Unique Steady-State Squeezing in a Driven Quantum Rabi Model

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Squeezing is essential to many quantum technologies and our understanding of quantum physics. Here, we show a novel type of steady-state squeezing that can be generated in the closed and open quantum Rabi as well as Dicke model. To this end, we eliminate the spin dynamics which effectively leads to an abstract harmonic oscillator whose eigenstates are squeezed with respect to the noninteracting harmonic oscillator. By driving the system, we generate squeezing which has the unique property of time-independent uncertainties and squeezed dynamics. Such squeezing might find applications in continuous backaction evading measurements and should already be observable in optomechanical systems and Coulomb crystals.

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Introduction.—Squeezing [1,2] relies on redistributing quantum uncertainties between two noncommuting observables. The primary example is the squeezing of light [3], where the uncertainties are redistributed between the strength of electric and magnetic fields with respect to a coherent state where the uncertainties are equal. Squeezing is a precious quantum resource, as it is rather robust to decoherence and dissipation. For this reason, it finds applications in many quantum technologies with the most prominent ones being high-precision measurements [4–15] and entanglement-based quantum key distribution [16–19]. On the other hand, squeezing is a form of quantum correlation important in the context of quantum phase transitions [20] and is used to study the fundamental aspects of quantum physics [21].

The quantum Rabi model [22,23] is a paradigmatic model in physics that describes a quantized harmonic oscillator coupled to a two-level system, and its Hamiltonian reads ($\hbar = 1$)

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{\Omega}{2}\hat{\sigma}_z + \frac{g}{2}(\hat{a} + \hat{a}^\dagger)\hat{\sigma}_x, \tag{1}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators for a harmonic oscillator with frequency ω. The Pauli matrices $\hat{\sigma}_z$ and $\hat{\sigma}_x$ describe the two-level system (here, interchangeably referred to as spin) with frequency Ω, and g is the interaction strength between the two subsystems. The interaction term can be rewritten with the help of spin raising and lowering operators, $\hat{\sigma}_x = (\hat{\sigma}_+ + \hat{\sigma}_-)$, into two terms

$$(\hat{a} + \hat{a}^\dagger)\hat{\sigma}_x = (\hat{a}\hat{\sigma}_- + \hat{a}^\dagger\hat{\sigma}_+) + (\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-). \tag{2}$$

The first one is typically referred to as the counter-rotating term and the second one is the rotating term. Neglecting the fast oscillating counter-rotating term leads to the Jaynes-Cummings model [24,25] which is the backbone of modern quantum optics. This (rotating wave) approximation is valid for $g \ll \omega, \Omega$ and $|\Omega - \omega| \ll |\Omega + \omega|$, however, it is not able to capture every aspect of the rich and intriguing physics close to the critical point of the quantum Rabi model ($g \sim g_c \equiv \sqrt{\omega\Omega}$) [20] (for a detailed analysis of the critical behavior under the rotating wave approximation see Ref. [26]).

In order to see why the vicinity of the critical point is interesting, we eliminate the dynamics of the spin using the Schrieffer-Wolff transformation [27]. Under the assumption of $1 - g^2/g_c^2 \gg (\omega/\Omega)^{2/3}$ [28], this leads to

$$\hat{H}_a = \omega \hat{a}^\dagger \hat{a} - \frac{g^2}{4\Omega}(\hat{a} + \hat{a}^\dagger)^2, \tag{3}$$

which is a squeezing Hamiltonian with eigenstates

$$|\psi_n\rangle = \exp\left\{\frac{1}{2}(\xi^* \hat{a}^2 - \xi \hat{a}^{\dagger 2})\right\}|n\rangle, \tag{4}$$

where $\xi = \frac{1}{4}\ln\{1 - g^2/g_c^2\}$ is the squeezing parameter and $|n\rangle$ are the Fock states. Note that the above Hamiltonian can be diagonalized by introducing an operator $\hat{c} = \cosh(\xi)\hat{a} + \sinh(\xi)\hat{a}^\dagger$ which characterizes the eigenmode of a strongly interacting system with frequency $\omega\sqrt{1 - g^2/g_c^2}$ [29]. In other words, the spin can be thought of as a mediator of interactions between harmonic oscillator excitations in the limit of $\omega \ll \Omega$.

To interpret this squeezing, we introduce the operators $\hat{x} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2\omega}$ and $\hat{p} = \sqrt{\omega}(\hat{a} - \hat{a}^\dagger)/\sqrt{-2}$ such that

$$\hat{H}_a = \frac{\hat{p}^2}{2} + \frac{\omega^2}{2}\left(1 - \frac{g^2}{g_c^2}\right)\hat{x}^2. \tag{5}$$

This Hamiltonian describes an abstract harmonic oscillator with a modified frequency $\omega\sqrt{1-g^2/g_c^2}$ and a unit mass. Approaching the critical point amounts amounts to opening the abstract harmonic oscillator which leads to squeezing with respect to the noninteracting harmonic oscillator [30] $\omega\hat{a}^\dagger\hat{a}$ (the ground state of a given harmonic oscillator is a squeezed ground state of a harmonic oscillator with a different frequency). This means that, if the measurement is performed in the basis of the noninteracting harmonic oscillator described by $\hat{x}$ and $\hat{p}$, increasing the coupling will lead to redistribution of uncertainties between $\hat{x}$ and $\hat{p}$. Therefore, the lower the effective frequency, the larger the spread $\Delta\hat{x}$. In particular, for $g = g_c$ the spread $\Delta\hat{x}$ is infinite and $\Delta\hat{p}$ becomes 0 as the Hamiltonian describes an abstract free particle whose eigenstates are that of the abstract momentum.

In this Letter, we present how a driven (and dissipative) quantum Rabi model close to the critical point can generate (steady-state) squeezing of the harmonic oscillator excitations and its dynamics. Such squeezing has the unique property of time-independent uncertainties which might be crucial for several quantum technologies. We start with the closed quantum Rabi model, subsequently, we consider an open and driven system, and finally, we show how increasing the number of spins (Dicke model) leads to enhanced squeezing. We conclude by identifying potential applications of the presented squeezing mechanism and discussing possible platforms for the implementation of the protocol.

Kicked quantum Rabi model.—In order to excite the harmonic oscillator, we include a drive term to the quantum Rabi Hamiltonian (1)

$$\hat{H}_d = \eta(\hat{a}e^{i\omega_d t} + \hat{a}^\dagger e^{-i\omega_d t}), \tag{6}$$

where η is the strength of the drive and ω_d is its frequency. Although such a driving term is characteristic of laser-pumped cavities [31], it can also describe driving of other harmonic oscillators. In an isolated system, the drive will excite the system indefinitely, therefore, we assume a strong short pulse (kick) in this case. Such a drive acts as a displacement operator $\hat{D}(\alpha) = \exp(\hat{a}\alpha + \hat{a}^\dagger\alpha^*)$ which displaces an initial vacuum state by α, creating a coherent state $|\alpha\rangle$. If the coupling strength g is equal to 0, the coherent state will rotate around the origin of the phase space with frequency ω at a fixed radius $|\alpha|$. Adiabatically increasing the coupling strength toward the critical point will then change the frequency of the abstract harmonic oscillator, leading to the change of the orbit from circular to elliptical and will redistribute the uncertainties between the quadrature operators $\hat{X} = (\hat{a} + \hat{a}^\dagger)/2$ and $\hat{P} = (\hat{a} - \hat{a}^\dagger)/2i$ (see Fig. 1). The final state can be easily found by constructing a coherent state out of squeezed Fock states

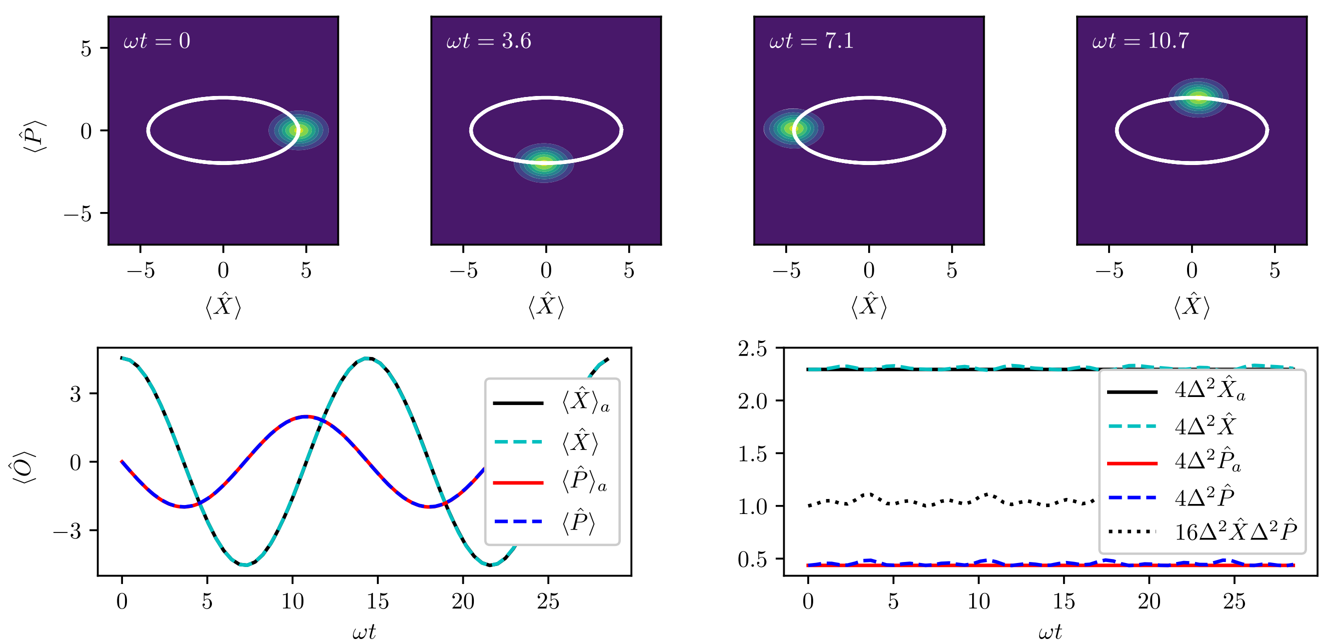

FIG. 1. Time evolution of the squeezed state for the kicked quantum Rabi model. The top panel shows the Husimi Q function at various times including the phase-space trajectory. The bottom left panel depicts the mean values $\langle\hat{X}\rangle$ and $\langle\hat{P}\rangle$ (subscript a indicates the approximated abstract oscillator Hamiltonian) and the bottom right panel the squeezing of $\hat{X}$ and $\hat{P}$. The orbit (white line) is equally squeezed as the time-independent squeezed uncertainties. The dynamics described by the full quantum Rabi model (dashed lines) and the effective (solid lines) Hamiltonian agree very well, the visible wiggles for the squeezing appear because the simulation parameters are on the verge of the approximation breakdown. The parameters are $\Omega/\omega = 10^5$, $g/g_c = 0.9$, and $\alpha = 3$.

of the harmonic mode $\hat{a}$. The time evolution of such a squeezed coherent state then becomes

$$|\psi(t)\rangle = e^{\frac{-|\alpha|^2}{2}} \sum_{n=0}^{\infty} e^{-in\omega\sqrt{1-\frac{g^2}{g_c^2}}t} \frac{\alpha^n}{\sqrt{n!}} e^{\frac{1}{2}(\xi^* \hat{a}^2 - \xi \hat{a}^{\dagger 2})} |n\rangle. \quad (7)$$

The equation of the new (squeezed) phase-space orbit can be found by calculating the average value of $\hat{X}$ and $\hat{P}$

$$\langle \hat{X} \rangle = \alpha \cos\left(\omega\sqrt{1-\frac{g^2}{g_c^2}}t\right) \exp(-\xi),$$
$$\langle \hat{P} \rangle = -\alpha \sin\left(\omega\sqrt{1-\frac{g^2}{g_c^2}}t\right) \exp(\xi). \quad (8)$$

The uncertainties in $\hat{X}$ and $\hat{P}$ can be calculated in the same way

$$\Delta^2 \hat{X} = \frac{1}{4} \exp(-2\xi),$$
$$\Delta^2 \hat{P} = \frac{1}{4} \exp(2\xi), \quad (9)$$

and turn out to be squeezed and time independent. In contrast, creating squeezing in other ways, for example with a position measurement, squeezes the state but not the harmonic oscillator. Therefore, the squeezing experiences additional rotation which leads, for instance, to measurement backaction [32–35] and consequently to the standard quantum limit of measurement precision [36]. In our approach, we adiabatically squeeze the abstract harmonic oscillator by changing its frequency which also squeezes the state. As a result, the state does not experience any rotations around its own axis at the expense of exhibiting equally squeezed orbit and uncertainties [see Fig. 1, Eqs. (8) and (9)].

In practice, obtaining large and detectable squeezing of a macroscopic coherent state ($|\alpha| \gg 1$) might be difficult with a single two-level system, as it would require an extremely large ratio Ω/ω to prevent the spin from getting excited (the effective Hamiltonian will no longer be valid). One way to artificially increase the atomic frequency in the quantum Rabi model is by enlarging the number of spins (Dicke model) which can be naively understood as changing the frequency from Ω to $N\Omega$ [37], with N being the number of two-level systems (see Fig. 3). In the limiting case $N \to \infty$, the large spin can be replaced by another harmonic oscillator by means of the Holstein-Primakoff transformation [38].

Driven-dissipative quantum Rabi model.—In an open quantum system, the dissipation will eventually bring the state of the system to the ground state. In order to prevent this, we continuously drive the system. Since the effective Hamiltonian is quadratic, the system is described by a Gaussian state, and we expect that the dynamics of the full quantum Rabi Hamiltonian is also well approximated by Gaussian physics. Therefore, we use a second-order mean-field description [39], which leads to a closed set of equations [40]. The effect of harmonic oscillator dissipation with rate κ is taken into account by means of the Lindblad master equation

$$\mathcal{L}[\hat{\rho}] = -i[\hat{H} + \hat{H}_d, \hat{\rho}] + \kappa\left(\hat{c}\,\hat{\rho}\,\hat{c}^\dagger - \frac{1}{2}\{\hat{c}^\dagger \hat{c}, \hat{\rho}\}\right). \quad (10)$$

Note that, once the coupling is strong, the jump operators have to be redefined [41–44]. The role of the jump operators for a dissipative process is to bring the (undriven) system to a unique ground state. For strongly interacting systems, the ground state has much more energy than the ground state of a noninteracting system. In other words, the ground state of strongly interacting systems is a highly excited state of a noninteracting system. Therefore, naively using the jump operators for the noninteracting systems would lead to unphysical behavior as, for instance, extracting energy from the ground state [29]. A similar argument holds for the driving term. Once the coupling is strong, we are no longer driving the bare mode described by $\hat{a}$ but a new (dressed) mode described by $\hat{c}$.

After adiabatic elimination of the spin dynamics (equivalent to the Schrieffer-Wolff transformation for the closed system), we can find the effective Hamiltonian

$$\hat{H}_e = \omega \hat{a}^\dagger \hat{a} - \frac{g^2}{4\Omega}(\hat{a} + \hat{a}^\dagger)^2 + \eta(\hat{c}e^{i\omega_d t} + \hat{c}^\dagger e^{-i\omega_d t}), \quad (11)$$

which describes the abstract harmonic oscillator with an additional modified drive term. Figure 2 depicts the time evolution of the squeezing for the driven dissipative case. At the top and lower left panel, we see that the introduction of drive and dissipation does not qualitatively change the results. In the driven-dissipative case, the amplitude of the phase space oscillations is related to $2\eta/\kappa$. The lower right panel shows the almost time-independent squeezing after some settling time. The time evolution described by the adiabatically eliminated Hamiltonian (solid lines) agrees very well with that of the full quantum Rabi Hamiltonian (dashed lines) including the spin degree of freedom. For non-negligible spin excitation, this approximation breaks down, furthermore, the uncertainties also become strongly time dependent. This can be suppressed by increasing the number of spins N. In Fig. 3, we show the time dependence of the squeezing (left) and spin excitation (right) for different N. For a sufficiently large number of spins, the uncertainties become time independent. Including weak dissipation of the spin can also suppress the excitation and lead to time-

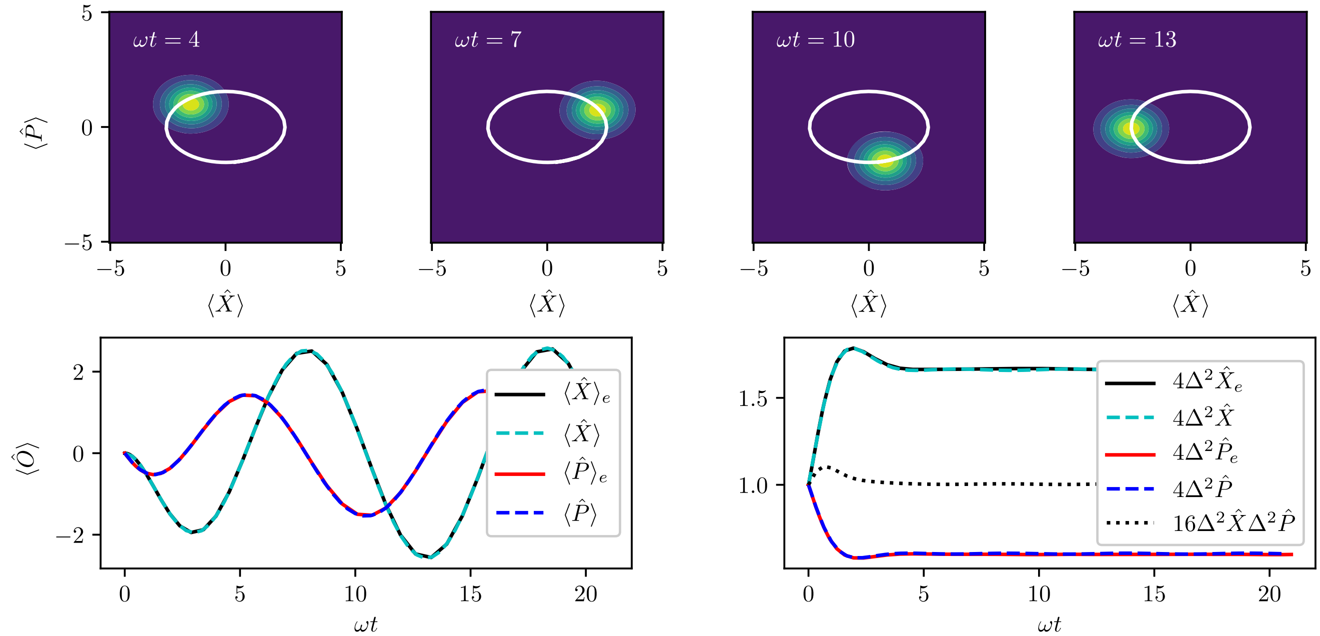

FIG. 2. Time evolution of the squeezed state for the driven-dissipative quantum Rabi model. The top panel shows the Husimi Q function at various times including the phase-space trajectory. The bottom left panel depicts the mean values $\langle\hat{X}\rangle$ and $\langle\hat{P}\rangle$ (subscript e indicates the adiabatically eliminated abstract oscillator Hamiltonian) and the bottom right panel the squeezing of $\hat{X}$ and $\hat{P}$. The uncertainties reach a steady state (no need for the adiabatic time evolution) with only minor time dependence. The dynamics described by the full quantum Rabi model (dashed lines) and the effective (solid lines) Hamiltonian agree very well. The parameters are $\Omega/\omega = 2 \times 10^3$, $g/g_c = 0.8$, $\omega_d/\omega = \sqrt{1 - g^2/g_c^2}$, $\eta/\omega = 1$, and $\kappa/\omega = 1$.

independent uncertainties. However, strong damping can prevent the system from generating squeezing [29].

We would like to point out that the direction of squeezing depends on the type of coupling $g(\hat{a}e^{i\theta} + \hat{a}^\dagger e^{-i\theta})\hat{\sigma}_x$, and it is related to θ (this also holds for the closed system). For instance, choosing θ to be $\pi/2$ would result in a harmonic oscillator where the $\hat{X}$ quadrature is squeezed and not antisqueezed, as in this Letter. In principle, by adjusting θ, one could obtain squeezing in an arbitrary quadrature direction. This, in turn, suggests that it should be possible to observe motion of a harmonic oscillator where, at the point of maximal displacement, the oscillator has the maximal momentum, for example, by tilting the axis by $\pi/4$. Such behavior is unfathomable for a classical harmonic oscillator.

Squeezing detection.—So far, the description was general, and we did not specify the harmonic oscillator and the underlying physical system. In this section, we discuss whether it is possible to observe steady-state squeezing and how to do it. In the case of a mechanical oscillator whose internal degree of freedom is coupled to the center-of-mass motion (phonon mode), the squeezing could simply be observed by measuring the position or momentum of the center of mass. In this case, the squeezing manifests itself in decreased or increased uncertainty of position and momentum. Such squeezing should already be realizable

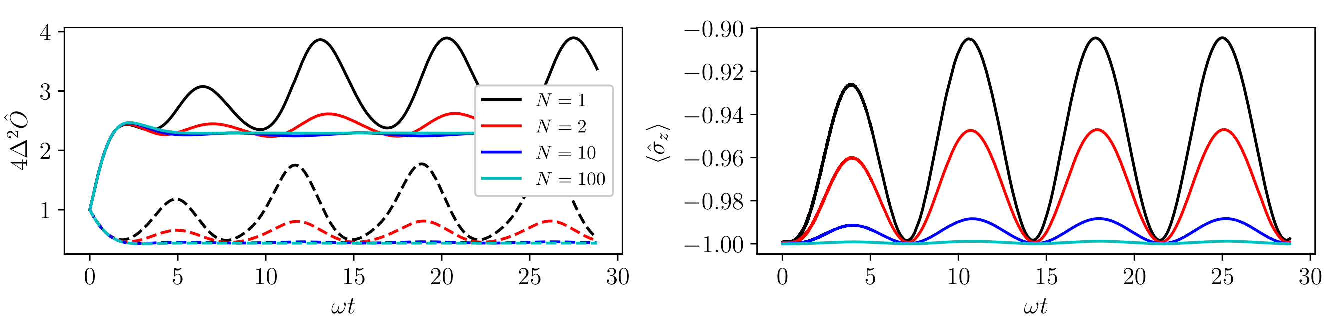

FIG. 3. Steady state squeezing for large number of spins. The left panel shows the squeezing of $\hat{X}$ (solid lines) and $\hat{P}$ (dashed lines) and the right side shows the excitation of one spin. For large number of spins N, the excitation per spin $\langle\hat{\sigma}_z\rangle$ is decreased and the uncertainties reach a steady state (cyan curve). The parameters are $\Omega/\omega = 2 \times 10^3$, $g/g_c = 0.9$, $\omega_d/\omega = \sqrt{1 - g^2/g_c^2}$, $\eta/\omega = 4$, and $\kappa/\omega = 1$.

in optomechanical systems [45,46] and ions interacting with a common phonon mode [47,48] by weakly driving the ground state close to the critical point. The crucial element that allows us to observe squeezing in these systems is the well-defined observables $\hat{x}$ and $\hat{p}$ for the uncoupled mechanical oscillator.

For the electromagnetic resonators, the measurement of squeezing is different. In this case, the squeezing manifests itself in a changed resonance frequency of the resonator. This means that the definition of $\hat{x}$ and $\hat{p}$ depends on the frequency of the resonator. Therefore, it is impossible to perform a measurement in the basis of the noninteracting harmonic oscillator [29]. The cavity in which atoms are strongly coupled to a single mode of radiation cannot generate squeezed light simply by driving it close to the critical point of the phase transition. From another perspective, light-matter interactions change the index of refraction and, hence, the resonance frequency which can be understood as squeezing of the electromagnetic field by changing its frequency.

Conclusions.—We have presented unique steady-state squeezing in the closed and open quantum Rabi model. In both cases, we obtain steady-state squeezing with time-independent variances and a squeezed trajectory in the phase-space picture defined by the noninteracting harmonic oscillator. Such squeezing can find applications in many quantum technologies, in particular, in quantum back-action-free continuous measurements [36,49,50] and driven-dissipative [51,52] critical [53–64] metrology. In order to understand squeezing, we introduced an effective Hamiltonian describing an abstract harmonic oscillator which we obtain by eliminating the dynamics of the two-level system. A promising extension of this proposal is the possibility of directly driving the spin and subsequently eliminating its dynamics in the dispersive regime to see steady-state squeezing.

Since a quantum harmonic oscillator coupled to a two-level (or multiple-level) system can be used to describe many systems, the proposed method could be tested in a variety of physical platforms including mechanical resonators [45,46,65–67], spin-orbit coupled quantum gases [28,68], ions coupled to phonons [69], Coulomb crystals [47,48], and even electrons trapped on a surface of liquid helium [70]. We predict, that the most promising system for the implementation of the described steady-state squeezing would be linear optomechanical systems in the far red-detuned and ultrastrong coupling regimes [71]. Also, systems composed of N trapped ions interacting with a single phononic mode (realizing the Dicke model) might be a perfect platform to create steady-state squeezing as it should be relatively easy to enter the regime where the effective Hamiltonian is valid.

In principle, it should also be possible to observe steady-state squeezing on the other side of the critical point $g > g_c$, where the system can be described by a slightly modified effective Hamiltonian (3) (see Ref. [20] for details). From a practical point of view, however, the generation of such squeezing would require an extremely large detuning $\Omega \gg \omega$ and would generate a macroscopic number of excitations. Also, beyond the critical point, the ground state is double degenerate, and we expect that this degeneracy further hinders the possibility of observing the steady-state squeezing with constant variances.

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[1] C. Gardiner and P. Zoller, *Quantum Noise: A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics* (Springer Science & Business Media, New York, 2004).
[2] S. Zubairy, Quantum squeezing, J. Opt. B **7**, 156 (2005).
[3] D. F. Walls, Squeezed states of light, Nature (London) **306**, 141 (1983).
[4] P. Grangier, R. E. Slusher, B. Yurke, and A. LaPorta, Squeezed-light–enhanced polarization interferometer, Phys. Rev. Lett. **59**, 2153 (1987).
[5] M. Xiao, L.-A. Wu, and H. J. Kimble, Precision measurement beyond the shot-noise limit, Phys. Rev. Lett. **59**, 278 (1987).
[6] E. S. Polzik, J. Carri, and H. J. Kimble, Spectroscopy with squeezed light, Phys. Rev. Lett. **68**, 3020 (1992).
[7] LIGO Scientific Collaboration, A gravitational wave observatory operating beyond the quantum shot-noise limit, Nat. Phys. **7**, 962 (2011).
[8] H. Grote, K. Danzmann, K. L. Dooley, R. Schnabel, J. Slutsky, and H. Vahlbruch, First long-term application of squeezed states of light in a gravitational-wave observatory, Phys. Rev. Lett. **110**, 181101 (2013).
[9] R. Schnabel, Squeezed states of light and their applications in laser interferometers, Phys. Rep. **684**, 1 (2017).
[10] LIGO Scientific Collaboration, Quantum-enhanced Advanced LIGO detectors in the era of gravitational-wave astronomy, Phys. Rev. Lett. **123**, 231107 (2019).
[11] Virgo Collaboration, Increasing the astrophysical reach of the Advanced VIRGO detector via the application of squeezed vacuum states of light, Phys. Rev. Lett. **123**, 231108 (2019).
[12] LIGO Scientific Collaboration and Virgo Collaboration, Gwtc-2: Compact binary coalescences observed by LIGO and VIRGO during the first half of the third observing run, Phys. Rev. X **11**, 021053 (2021).
[13] J. Qin, Y.-H. Deng, H.-S. Zhong, L.-C. Peng, H. Su, Y.-H. Luo, J.-M. Xu, D. Wu, S.-Q. Gong, H.-L. Liu, H. Wang, M.-C. Chen, L. Li, N.-L. Liu, C.-Y. Lu, and J.-W. Pan, Unconditional and robust quantum metrological advantage beyond NOON states, Phys. Rev. Lett. **130**, 070801 (2023).

[14] E. Pedrozo-Peñafiel, S. Colombo, C. Shu, A. F. Adiyatullin, Z. Li, E. Mendez, B. Braverman, A. Kawasaki, D. Akamatsu, Y. Xiao, and V. Vuletić, Entanglement on an optical atomic-clock transition, Nature (London) **588**, 414 (2020).

[15] S. Colombo, E. Pedrozo-Peñafiel, A. F. Adiyatullin, Z. Li, E. Mendez, C. Shu, and V. Vuletić, Time-reversal-based quantum metrology with many-body entangled states, Nat. Phys. **18**, 925 (2022).

[16] T. Gehring, V. Händchen, J. Duhme, F. Furrer, T. Franz, C. Pacher, R. F. Werner, and R. Schnabel, Implementation of continuous-variable quantum key distribution with composable and one-sided-device-independent security against coherent attacks, Nat. Commun. **6**, 8795 (2015).

[17] P. Wang, X. Wang, and Y. Li, Continuous-variable measurement-device-independent quantum key distribution using modulated squeezed states and optical amplifiers, Phys. Rev. A **99**, 042309 (2019).

[18] I. Derkach, V. C. Usenko, and R. Filip, Squeezing-enhanced quantum key distribution over atmospheric channels, New J. Phys. **22**, 053006 (2020).

[19] N. Hosseinidehaj, M. S. Winnel, and T. C. Ralph, Simple and loss-tolerant free-space quantum key distribution using a squeezed laser, Phys. Rev. A **105**, 032602 (2022).

[20] M.-J. Hwang, R. Puebla, and M. B. Plenio, Quantum phase transition and universal dynamics in the Rabi model, Phys. Rev. Lett. **115**, 180404 (2015).

[21] O. Thearle, J. Janousek, S. Armstrong, S. Hosseini, M. Schünemann (Mraz), S. Assad, T. Symul, M. R. James, E. Huntington, T. C. Ralph, and P. K. Lam, Violation of Bell's inequality using continuous variable measurements, Phys. Rev. Lett. **120**, 040406 (2018).

[22] D. Braak, Q.-H. Chen, M. T. Batchelor, and E. Solano, Semi-classical and quantum Rabi models: In celebration of 80 years, J. Phys. A **49**, 300301 (2016).

[23] Q. Xie, H. Zhong, M. T. Batchelor, and C. Lee, The quantum Rabi model: Solution and dynamics, J. Phys. A **50**, 113001 (2017).

[24] B. W. Shore and P. L. Knight, The Jaynes-Cummings model, J. Mod. Opt. **40**, 1195 (1993).

[25] J. Larson and T. Mavrogordatos, *The Jaynes–Cummings Model and Its Descendants* (IOP Publishing, Bristol, 2021), pp. 2053–2563, 10.1088/978-0-7503-3447-1.

[26] M.-J. Hwang and M. B. Plenio, Quantum phase transition in the finite Jaynes-Cummings lattice systems, Phys. Rev. Lett. **117**, 123602 (2016).

[27] S. Bravyi, D. P. DiVincenzo, and D. Loss, Schrieffer–Wolff transformation for quantum many-body systems, Ann. Phys. (N.Y.) **326**, 2793 (2011).

[28] K. Gietka, Harnessing center-of-mass excitations in quantum metrology, Phys. Rev. Res. **4**, 043074 (2022).

[29] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.131.223604 for effective model derivation, the correct jump operators, and a summary of the effect of spin dissipation.

[30] K. Gietka, Squeezing by critical speeding up: Applications in quantum metrology, Phys. Rev. A **105**, 042620 (2022).

[31] H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Cold atoms in cavity-generated dynamical optical potentials, Rev. Mod. Phys. **85**, 553 (2013).

[32] K. Bencheikh, J. A. Levenson, P. Grangier, and O. Lopez, Quantum nondemolition demonstration via repeated backaction evading measurements, Phys. Rev. Lett. **75**, 3422 (1995).

[33] C. F. Ockeloen-Korppi, E. Damskägg, J.-M. Pirkkalainen, A. A. Clerk, M. J. Woolley, and M. A. Sillanpää, Quantum backaction evading measurement of collective mechanical modes, Phys. Rev. Lett. **117**, 140401 (2016).

[34] M. Brunelli, D. Malz, and A. Nunnenkamp, Conditional dynamics of optomechanical two-tone backaction-evading measurements, Phys. Rev. Lett. **123**, 093602 (2019).

[35] C. Troullinou, R. Jiménez-Martínez, J. Kong, V. G. Lucivero, and M. W. Mitchell, Squeezed-light enhancement and backaction evasion in a high sensitivity optically pumped magnetometer, Phys. Rev. Lett. **127**, 193601 (2021).

[36] C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmermann, On the measurement of a weak classical force coupled to a quantum-mechanical oscillator, I. Issues of principle, Rev. Mod. Phys. **52**, 341 (1980).

[37] K. Gietka and T. Busch, Inverted harmonic oscillator dynamics of the nonequilibrium phase transition in the Dicke model, Phys. Rev. E **104**, 034132 (2021).

[38] T. Holstein and H. Primakoff, Field dependence of the intrinsic domain magnetization of a ferromagnet, Phys. Rev. **58**, 1098 (1940).

[39] D. Plankensteiner, C. Hotter, and H. Ritsch, QuantumCumulants.jl: A JULIA framework for generalized mean-field equations in open quantum systems, Quantum **6**, 617 (2022).

[40] https://qojulia.github.io/QuantumCumulants.jl/stable/examples/unique_squeezing/.

[41] F. Beaudoin, J. M. Gambetta, and A. Blais, Dissipation and ultrastrong coupling in circuit QED, Phys. Rev. A **84**, 043832 (2011).

[42] M.-J. Hwang, P. Rabl, and M. B. Plenio, Dissipative phase transition in the open quantum Rabi model, Phys. Rev. A **97**, 013825 (2018).

[43] M. Cattaneo, G. L. Giorgi, S. Maniscalco, and R. Zambrini, Local versus global master equation with common and separate baths: Superiority of the global approach in partial secular approximation, New J. Phys. **21**, 113045 (2019).

[44] P. Forn-Díaz, L. Lamata, E. Rico, J. Kono, and E. Solano, Ultrastrong coupling regimes of light-matter interaction, Rev. Mod. Phys. **91**, 025005 (2019).

[45] A. A. Rakhubovsky, D. W. Moore, U. Delić, N. Kiesel, M. Aspelmeyer, and R. Filip, Detecting nonclassical correlations in levitated cavity optomechanics, Phys. Rev. Appl. **14**, 054052 (2020).

[46] H. Rudolph, U. Delić, M. Aspelmeyer, K. Hornberger, and B. A. Stickler, Force-gradient sensing and entanglement via feedback cooling of interacting nanoparticles, Phys. Rev. Lett. **129**, 193602 (2022).

[47] R. C. Thompson, Ion Coulomb crystals, Contemp. Phys. **56**, 63 (2015).

[48] A. Safavi-Naini, R. J. Lewis-Swan, J. G. Bohnet, M. Gärttner, K. A. Gilmore, J. E. Jordan, J. Cohn, J. K. Freericks, A. M. Rey, and J. J. Bollinger, Verification of a many-ion simulator of the Dicke model through slow quenches across a phase transition, Phys. Rev. Lett. **121**, 040503 (2018).

[49] J. B. Hertzberg, T. Rocheleau, T. Ndukum, M. Savva, A. A. Clerk, and K. C. Schwab, Back-action-evading

measurements of nanomechanical motion, Nat. Phys. **6**, 213 (2010).

[50] C. B. Møller, R. A. Thomas, G. Vasilakis, E. Zeuthen, Y. Tsaturyan, M. Balabas, K. Jensen, A. Schliesser, K. Hammerer, and E. S. Polzik, Quantum back-action-evading measurement of motion in a negative mass reference frame, Nature (London) **547**, 191 (2017).

[51] V. P. Pavlov, D. Porras, and P. A. Ivanov, Quantum metrology with critical driven-dissipative collective spin system, Phys. Scr. **98**, 095103 (2023).

[52] T. Ilias, D. Yang, S. F. Huelga, and M. B. Plenio, Criticality-enhanced electromagnetic field sensor with single trapped ions, arXiv:2304.02050.

[53] S. S. Mirkhalaf, E. Witkowska, and L. Lepori, Supersensitive quantum sensor based on criticality in an antiferromagnetic spinor condensate, Phys. Rev. A **101**, 043609 (2020).

[54] K. Gietka, F. Metz, T. Keller, and J. Li, Adiabatic critical quantum metrology cannot reach the Heisenberg limit even when shortcuts to adiabaticity are applied, Quantum **5**, 489 (2021).

[55] R. Liu, Y. Chen, M. Jiang, X. Yang, Z. Wu, Y. Li, H. Yuan, X. Peng, and J. Du, Experimental critical quantum metrology with the Heisenberg scaling, npj Quantum Inf. **7**, 170 (2021).

[56] S. S. Mirkhalaf, D. Benedicto Orenes, M. W. Mitchell, and E. Witkowska, Criticality-enhanced quantum sensing in ferromagnetic Bose-Einstein condensates: Role of readout measurement and detection noise, Phys. Rev. A **103**, 023317 (2021).

[57] T. Ilias, D. Yang, S. F. Huelga, and M. B. Plenio, Criticality-enhanced quantum sensing via continuous measurement, PRX Quantum **3**, 010354 (2022).

[58] L. Garbe, O. Abah, S. Felicetti, and R. Puebla, Critical quantum metrology with fully-connected models: From Heisenberg to Kibble–Zurek scaling, Quantum Sci. Technol. **7**, 035010 (2022).

[59] K. Gietka, L. Ruks, and T. Busch, Understanding and improving critical metrology. Quenching superradiant light-matter systems beyond the critical point, Quantum **6**, 700 (2022).

[60] Z.-J. Ying, S. Felicetti, G. Liu, and D. Braak, Critical quantum metrology in the non-linear quantum Rabi model, Entropy **24**, 1015 (2022).

[61] D.-S. Ding, Z.-K. Liu, B.-S. Shi, G.-C. Guo, K. Mølmer, and C. S. Adams, Enhanced metrology at the critical point of a many-body Rydberg atomic system, Nat. Phys. **18**, 1447 (2022).

[62] E. Aybar, A. Niezgoda, S. S. Mirkhalaf, M. W. Mitchell, D. Benedicto Orenes, and E. Witkowska, Critical quantum thermometry and its feasibility in spin systems, Quantum **6**, 808 (2022).

[63] L. Garbe, O. Abah, S. Felicetti, and R. Puebla, Exponential time-scaling of estimation precision by reaching a quantum critical point, Phys. Rev. Res. **4**, 043061 (2022).

[64] K. Gietka and H. Ritsch, Squeezing and overcoming the Heisenberg scaling with spin-orbit coupled quantum gases, Phys. Rev. Lett. **130**, 090802 (2023).

[65] A. Kronwald, F. Marquardt, and A. A. Clerk, Arbitrarily large steady-state bosonic squeezing via dissipation, Phys. Rev. A **88**, 063833 (2013).

[66] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod. Phys. **86**, 1391 (2014).

[67] K. Dare, J. J. Hansen, I. Coroli, A. Johnson, M. Aspelmeyer, and U. Delić, Linear ultrastrong optomechanical interaction, arXiv:2305.16226.

[68] Y. Zhang, M. E. Mossman, T. Busch, P. Engels, and C. Zhang, Properties of spin–orbit-coupled Bose-Einstein condensates, Front. Phys. **11**, 118103 (2016).

[69] M. L. Cai, Z. D. Liu, W. D. Zhao, Y. K. Wu, Q. X. Mei, Y. Jiang, L. He, X. Zhang, Z. C. Zhou, and L. M. Duan, Observation of a quantum phase transition in the quantum Rabi model with a single trapped ion, Nat. Commun. **12**, 1126 (2021).

[70] K. M. Yunusova, D. Konstantinov, H. Bouchiat, and A. D. Chepelianskii, Coupling between Rydberg states and Landau levels of electrons trapped on liquid helium, Phys. Rev. Lett. **122**, 176802 (2019).

[71] K. Kustura, C. Gonzalez-Ballestero, A. d. l. R. Sommer, N. Meyer, R. Quidant, and O. Romero-Isart, Mechanical squeezing via unstable dynamics in a microcavity, Phys. Rev. Lett. **128**, 143601 (2022).

[72] S. Krämer, D. Plankensteiner, L. Ostermann, and H. Ritsch, QuantumOptics.jl: A JULIA framework for simulating open quantum systems, Comput. Phys. Commun. **227**, 109 (2018).