C.-P. Yuan^{3,§}

¹*School of Physics, Peking University, Beijing 100871, China*

²*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*

³*Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*



(Received 17 July 2023; revised 2 October 2023; accepted 14 November 2023; published 11 December 2023)

Electroweak dipole operators in the standard-model-effective-field theory (SMEFT) are important indirect probes of quantum effects of new physics beyond the standard model (SM), yet they remain poorly constrained by current experimental analyses for lack of interference with the SM amplitudes in constructing cross section observables. In this Letter, we point out that dipole operators flip fermion helicities and so are ideally studied through single transverse spin asymmetries. We illustrate this at a future electron-positron collider with transversely polarized beams, where such an effect exhibits as azimuthal $\cos\phi$ and $\sin\phi$ distributions which originate from the interference of the electron dipole operators with the SM and are linearly dependent on their Wilson coefficients. This new method can improve the current constraints on the electron dipole couplings by 1–2 orders of magnitude, without depending on other new physics operators, and can also simultaneously constrain both their real and imaginary parts, offering a new opportunity for probing potential CP -violating effects.

DOI: [10.1103/PhysRevLett.131.241801](https://doi.org/10.1103/PhysRevLett.131.241801)

Introduction.—The standard-model-effective-field theory (SMEFT) provides a powerful systematic bottom-up approach in particle physics to parametrize possible new physics (NP) effects beyond the standard model (SM) [1,2]. Measuring the corresponding Wilson coefficients to reveal hidden NP effects in the SM background has formed a major task of current and future colliders. Global SMEFT analyses with available data at the Large Hadron Collider (LHC) and other facilities have been carried out for various subsets of the dimension-six operators; see, e.g., Refs. [3–35]. Among them, the electroweak dipole operators of light fermions, which are likely to entail important information about heavy particle interactions, are poorly constrained experimentally [26,35–39]. Unlike the current interaction vertices in the SM, dipole operators flip fermion helicities, so their interference with the former is suppressed by the light fermion masses if one considers only unpolarized rate observables as in some typical analyses [25,40–48]. The leading contribution from such operators then starts at $\mathcal{O}(1/\Lambda^4)$, where Λ is the scale of the NP. This makes dipole operators harder to probe at a nominal experimental energy reach, should Λ be far from the electroweak scale.

To address this issue, in this Letter, we propose a novel approach to probing dimension-six light fermion dipole operators via single transverse spin asymmetries (SSAs). We shall demonstrate it with the measurement of electron dipole operators at a transversely polarized electron-positron collider, through the production processes of Zh , $Z\gamma$, W^+W^- , and $\mu^+\mu^-$ at a center of mass (c.m.) energy $\sqrt{s} = 250$ GeV for a future Higgs factory. Even though the dipole operators yield different helicity amplitude structures from the SM and so do not interfere with the latter in constructing unpolarized cross sections, the transverse beam spin effects introduce exactly the interference of opposite helicity states and so are directly probing the interference of the dipole and SM operators. This results in nontrivial azimuthal $\cos\phi$ and $\sin\phi$ distributions that are linearly dependent on the Wilson coefficients of the dipole operators and serve as useful probes at $\mathcal{O}(1/\Lambda^2)$. While such SSA observables are well known in the study of hadronic physics [49] and heavy fermion productions [50–53], it is the first time for them to be used to enhance the study of dipole operators at future electron-positron colliders.

Transverse spin and azimuthal distributions.—In the c.m. frame, the electron (positron) beam is chosen to be along the \hat{z} ($-\hat{z}$) axis. The spin-dependent amplitude square is

$$\Sigma(\phi, \mathbf{s}, \bar{\mathbf{s}}) = \rho_{\alpha_1\alpha'_1}(\mathbf{s})\rho_{\alpha_2\alpha'_2}(\bar{\mathbf{s}})\mathcal{M}_{\alpha_1\alpha_2}(\phi)\mathcal{M}_{\alpha'_1\alpha'_2}^*(\phi), \quad (1)$$

where the dependence on other kinematic variables has been suppressed. Here, α 's denote beam helicities, $\mathcal{M}_{\alpha_1\alpha_2}(\phi)$ is the

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

helicity amplitude of $e_{\alpha_1}^- e_{\alpha_2}^+$ scattering into the final state Zh , $Z\gamma$, W^+W^- , or $\mu^+\mu^-$ with a characteristic angle ϕ , which is taken as the azimuthal angle of Z , W^\pm , or μ^\pm in the e^+e^- c.m. frame, and we sum over repeated indices and final-state spins. The $\rho(s) = (1 + s \cdot \sigma)/2$ is the fermion spin density matrix with σ the Pauli matrices. This defines the spin vector s (\bar{s}) for the electron (positron) beam. With respect to the same \hat{x} and \hat{y} axes, we parametrize them as $s = (s_T, 0) = (b_T \cos \phi_0, b_T \sin \phi_0, 0)$ and $\bar{s} = (\bar{s}_T, 0) = (\bar{b}_T \cos \bar{\phi}_0, -\bar{b}_T \sin \bar{\phi}_0, 0)$, where b_T (\bar{b}_T) > 0 is the magnitude of the electron's (positron's) transverse spin and ϕ_0 ($\bar{\phi}_0$) its azimuthal orientation. We have neglected effects from nonzero helicity polarization. Similar azimuthal distributions can arise from the interplay of a longitudinally polarized beam and a transversely polarized beam; we leave that study for the future.

Since b_T and \bar{b}_T enter Eq. (1) in a linear way through the density matrices, $\Sigma(\phi, s, \bar{s})$ can be decomposed as

$$\Sigma(\phi, s, \bar{s}) = \Sigma_{UU} + b_T \Sigma_{TU}(\phi) + \bar{b}_T \Sigma_{UT}(\phi) + b_T \bar{b}_T \Sigma_{TT}(\phi). \quad (2)$$

The Σ_{UU} does not depend on the beam polarization nor the azimuthal angle ϕ , while Σ_{TU} and Σ_{UT} depend singly on the transverse spin of the electron and positron beams, respectively, Σ_{TT} captures the transverse beam spin correlation, and they contain nontrivial ϕ dependence. By using $\mathcal{M}_{\alpha_1\alpha_2}(\phi) = e^{i(\alpha_1 - \alpha_2)\phi} \mathcal{A}_{\alpha_1\alpha_2}$, with $\mathcal{A}_{\alpha_1\alpha_2}$ being independent of ϕ , the azimuthal dependence of $\Sigma_{TU, UT, TT}$ can be obtained from Eq. (1) as [54,55]

$$\begin{aligned} \Sigma_{TU} &= \frac{1}{2} \text{Re} [e^{i(\phi - \phi_0)} (\mathcal{A}_{+-} \mathcal{A}_{--}^* + \mathcal{A}_{++} \mathcal{A}_{+-}^*)], \\ \Sigma_{UT} &= \frac{1}{2} \text{Re} [e^{i(\phi - \bar{\phi}_0)} (\mathcal{A}_{+-} \mathcal{A}_{++}^* + \mathcal{A}_{--} \mathcal{A}_{-+}^*)], \\ \Sigma_{TT} &= \frac{1}{2} \text{Re} [e^{-i(\phi_0 - \bar{\phi}_0)} \mathcal{A}_{++} \mathcal{A}_{--}^* + e^{i(2\phi - \phi_0 - \bar{\phi}_0)} \mathcal{A}_{+-} \mathcal{A}_{-+}^*], \end{aligned} \quad (3)$$

where the subscript \pm refers to $\alpha = \pm 1/2$. So Σ_{TU} and Σ_{UT} contain $\cos \phi$ and $\sin \phi$ distributions, whereas Σ_{TT} gives $\cos 2\phi$ and $\sin 2\phi$ distributions.

Equation (3) entails the double roles of the transverse spins b_T and \bar{b}_T . First, they break the rotational invariance around the beam axis and induce nontrivial azimuthal distributions. Second, the single spin observables Σ_{TU} and Σ_{UT} arise from the interference of amplitudes that differ by one single electron helicity flip. In the SM, this can happen for the gauge interactions only via a mass insertion and is then suppressed by the electron mass. Within the SMEFT, in contrast, a helicity flip can be induced by dipole operators of the electron without such mass suppression. Hence, the unique $\cos \phi$ and $\sin \phi$ distributions are linearly dependent on the corresponding

Wilson coefficients. The Σ_{TT} part, on the other hand, is the interference of amplitudes that differ by double helicity flips and so can happen for both the SM and SMEFT operators, depending quadratically on the dipole couplings.

SSAs from electron dipole operators.—We consider the SMEFT dipole operator in this Letter:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{\sqrt{2}} \bar{\ell}_L \sigma^{\mu\nu} (g_1 \Gamma_B^e B_{\mu\nu} + g_2 \Gamma_W^e \sigma^a W_{\mu\nu}^a) \frac{H}{v^2} e_R + \text{H.c.}, \quad (4)$$

where ℓ_L (e_R) is the first-generation left (right) -handed lepton doublet (singlet) field and H is the Higgs doublet field, with $v = 246$ GeV being the vacuum expectation value. $B_{\mu\nu}$ and $W_{\mu\nu}^a$ are the gauge field strength tensors of the $U(1)_Y$ and $SU(2)$, respectively, with g_1 and g_2 the corresponding gauge coupling strengths. The dimensionless Wilson coefficients Γ_B^e and Γ_W^e quantify the dipole operator coupling strengths.

At tree level, the amplitudes $\mathcal{A}_{\alpha_1\alpha_2}$ in Eq. (3) are real up to possible complex phases of Γ_B^e and Γ_W^e . It is then evident that the real parts of Γ_B^e and Γ_W^e generate only $\cos(\phi - \phi_0)$ in Σ_{TU} and $\cos(\phi - \bar{\phi}_0)$ in Σ_{UT} . With nonzero imaginary parts, they would break the CP symmetry and also induce $\sin(\phi - \phi_0)$ and $\sin(\phi - \bar{\phi}_0)$. Beyond tree level, quantum loop effects would also generate imaginary parts to the amplitudes $\mathcal{A}_{\alpha_1\alpha_2}$, which also contribute to the sine (cosine) modulations for real (imaginary) parts of the dipole couplings; the magnitude is generally suppressed [51] and will be neglected in this Letter. For convenience, in the following discussion, we define the electron dipole couplings to the photon and Z boson, $\Gamma_\gamma^e = \Gamma_W^e - \Gamma_B^e$ and $\Gamma_Z^e = c_W^2 \Gamma_W^e + s_W^2 \Gamma_B^e$, respectively, where $s_W \equiv \sin \theta_W$ and $c_W \equiv \cos \theta_W$, with θ_W being the weak mixing angle.

Given the simple dependence on ϕ_0 and $\bar{\phi}_0$ in Eq. (3), it is sufficient to consider two experimental setups: (i) aligned-spin setup $\phi_0 = \bar{\phi}_0 = 0$ and (ii) opposite-spin setup $(\phi_0, \bar{\phi}_0) = (0, \pi)$. Then, the azimuthal distribution of a two-body final state i with transversely polarized lepton beams can be written as

$$\begin{aligned} \frac{2\pi d\sigma^i}{\sigma^i d\phi} &= 1 + A_R^i(b_T, \bar{b}_T) \cos \phi + A_I^i(b_T, \bar{b}_T) \sin \phi \\ &\quad + b_T \bar{b}_T B^i \cos 2\phi + \mathcal{O}(1/\Lambda^4), \end{aligned} \quad (5)$$

where, as examples, $i = Zh$, $Z\gamma$, W^+W^- , or $\mu^+\mu^-$ and, for simplicity, we have integrated over $\cos \theta$. The ϕ angle is taken as the azimuthal angle of $Z/W^\pm/\mu^\pm$ in the e^+e^- c.m. frame. For the $Z\gamma$ production, we require the transverse momentum of the photon $p_T^\gamma > 10$ GeV to avoid collinear singularity. As indicated from Eqs. (2) and (3), the coefficients $A_{R,I}^i$ depend linearly on b_T , \bar{b}_T , and the dipole couplings Γ_γ^e and Γ_Z^e , with A_R^i proportional to their real parts

and A_I^i their imaginary parts. To the accuracy of $\mathcal{O}(1/\Lambda^2)$, the coefficient B^i has no dependence on the dipole couplings. It receives contribution only from the SM (and other nondipole NP) interactions through the second term of Σ_{TT} in Eq. (3), while the first term vanishes in the massless lepton approximation. The azimuthal $\cos\phi$ and $\sin\phi$ distributions can, therefore, be used to give sensitive constraints on the dipole operators.

To extract the coefficients A_R^i and A_I^i , it is convenient to define the SSA observables:

$$\begin{aligned} A_{LR}^i &= \frac{\sigma^i(\cos\phi > 0) - \sigma^i(\cos\phi < 0)}{\sigma^i(\cos\phi > 0) + \sigma^i(\cos\phi < 0)} = \frac{2}{\pi} A_R^i, \\ A_{UD}^i &= \frac{\sigma^i(\sin\phi > 0) - \sigma^i(\sin\phi < 0)}{\sigma^i(\sin\phi > 0) + \sigma^i(\sin\phi < 0)} = \frac{2}{\pi} A_I^i, \end{aligned} \quad (6)$$

which can be referred to as “left-right” and “up-down” asymmetries, respectively, for the event distribution in the transverse plane. Here, $\sigma^i(\cos\phi > 0)$ is the integrated cross section with $\cos\phi > 0$, etc. By this construction, the coefficient B^i , which can receive contribution from the SM and other nondipole NP operators, does not contribute to the SSAs in Eq. (6), so that this method is exclusively probing the electron dipole operators. By combining the measurements of A_{LR} and A_{UD} , we can simultaneously determine the real and imaginary parts of the dipole couplings, allowing one to also probe CP -violating effects.

An interesting constraint follows from properties of the CP transformation, under which the initial state e^+ and e^- exchanges only their spins, and the final states W^+W^- and $\mu^+\mu^-$ are left invariant, whereas Zh and $Z\gamma$ flip their momentum directions. As a result,

$$A^i(s_T, \bar{s}_T; \Gamma_{Z,\gamma}^e) = \pm A^i(\bar{s}_T, s_T; \Gamma_{Z,\gamma}^{e*}), \quad (7)$$

which holds for both A_R^i and A_I^i , with “+” for $i = W^+W^-$ or $\mu^+\mu^-$ and “-” for $i = Zh$ or $Z\gamma$. This constrains the coefficients A_R^i and A_I^i to have definite dependence on the beam spins, namely,

$$\begin{aligned} (A_R^{WW,\mu\mu}, A_I^{Zh,Z\gamma}) &\propto s_T + \bar{s}_T, \\ (A_I^{WW,\mu\mu}, A_R^{Zh,Z\gamma}) &\propto s_T - \bar{s}_T. \end{aligned} \quad (8)$$

Therefore, we can take advantage of both beams being polarized to enhance the signals of SSAs. For probing $A_R^{WW,\mu\mu}$ and $A_I^{Zh,Z\gamma}$, we shall use the aligned-spin setup, while $A_I^{WW,\mu\mu}$ and $A_R^{Zh,Z\gamma}$ are better measured with the opposite-spin setup.

Numerical results and discussion.—Now we present the projected constraints of the SSAs in Eq. (6) on the dipole couplings $\Gamma_{Z,\gamma}^e$ at an e^+e^- collider. Since each final-state particle is color neutral, the parton showering and hadronization effects in their hadronic decay channels will not be

correlated with the overall azimuthal distributions and so will not affect the SSAs, nor will the detector effects, as we have verified explicitly.

The Zh and $Z\gamma$ events at lepton colliders can be identified by the recoil mass method. We combine all decay modes of the Z boson in the $Z\gamma$ channel, and, for the Zh process, we include all decay modes of the Higgs boson and $Z \rightarrow \ell^+\ell^-/jj$ with $\ell = e, \mu$, and τ . To measure the ϕ distribution in the W^+W^- production, we select the decay channel $W^+W^- \rightarrow jj\ell^\pm\nu_\ell(\bar{\nu}_\ell)$. The kinematic cuts for the decay products of final states can change the total event numbers but shall not significantly alter the azimuthal angle distribution and will be neglected in the following analysis. The statistical uncertainties of the SSAs in Eq. (6) are given by

$$\delta A_{LR,UD}^i = \sqrt{\frac{1 - (A_{LR,UD}^i)^2}{N^i}} \simeq \frac{1}{\sqrt{N^i}}, \quad (9)$$

where N^i is the number of the selected events of the final state i for a certain spin setup after the kinematic cuts and the “ \simeq ” in the second step takes the approximation $A_{LR,UD}^i \simeq 0$ in the SM. We assume the systematic uncertainties are canceled in the SSAs in this Letter [56].

While there are several options in the market for a future lepton collider with a c.m. energy $\sqrt{s} = 240\text{--}250$ GeV, such as the Circular Electron Positron Collider (CEPC) [57], the Future Circular Collider (FCC-ee) [58], and the International Linear Collider (ILC) [56], none of them have explored the potential of using transversely polarized beams. As a benchmark study, we adopt the polarization parameters $(b_T, \bar{b}_T) = (0.8, 0.3)$ for our analysis [56].

Figure 1 presents the expected constraints on the real and imaginary parts of the dipole couplings $\Gamma_{Z,\gamma}^e$ at 68% confidence level (C.L.) for the Zh (red bands), $Z\gamma$ (green bands), W^+W^- (blue bands), and $\mu^+\mu^-$ (purple bands) channels with an integrated luminosity of $\mathcal{L} = 5$ ab $^{-1}$ at $\sqrt{s} = 250$ GeV for each spin setup, under the SM assumption. The linear dependence of the SSAs in Eq. (6) on the dipole couplings lead to linearly shaped confidence regions, among which the Zh process is solely constraining Γ_Z^e , whereas the other channels are probing almost orthogonal linear combinations of Γ_Z^e and Γ_γ^e . As expected from Eq. (8), the constraints of the aligned (opposite) -spin setup on the real parts of Γ_Z^e and Γ_γ^e are the same as the opposite (aligned) -spin setup on their imaginary parts. It also confirms that the aligned (opposite) -spin setup gives better constraints on $\text{Re}\Gamma_{Z,\gamma}^e$ for the W^+W^- and $\mu^+\mu^-$ (Zh and $Z\gamma$) channels and conversely for $\text{Im}\Gamma_{Z,\gamma}^e$.

For the $Z\gamma$ and $\mu^+\mu^-$ processes, parity symmetry requires the dependence of A_R (A_I) on $\text{Re}\Gamma_\gamma^e$ ($\text{Im}\Gamma_\gamma^e$) to be also proportional to the vector component of the SM $Ze\bar{e}$ coupling, $g_V^e = -1/2 + 2s_W^2 \simeq -0.038$, which is nearly

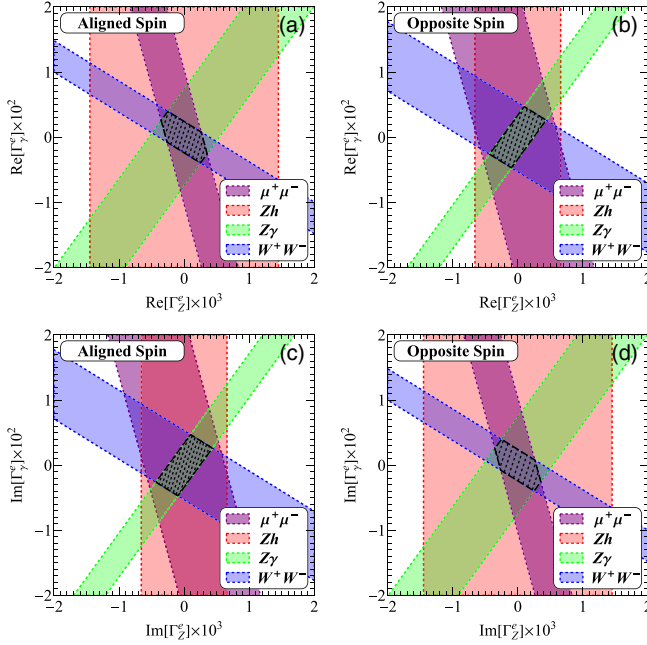


FIG. 1. Expected constraints on the real and imaginary parts of the electron dipole couplings $\Gamma_{Z,\gamma}^e$ from the SSAs $A_{LR,UD}$. The black bands are combinations of the four production channels.

zero. As a result, the sensitivity to Γ_Z^e is much stronger than Γ_γ^e . A similar conclusion holds in the W^+W^- channel due to the larger WWZ coupling than $WW\gamma$ and that the translation $\Gamma_W^e = \Gamma_Z^e + s_W^2 \Gamma_\gamma^e$ gives a heavier weight on Γ_Z^e . After combining the four processes (black bands in Fig. 1), we have the same constraints for the real and imaginary parts of the dipole couplings, with typical upper limits of the order of $\mathcal{O}(0.01\%)$ for Γ_Z^e and $\mathcal{O}(0.1\%)$ for Γ_γ^e .

Without the use of such SSAs, dipole operators contribute only to cross sections at $\mathcal{O}(1/\Lambda^4)$ and are also hard to be disentangled from other NP operators, which severely limits the ability to constrain their Wilson coefficients. This is true for the studies in the literature using the Drell-Yan process at the LHC [26] and Z-pole data at the LEP [36,59], which constrain $|\Gamma_{Z,\gamma}^e|$ within $\mathcal{O}(1\%)$ only when one operator is considered at a time. They also lack the sensitivity to distinguish the real and imaginary parts of dipole couplings. This makes the SSAs at the transversely polarized e^+e^- collider a unique opportunity to constrain the dipole operators.

Before closing this section, we note that the real and imaginary parts of Γ_γ^e can contribute to the electron's anomalous magnetic (MDM) and electric dipole moment (EDM), respectively, and have been severely constrained by the experimental measurements. It was found that $\text{Re}\Gamma_\gamma^e \sim \mathcal{O}(10^{-6})$ and $\text{Im}\Gamma_\gamma^e \sim \mathcal{O}(10^{-13})$ [60], for arising from tree-level contributions. It is also important to note that the photon and Z dipole operators can mix under renormalization at the loop level. As a result, the measurements of electron MDM and EDM can provide strong

constraints on the Z dipole interactions when considering one operator at a time, which yields $\text{Re}\Gamma_Z^e \sim \mathcal{O}(10^{-2})$ and $\text{Im}\Gamma_Z^e \sim \mathcal{O}(10^{-9})$ [60–62]. However, the conclusions drawn from these constraints heavily depend on the theoretical assumptions made in the analysis. For instance, the presence of four-fermion operators can also contribute to the electron MDM and EDM at the loop level, which could potentially affect the accuracy and reliability of the extracted values from data. In contrast, the measurements of SSAs proposed in this Letter do not depend on other dimension-six SMEFT operators except the dipole operators listed in Eq. (4). Furthermore, the expected limits for the parameter $\text{Re}\Gamma_Z^e$ (cf. Fig. 1) would be 2 orders of magnitude stronger than the constraints derived from the low-energy measurements of the electron MDM. Finally, we notice that the Yukawa-type operator $H^\dagger H \bar{\ell}_L H e_R / \Lambda^2$ could also contribute to the W^+W^- process, whose effect, however, is found to be negligibly small, as compared to the effect of dipole operators.

Conclusions.—In this Letter, we proposed a novel approach to sensitively probing new physics dipole operators via single transverse spin asymmetry observables, using as a concrete example the constraining power of such observables on the electron dipole operators at a transversely polarized electron-positron collider. By inducing a single fermion helicity flip, the interference of such operators with the SM generates unique $\cos\phi$ and $\sin\phi$ distributions that are linearly dependent on their Wilson coefficients, with no contribution from the SM and other new physics operators. This gives much stronger sensitivity to the electron dipole operators than other approaches in Drell-Yan and Z-pole processes, at the LHC and LEP, respectively, by 1–2 orders of magnitude. It also allows one to simultaneously determine the real and imaginary parts of those couplings, giving the opportunity to directly study potential CP -violating effects. Our approach can be readily applied to similar studies at a muon collider [63,64] and forthcoming electron-ion collider [65] when beams are transversely polarized. With measurements of final-state spins [66,67], it can also be applied to the studies at unpolarized colliders such as the LHC.

X.-K. W. is supported in part by the National Science Foundation of China under Grants No. 11725520, No. 11675002, and No. 12235001. B. Y. is supported by the IHEP under Contract No. E25153U1. Z. Y. and C.-P. Y. are supported by the U.S. National Science Foundation under Grants No. PHY-2013791 and No. PHY-2310291, and C.-P. Y. is grateful for the support from the Wu-Ki Tung endowed chair in particle physics.

*Corresponding author: yanbin@ihep.ac.cn

†xinkaiwen@pku.edu.cn

‡yuzhite@msu.edu

§yuanch@msu.edu

- [1] W. Buchmuller and D. Wyler, *Nucl. Phys.* **B268**, 621 (1986).
- [2] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, *J. High Energy Phys.* **10** (2010) 085.
- [3] C. Englert, A. Freitas, M. M. Mühlleitner, T. Plehn, M. Rauch, M. Spira, and K. Walz, *J. Phys. G* **41**, 113001 (2014).
- [4] A. Falkowski, *Pramana* **87**, 39 (2016).
- [5] T. Corbett, O. J. P. Eboli, D. Goncalves, J. Gonzalez-Fraile, T. Plehn, and M. Rauch, *J. High Energy Phys.* **08** (2015) 156.
- [6] Q.-H. Cao, B. Yan, J.-H. Yu, and C. Zhang, *Chin. Phys. C* **41**, 063101 (2017).
- [7] Q.-H. Cao and B. Yan, *Phys. Rev. D* **92**, 094018 (2015).
- [8] Q.-H. Cao, B. Yan, D.-M. Zhang, and H. Zhang, *Phys. Lett. B* **752**, 285 (2016).
- [9] Q.-H. Cao, G. Li, B. Yan, D.-M. Zhang, and H. Zhang, *Phys. Rev. D* **96**, 095031 (2017).
- [10] V. Cirigliano, W. Dekens, J. de Vries, and E. Mereghetti, *Phys. Rev. D* **94**, 034031 (2016).
- [11] S. Alioli, W. Dekens, M. Girard, and E. Mereghetti, *J. High Energy Phys.* **08** (2018) 205.
- [12] G. Durieux, M. Perelló, M. Vos, and C. Zhang, *J. High Energy Phys.* **10** (2018) 168.
- [13] C. Degrande, F. Maltoni, K. Mimasu, E. Vryonidou, and C. Zhang, *J. High Energy Phys.* **10** (2018) 005.
- [14] E. Vryonidou and C. Zhang, *J. High Energy Phys.* **08** (2018) 036.
- [15] G. Durieux, J. Gu, E. Vryonidou, and C. Zhang, *Chin. Phys. C* **42**, 123107 (2018).
- [16] Q.-H. Cao, L.-X. Xu, B. Yan, and S.-H. Zhu, *Phys. Lett. B* **789**, 233 (2019).
- [17] J. De Blas, G. Durieux, C. Grojean, J. Gu, and A. Paul, *J. High Energy Phys.* **12** (2019) 117.
- [18] I. Brivio, S. Bruggisser, F. Maltoni, R. Moutafis, T. Plehn, E. Vryonidou, S. Westhoff, and C. Zhang, *J. High Energy Phys.* **02** (2020) 131.
- [19] N. P. Hartland, F. Maltoni, E. R. Nocera, J. Rojo, E. Slade, E. Vryonidou, and C. Zhang, *J. High Energy Phys.* **04** (2019) 100.
- [20] Y. Du, H.-L. Li, J. Tang, S. Vihonen, and J.-H. Yu, *J. High Energy Phys.* **03** (2021) 019.
- [21] S. Alioli, R. Boughezal, E. Mereghetti, and F. Petriello, *Phys. Lett. B* **809**, 135703 (2020).
- [22] V. Cirigliano, K. Fuyuto, C. Lee, E. Mereghetti, and B. Yan, *J. High Energy Phys.* **03** (2021) 256.
- [23] J. J. Ethier, G. Magni, F. Maltoni, L. Mantani, E. R. Nocera, J. Rojo, E. Slade, E. Vryonidou, and C. Zhang (SMEFIt Collaboration), *J. High Energy Phys.* **11** (2021) 089.
- [24] V. Miralles, M. M. López, M. M. Llácer, A. Peñuelas, M. Perelló, and M. Vos, *J. High Energy Phys.* **02** (2022) 032.
- [25] B. Yan, *Phys. Lett. B* **822**, 136709 (2021).
- [26] R. Boughezal, E. Mereghetti, and F. Petriello, *Phys. Rev. D* **104**, 095022 (2021).
- [27] V. Cirigliano, W. Dekens, J. de Vries, K. Fuyuto, E. Mereghetti, and R. Ruiz, *J. High Energy Phys.* **08** (2021) 103.
- [28] Q.-H. Cao, H.-r. Jiang, and G. Zeng, *Chin. Phys. C* **45**, 093110 (2021).
- [29] Y. Du, H.-L. Li, J. Tang, S. Vihonen, and J.-H. Yu, *Phys. Rev. D* **105**, 075022 (2022).
- [30] Y. Liao, X.-D. Ma, and H.-L. Wang, *Chin. Phys. C* **45**, 073102 (2021).
- [31] Y. Liu, Y. Wang, C. Zhang, L. Zhang, and J. Gu, *Chin. Phys. C* **46**, 113105 (2022).
- [32] J. de Blas, Y. Du, C. Grojean, J. Gu, V. Miralles, M. E. Peskin, J. Tian, M. Vos, and E. Vryonidou, in *Proceedings of the Snowmass 2021* (2022); [arXiv:2206.08326](https://arxiv.org/abs/2206.08326).
- [33] S. Dawson and P. P. Giardino, *Phys. Rev. D* **105**, 073006 (2022).
- [34] A. Greljo, A. Palavrić, and A. E. Thomsen, *J. High Energy Phys.* **10** (2022) 010.
- [35] C. Grunwald, G. Hiller, K. Kröninger, and L. Nollen, [arXiv:2304.12837](https://arxiv.org/abs/2304.12837).
- [36] R. Escribano and E. Masso, *Nucl. Phys.* **B429**, 19 (1994).
- [37] E. da Silva Almeida, N. Rosa-Agostinho, O. J. P. Éboli, and M. C. Gonzalez-Garcia, *Phys. Rev. D* **100**, 013003 (2019).
- [38] Q.-H. Cao, H.-R. Jiang, B. Li, Y. Liu, and G. Zeng, *Chin. Phys. C* **45**, 093108 (2021).
- [39] R. Boughezal, D. de Florian, F. Petriello, and W. Vogelsang, *Phys. Rev. D* **107**, 075028 (2023).
- [40] M. Beneke, D. Boito, and Y.-M. Wang, *J. High Energy Phys.* **11** (2014) 028.
- [41] N. Craig, J. Gu, Z. Liu, and K. Wang, *J. High Energy Phys.* **03** (2016) 050.
- [42] S.-F. Ge, H.-J. He, and R.-Q. Xiao, *J. High Energy Phys.* **10** (2016) 007.
- [43] H. Khanpour and M. Mohammadi Najafabadi, *Phys. Rev. D* **95**, 055026 (2017).
- [44] W. H. Chiu, S. C. Leung, T. Liu, K.-F. Lyu, and L.-T. Wang, *J. High Energy Phys.* **05** (2018) 081.
- [45] G. Durieux, C. Grojean, J. Gu, and K. Wang, *J. High Energy Phys.* **09** (2017) 014.
- [46] T. Barklow, K. Fujii, S. Jung, R. Karl, J. List, T. Ogawa, M. E. Peskin, and J. Tian, *Phys. Rev. D* **97**, 053003 (2018).
- [47] H.-D. Li, C.-D. Lü, and L.-Y. Shan, *Chin. Phys. C* **43**, 103001 (2019).
- [48] K. Rao, S. D. Rindani, and P. Sarmah, *Nucl. Phys.* **B950**, 114840 (2020).
- [49] D. Boer, [arXiv:hep-ph/0312149](https://arxiv.org/abs/hep-ph/0312149).
- [50] G. L. Kane, J. Pumplin, and W. Repko, *Phys. Rev. Lett.* **41**, 1689 (1978).
- [51] G. L. Kane, G. A. Ladinsky, and C. P. Yuan, *Phys. Rev. D* **45**, 124 (1992).
- [52] C. P. Yuan, *Phys. Rev. D* **45**, 782 (1992).
- [53] W. G. D. Dharmaratna and G. R. Goldstein, *Phys. Rev. D* **53**, 1073 (1996).
- [54] K.-i. Hikasa, *Phys. Rev. D* **33**, 3203 (1986).
- [55] G. Moortgat-Pick *et al.*, *Phys. Rep.* **460**, 131 (2008).
- [56] H. Baer *et al.* (ILC Collaboration), [arXiv:1306.6352](https://arxiv.org/abs/1306.6352).
- [57] M. Dong *et al.* (CEPC Study Group), [arXiv:1811.10545](https://arxiv.org/abs/1811.10545).
- [58] A. Abada *et al.* (FCC Collaboration), *Eur. Phys. J. Spec. Top.* **228**, 261 (2019).
- [59] S. Schael *et al.* (ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group), *Phys. Rep.* **427**, 257 (2006).
- [60] J. Aebischer, W. Dekens, E. E. Jenkins, A. V. Manohar, D. Sengupta, and P. Stoffer, *J. High Energy Phys.* **07** (2021) 107.

- [61] R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, *J. High Energy Phys.* **04** (2014) 159.
- [62] J. Kley, T. Theil, E. Venturini, and A. Weiler, *Eur. Phys. J. C* **82**, 926 (2022).
- [63] H. Al Ali *et al.*, *Rep. Prog. Phys.* **85**, 084201 (2022).
- [64] C. Accettura *et al.*, *Eur. Phys. J. C* **83**, 864 (2023).
- [65] R. Abdul Khalek *et al.*, *Nucl. Phys.* **A1026**, 122447 (2022).
- [66] Z. Yu and C. P. Yuan, *Phys. Rev. Lett.* **129**, 112001 (2022).
- [67] Z. Yu, K. A. Mohan, and C. P. Yuan, arXiv:2211.00845.