

Dynamical Localization Transition of String Breaking in Quantum Spin Chains

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The fission of a string connecting two charges is an astounding phenomenon in confining gauge theories. The dynamics of this process have been studied intensively in recent years, with plenty of numerical results yielding a dichotomy: the confining string can decay relatively fast or persist up to extremely long times. Here, we put forward a dynamical localization transition as the mechanism underlying this dichotomy. To this end, we derive an effective string breaking description in the light-meson sector of a confined spin chain and show that the problem can be regarded as a dynamical localization transition in Fock space. Fast and suppressed string breaking dynamics are identified with delocalized and localized behavior, respectively. We then provide a further reduction of the dynamical string breaking problem onto a quantum impurity model, where the string is represented as an “impurity” immersed in a meson bath. It is shown that this model features a localization-delocalization transition, giving a general and simple physical basis to understand the qualitatively distinct string breaking regimes. These findings are directly relevant for a wider class of confining lattice models in any dimension and could be realized on present-day Rydberg quantum simulators.

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Introduction.—The efficient implementation of gauge theories is a central target in quantum simulation [1–6], with some remarkable experimental realizations achieved in recent years [7–18]. However, the intrinsic structure of gauge theory still poses formidable technical challenges. Simultaneously, quantum spin chains, which are more amenable to quantum simulation, have been shown to be a versatile platform to emulate lattice gauge theory phenomenology. This has led to recent intensive efforts to investigate the structure of the gauge vacuum and out-of-equilibrium transport properties under the influence of confinement in this setting [19–40]. Yet, various aspects of such phenomena remain to be elucidated. In particular, numerical studies of dynamical string breaking—where a string connecting two charges decays due to pair production [41,42]—suggest a dichotomy for the fate of the confining string: its fission can occur relatively fast or be substantially delayed.

In this Letter, we discuss how these observations can be interpreted in terms of an *underlying* dynamical localization transition. In this picture, the *localized phase* corresponds to a regime with a long-lived (prethermal) string,

while the *delocalized phase* to fast string breaking. First, we show via exact diagonalization in quantum Ising chains that two qualitatively different string dynamics are separated by a sharp threshold in the long-time behavior of dynamical quantities. In particular, we study the survival probability and the half-chain entanglement entropy, with the former quantity serving as a direct diagnostics of string breaking. We then derive an effective model for the breaking of a short string by projecting onto a reduced subspace that captures resonant decay channels in the limit of vanishing transverse field. Within this effective description, string breaking can be understood as a dynamical localization problem in Fock space. Next, this description is heuristically generalized to a quantum impurity model, where the string is effectively represented by a few-level system coupled to a meson bath. We show that this model features a dynamical localization-delocalization transition, with both sides of the transition explaining the observed string breaking regimes. This description, independent of microscopic details, provides a general and simple physical basis to understand dynamical string breaking. Finally, we discuss how our results can be applied to a wider class of confining lattice models in any dimension, and potential implementations with Rydberg quantum simulators.

String dynamics in quantum Ising chains.—We consider the quantum Ising model in both transverse (h_x) and longitudinal (h_z) fields, whose Hamiltonian for L spins on the ring reads

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$$\hat{H} = -J \sum_{i=1}^L \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - h_x \sum_{i=1}^L \hat{\sigma}_i^x - h_z \sum_{i=1}^L \hat{\sigma}_i^z, \quad (1)$$

where $\hat{\sigma}_i^{x/z}$ are the Pauli matrices at site i , and $J > 0$ is the strength of a ferromagnetic coupling. The model (1) is of paramount importance in various fields—from statistical mechanics and condensed matter [43,44] to high-energy physics [45–47]. Further, it can be naturally realized in present-day Rydberg quantum simulators [48–53], and solid-state materials [54]. Both integrability and \mathbb{Z}_2 symmetry are broken by a finite h_z , which induces a confining potential between pairs of domain wall (DW) excitations (provided that $h_x < J$). In this scenario, pairs of DWs form bound, mesonlike states. String breaking dynamics can then be probed by studying the stability of one such object under the unitary evolution generated by the Hamiltonian (1). Below, we review the main aspects of this process (see also Ref. [24]), in the confining regime with controlled quantum fluctuations $h_x \ll J$.

A dichotomy between distinct string breaking dynamics is revealed in a simple experimentally feasible quantum quench protocol. The system is initially prepared in a state with an Ising electric-field string of $\ell \downarrow$ spins (in the σ^z basis) that connects two DWs, on top of the vacuum, i.e., $|\psi_{\text{string}}(\ell)\rangle \equiv |\cdots \uparrow \downarrow_{i_0} \downarrow \cdots \downarrow \downarrow_{(i_0+\ell-1)} \uparrow \cdots\rangle$. Next, the real-time evolution of the system in Eq. (1) is studied at finite h_x/J and h_z/J . Two qualitatively different dynamical string breaking scenarios are illustrated in Figs. 1(a) and 1(b), for an initial string of length $\ell = 4$. The dynamics are shown in terms of the local DW density, $\nu_i(t) = \frac{1}{2} \langle \hat{I} - \hat{\sigma}_i^z(t) \hat{\sigma}_{i+1}^z(t) \rangle$, defined on the bonds between consecutive lattice sites. In Fig. 1(a), a rapid production of new DW pairs occurs inside the string, eventually leading to its decay and emission of lighter mesons. A subsequent proliferation of DW pairs throughout the whole chain eventually restores translation invariance, in agreement with the fact that the system (1) is ergodic and thermalizing at late times [55]. This *fast* string breaking dynamics can be understood as a consequence of underlying resonances that arise for commensurable (J, h_z) [24,56]. In sharp contrast, the rapid string breaking dynamics is surprisingly absent in Fig. 1(b), up to the accessed long time $O(10^2 J^{-1})$, which is also beyond the timescale for light meson kinetics $t \gg J/h_x^2$. Based on general thermalization arguments (as mentioned above), the latter regime must be understood only as a prethermal phenomenon [34].

The scenarios above have been observed in both quantum spin models [21,24,25] and low-dimensional lattice gauge theories [57–62]. However, a general picture of how these systems cross from one regime over to the other remains to be provided. As a first step in this quest, we study the long-time behavior of the string survival probability

$$P(t) = |\langle \psi_{\text{string}} | \psi(t) \rangle|^2, \quad (2)$$

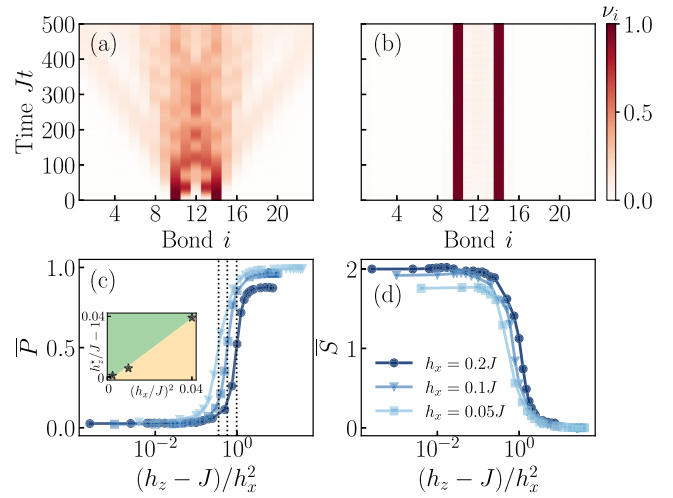


FIG. 1. String breaking dynamics in quantum Ising chains. (a) [$h_z = J$] Fast and (b) [$h_z = 1.1J$] suppressed string breaking dynamics in quantum Ising chains [Eq. (1)], in terms of the spatiotemporally resolved domain wall density $\nu_i(t)$. In both cases, $L = 24$, $h_x = 0.2J$, and $\ell = 4$ (initial string length). Long-time behavior of (c) the survival probability [Eq. (2)] and (d) half-chain entanglement entropy [Eq. (3)], for various values of the magnetic fields and $L = 16$, $\ell = 4$. A sharp threshold, defined by the point h_z^*/J where $\bar{P} = 0.5$ [dotted lines and inset in (c)], separates the two string breaking regimes. Results obtained via exact diagonalization.

and the half-chain entanglement entropy

$$S(t) = S[\hat{\rho}_A(t)] = -\text{Tr}_A[\hat{\rho}_A(t) \ln \hat{\rho}_A(t)], \quad (3)$$

where $|\psi(t)\rangle$ is the time-evolved many-body wave function and $\hat{\rho}_A(t) = \text{Tr}_B[|\psi(t)\rangle\langle\psi(t)|]$ is the reduced density matrix computed on one half of the chain (cutting through the middle of the string and the opposite point on the periodic chain). We compute long-time averages as $\bar{\mathcal{O}} = (1/t_f - t_i) \int_{t_i}^{t_f} \mathcal{O}(t) dt$. In our calculations we take $Jt_f = 10^4$, and $Jt_i = 3Jt_{\text{sb}}$, where $Jt_{\text{sb}} \equiv [\pi/2(h_x/J)^2]$ is a typical timescale for string breaking [24]. The long-time averages of the quantities in Eqs. (2) and (3) are shown in Figs. 1(c) and 1(d), for various values of h_z/J and h_x/J . We observe a sharp threshold—defined by the point where $\bar{P} = 0.5$ —which roughly scales linearly with $(h_x/J)^2$ [inset in Fig. 1(c)], and separates a regime where the string breaks ($\bar{P} \sim 0$) from one in which it persists ($\bar{P} \sim 1$) up to the accessed timescales. The behavior of \bar{S} shows that string breaking is characterized by a significant amount of entanglement, while in *suppressed* string breaking dynamics entanglement production is strongly diminished.

String breaking as a localization problem in Fock space.—We now derive an effective description of the above phenomenology. Let us fix $\ell = 4$, as before. For the considered parameter regime $h_x \ll h_z \sim J$, one can systematically project out sectors of the Hilbert space that do

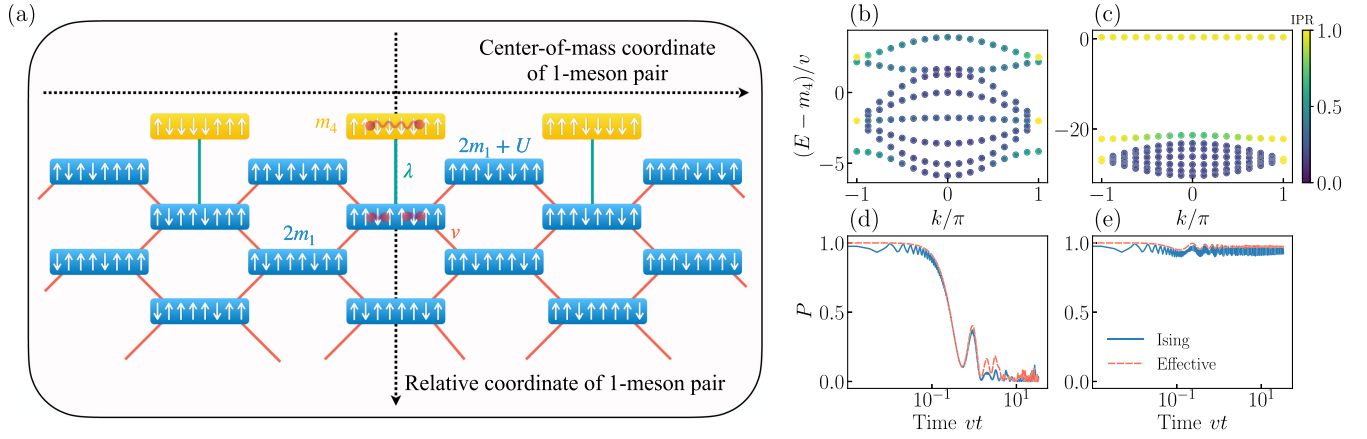


FIG. 2. Effective graph model for string breaking dynamics. (a) Schematic of the effective model for a short string ($\ell = 4$). Spin configurations in yellow represent string states (rest mass $m_4 = 12J$), whereas configurations in blue represent one-meson pairs with energy $2m_1$ and hopping amplitude $v = h_x^2/3J$ (red bonds). These are the configurations involved, to leading order, in the resonant decay of the string (green bonds). For illustration, confined DW pairs are depicted in some configurations as red dots joined by a wiggly line. String breaking can thus be thought of as a diffusion problem in the Fock-space graph. (b),(c) Energy spectrum of the effective model for $h_z = J$ and $h_z = 1.02J$, respectively. In (b) all bands have a similar energy, while in (c) there is a gap between the “string” band and the continuum of one-meson pairs. Color bar shows the IPR of individual eigenstates [Eq. (5)], exhibiting a strong localization of the string modes in the latter case. (d),(e) Time evolution of the survival probability [Eq. (2)], for the respective parameters, both in the effective and full Ising model. Note the log scale in the horizontal axis and the time in units of v . Parameters: $L = 16$, $h_x = 0.1J$.

not participate in resonant decay channels, by applying a Schrieffer-Wolff transformation [63,64] to (1), see Ref. [65] for details. Here, the relevant physical subspace is formed by the direct sum of the “string” sector and the “one-meson pair” sector, see Fig. 2(a). The former sector is spanned by the kets $|S_j\rangle = |\cdots \uparrow \uparrow \downarrow_j \downarrow \downarrow \downarrow \uparrow \uparrow \cdots\rangle$, with a string of size $\ell = 4$, labeled by the site index j of the first \downarrow spin. The second sector comprises configurations with exactly two one-meson particles: $|j, d\rangle = |\cdots \uparrow \uparrow \downarrow_j \uparrow \cdots \uparrow \downarrow_{(j+d)} \uparrow \uparrow \cdots\rangle$, where $2 \leq d \leq L/2$ (L even), is the relative distance between the two \downarrow spins.

The resulting effective model reads

$$\hat{H}_{\text{eff}} = \hat{H}_{\text{string}} + \hat{H}_{\text{mesons}} + \hat{H}_\lambda, \quad (4)$$

where \hat{H}_{string} gives the string rest mass [$\mathcal{E}_s \equiv m_4 = 12J$]; \hat{H}_{mesons} contains terms for hopping [$v = h_x^2/(3J)$], mass [$2m_1 = \mathcal{E}_s - 2v$], and repulsive contact interaction [$U = 9v/2$] of the one-meson particles; and \hat{H}_λ couples the two relevant sectors with amplitude $\lambda = -3v$; see Fig. 2(a) and Ref. [65] for details. The latter term is responsible for the processes of pair creation and recombination, and therefore, crucial for string breaking.

The energy spectrum of this model is shown in Figs. 2(b) and 2(c), for different choices of parameters. In Fig. 2(b) all bands are close in energy, while in Fig. 2(c) a large gap separates an isolated band (associated to string modes) from the rest. In the latter case, string modes are strongly localized. This is quantified by the color bar in Figs. 2(b) and 2(c), which shows the value of the inverse participation ratio (IPR) of individual energy eigenstates:

$$\text{IPR}(n) = \sum_a |\langle a|n\rangle|^4, \quad (5)$$

where $\{|n\rangle\}$ are eigenstates of \hat{H}_{eff} and $\{|a\rangle\}$ preferential basis states. Localized behavior of $|n\rangle$ occurs when $\text{IPR}(n) \simeq 1$, while $\text{IPR}(n)$ vanishes as $1/D$ in the maximally delocalized case, where D is the Hilbert space dimension [66].

The evolution of the survival probability [Eq. (2)], corresponding to the two cases above, is shown in Figs. 2(d) and 2(e). While in the former case, the string eventually breaks ($P \sim 0$), in the latter it survives ($P \sim 1$) up to long times. The spectra in Figs. 2(b) and 2(c) are hence identified with fast and suppressed string breaking dynamics, respectively. String breaking can thus be seen as a dynamical localization problem in the Fock-space graph in Fig. 2(a), where the string localizes if it is not resonantly coupled to the continuum of one-meson pairs. Quantitative agreement with the dynamics in the full Ising model is also observed in Figs. 2(d) and 2(e), which can be systematically improved by decreasing h_x/J [65].

Quantum impurity model picture.—The above picture resembles localization phenomena in quantum impurity models (QIMs) [67–69]. This is the basis for a further reduction of the string breaking problem. Let us consider an elementary string breaking (fusion) process: $(\cdots \uparrow \uparrow \downarrow \downarrow \downarrow \downarrow \uparrow \uparrow \cdots)_i \leftrightarrow (\cdots \uparrow \uparrow \downarrow \downarrow \downarrow \uparrow \uparrow \uparrow \cdots)_{ii} \leftrightarrow (\cdots \uparrow \uparrow \downarrow \downarrow \downarrow \uparrow \uparrow \cdots \uparrow \downarrow \uparrow \cdots)_{iii}$, where a string (i) gets cut near its edges via pair creation, yielding a metastable configuration (ii), and eventually, a shorter string plus a one meson (iii) [58,62]. We encode the different configurations

of this basic process in the internal states of a spin-1 system (“impurity”). Concretely, we map the symmetric and antisymmetric string states $(1/\sqrt{2})[|\psi_{\text{string}}\rangle \pm |\psi_{\text{meta}}\rangle]$, onto the impurity states $|S^z = \pm 1\rangle$, respectively, and the state where the string has been cut and a lighter meson radiated onto $|S^z = 0\rangle$. The impurity is also locally coupled to a meson bath in analogy to the picture in Fig. 2(a). This motivates a QIM with Hamiltonian

$$\hat{H}_{\text{QIM}} = \hat{H}_{\text{imp}} + \hat{H}_{\text{bath}} + \hat{H}_{\text{coup}}, \quad (6)$$

where $\hat{H}_{\text{imp}} = (M - \mu)(\hat{S}^z)^2 + \Lambda \hat{S}^z$, contains the string mass term M , a chemical potential μ accounting for higher-order corrections, and a $\Lambda > 0$ term, directly related to string breaking (fusion); $\hat{H}_{\text{bath}} = \sum_{j=1}^N [-T(\hat{b}_j^\dagger \hat{b}_{j+1} + \text{H.c.}) + (M - 2T)\hat{b}_j^\dagger \hat{b}_j]$, describes a bath of light mesons represented by hard-core bosons with creation (annihilation) operators \hat{b}_j^\dagger (\hat{b}_j), on a chain with N sites, with hopping amplitude $-T$ and maximal kinetic energy $M - 2T$; and $\hat{H}_{\text{coup}} = -T[(1 - (\hat{S}^z)^2)\hat{S}^x \hat{b}_1^\dagger + \text{H.c.}]$, couples the impurity with the bath such that if $|S^z = 0\rangle$ a meson at site 1 is created, and whenever $|S^z = \pm 1\rangle$ a meson at that site is annihilated [70].

A schematic of this mapping is shown in Fig. 3(a) for a short string that can decay into two shorter strings. The latter can be emitted into a meson bath, if the impurity-bath coupling is resonant, leaving the impurity in its “vacuum” state. Otherwise, the shorter strings can recombine back into a longer string, avoiding its decay. This QIM picture thus offers a distilled abstraction of the effective graph model in Fig. 2(a). We note, however, that the mapping between these two models is not exact. Yet, as shown below, both models have significant similarities both in the behavior of their eigenstates as well as in the dynamics of the impurity and the string.

The QIM in Eq. (6) features a localization-delocalization transition, explicitly shown in the single-meson limit. In this limit the spin can be replaced by two hard-core bosons, and due to particle number conservation, exact diagonalization is possible for large system sizes [65]. Focusing on the IPR of individual eigenstates, we see that the impurity mode can abruptly localize when varying Λ/T above a critical Λ_c/T , see Fig. 3(b), while the IPR of bulk eigenstates always vanishes (not shown). A standard finite-size scaling analysis [72], see Fig. 3(c), yields $\Lambda_c/T = 1.41(1)$ and critical exponents $\zeta = -1.02(5)$, $\nu = 1.00(5)$, for the considered parameters.

Such localization-delocalization transition underlies and governs two qualitatively different spin dynamics, see Fig. 3(d). Here we plot the long-time averaged spin autocorrelation function $\mathcal{M}(\Lambda) = \lim_{t \rightarrow \infty} (1/t) \int_0^t dt' \langle \hat{S}^z(t') \hat{S}^z(0) \rangle_\Lambda$, where $\langle \cdots \rangle_\Lambda$ denotes the expectation value at a given Λ/T . This quantity plays an equivalent role to the survival probability for the spin chain [Eq. (2)], and likewise, it

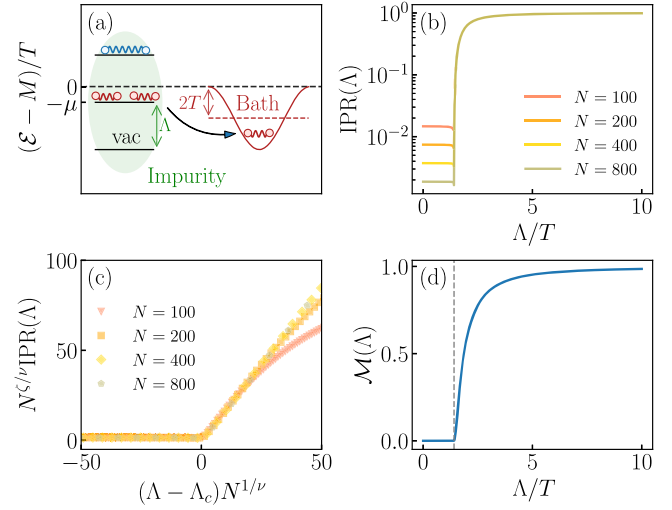


FIG. 3. Localization-delocalization transition in the QIM. (a) Minimal string breaking (fusion) as a three-level system (impurity), coupled to a bath. The impurity-bath coupling may or may not be resonant, yielding hybridization (as depicted here)—corresponding to string decay—or localization, respectively. (b) IPR of the impurity mode as a function of Λ/T , for various system sizes, displaying a localization-delocalization transition. (c) Data collapse of the data in (b) using a standard finite-size scaling ansatz within the package PYFSSA [71], yielding $\Lambda_c/T = 1.41(1)$ and critical exponents $\zeta = 1.02(5)$, $\nu = 1.00(5)$. (d) Long-time average of the spin autocorrelation function $\mathcal{M}(\Lambda)$, as a function of Λ/T with $N = 800$. The dashed line indicates the transition point $\Lambda_c/T \approx 1.41$. Parameters: $M/T = 10$, $\mu/T = 2$.

vanishes on the delocalized side of the transition, while it approaches unity as we ramp up Λ/T , above the localization transition point.

Our conclusions are restricted to the lattice as we have only considered a bounded spectrum of excitations. We expect our observations to hold beyond the limit $h_x \ll J$, as long as there exist values of h_z/J for which certain decay channels lead to faster dynamics than in other regimes. We note that our effective descriptions are valid only within the prethermal timescale of the localized regime. Also, further localization transitions may occur around other resonance points of the spin chain, which could involve longer decay paths [24,32], and hence, would require us to consider an impurity with more internal levels. Regarding the Fock-space graph model, we note that adding higher-order corrections could reshape the transition path and change the criticality. Nevertheless, as the effect of such higher-order terms is just a renormalization of hopping amplitudes [25], we expect the physics to remain qualitatively unaltered far from the localization transition point and deep in the two phases.

Discussion and outlook.—We expect our main results to be relevant for a wider class of confining theories in one and higher dimensions. In effect, what seems to be crucial

in the applicability of the QIM picture is that the system retains rotational symmetry, with the radial coordinate effectively defining a one-dimensional problem, when integrating out the rotation degree of freedom [68]. Fermionic bound states (e.g., baryons) could also be accounted for by changing the statistics of the bath [68,69]. Finally, our observations can be experimentally realized with current quantum technologies. In particular, Rydberg atoms offer a well suited platform, in which both the initial string states and the target unitary dynamics can be implemented in a highly controllable way [48–53].

The data shown in the figures is available on Zenodo [73].

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