Glitches in Rotating Supersolids

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Glitches, spin-up events in neutron stars, are of prime interest, as they reveal properties of nuclear matter at subnuclear densities. We numerically investigate the glitch mechanism due to vortex unpinning using analogies between neutron stars and dipolar supersolids. We explore the vortex and crystal dynamics during a glitch and its dependence on the supersolid quality, providing a tool to study glitches from different radial depths of a neutron star. Benchmarking our theory against neutron-star observations, our work will open a new avenue for the quantum simulation of stellar objects from Earth.

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One of the greatest strengths of ultracold gases is their ability to simulate the behavior of widely disparate systems [1]. This extraordinary capability enables quantum gases to serve as powerful solvers for unmasking fundamental open questions concerning the underlying dynamics of complex physical systems. The range of fields where quantum gas simulators have found applications include metallic superconductivity and condensed matter systems, as well as nuclear matter. Among these examples, nuclear matter under the extreme conditions existing in neutron stars is the most elusive to direct microscopic observation [2–4].

Neutron stars are the densest stellar objects known today. They form through the core collapse of massive progenitor stars in supernovae type-II events, leading to their extreme densities in which a giant gravitational mass of a few solar masses is concentrated in just a tiny radius of about 10 km. Shortly after their birth, neutron stars cool down to temperatures of the order of keV. Compared to ultracold gases (peV), these temperatures are very high, yet much smaller than the MeV energy scale typical of nuclear matter. For this reason, neutron stars can be viewed as cold dense nuclear matter in which quantum effects become very important. The current most-widely-accredited descriptions to explain observations in such systems account for fermionic pairing and correlations in quantum many-body systems [5,6].

The 1967 discovery of pulsars [7]—highly magnetized and rapidly rotating neutron stars [8,9]—provided crucial hints of superfluidity and fermionic pairing in these stellar objects. Pulsars can be seen as nearly perfect clocks or regular radio emitters [10–12]. They emit photons in a narrow angular beam, similar to that from a lighthouse. This lighthouse effect results from the misalignment between the rotation and magnetization axes and leads to a secular loss of rotational energy with a corresponding slow decrease of the pulsar rotation frequency Ω. Remarkably, it has been observed that the rotation frequency of the pulsars occasionally shows anomalous jumps—called "glitches"—in the form of an abrupt speedup of the pulsar rotation followed by a slow relaxation close to its original value. It is precisely the observations of such pulsar glitches that have provided the first evidence of superfluidity in neutron-star interiors.

This surprising observation suggests that the interiors of neutron stars are indeed made up of several components and that one among them is irrotational or at least weakly coupled to the rigid rotation of pulsars. Natural candidates are superfluids and supersolids, respectively. In this scenario, quantized vortices, forming in the superfluid component, can stochastically unpin from the rigid crystalline component and change the star's angular momentum. Understanding whether this is a plausible mechanism requires addressing several key questions, including: how do superfluid vortices pin and unpin? How do unpinned vortices percolate through the crystalline structure? What information can be extracted from the glitch signal shape?

Tackling these questions from first principles is challenging, as the properties of the inner crust of neutron stars are model dependent. Moreover, we have only observational access to the neutron-star atmosphere; thus, the underlying dynamics are basically a black box. One possible way to improve our understanding of pulsar glitches is to reproduce them in a controllable laboratory, where we have full access to the entire system [13–15].

Thanks to rapid developments in quantum simulation, it is now possible to employ dipolar quantum gases—where supersolidity and rotational physics have recently been observed in circularly symmetric systems [16–19]—as analogous microscopic quantum systems. Here, we demonstrate exactly this and predict the existence of glitches in a rotating ultracold dipolar supersolid. We show how quantized vortices unpin from the crystalline structure of the supersolid and escape, transferring angular momentum. Varying the interactions, we observe that the glitch size may depend not only on the number of unpinned vortices, but also on the superfluid fraction and the supersolid internal dynamics.

We start by outlining some basic properties of neutron stars, and then we move to show the analogies with dipolar supersolids. Neutron stars are expected to possess a complex internal structure with a sequence of layers [4,20–26], as shown in Fig. 1(a). Beneath a micrometer-thick atmosphere, the first layer, the so-called outer crust, is expected to be a crystalline solid of neutron-rich ions and electrons that behave as a normal component. At its heart, the core of the neutron star is instead believed to be in a liquidlike phase with superfluid properties [27–32]. Here, the density exceeds the nuclear saturation density $\rho_{\rm sat}$, meaning that the nucleons are so closely packed that they overlap [33]. Sandwiched between the solid outer crust and the superfluid core, one finds the inner crust: Here, the density of neutrons exceeds the neutron drip density ρ_d so that it becomes energetically favorable for them to drip out. The most accredited theories describe this phase in terms of unbound superfluid neutron pairs with a periodic density modulation; see Figs. 1(a$_1$) and 1(a$_2$) and Ref. [36]. The coexistence of solid and superfluid in the inner crust can be viewed in modern terms as a supersolid phase. This, as we shall see, is a key ingredient for the widely accepted physical explanation of glitches, schematically depicted in Fig. 1(b), associated with a transfer of angular momentum between the inner and the outer crust [12,57–61].

In the low-energy sector, quantum phases with supersolid properties have recently been observed in various settings [16,17,62–66]. Particularly relevant for drawing analogies with neutron stars is the case of circular supersolids of dipolar atoms [17], on which we specifically concentrate in this work, as shown in Figs. 1(c) and 1(d). These systems are obtained by trapping and cooling highly magnetic atoms, like erbium or dysprosium, into quantum degenerate states known as dipolar Bose-Einstein condensates (BECs) [67,68]. The dipolar supersolid phase exists due to the competition of three types of interactions: a repulsive isotropic contact interaction, a momentum-dependent long-range and anisotropic dipole-dipole interaction, and a repulsive higher-order-density interaction arising from quantum fluctuations [69]. Supersolids are characterized by the existence of a superfluid connection between the crystal sites, controlled, in turn, by the strength of the short-range interactions, governed by the scattering length a_s, which plays the role of the radial depth of the

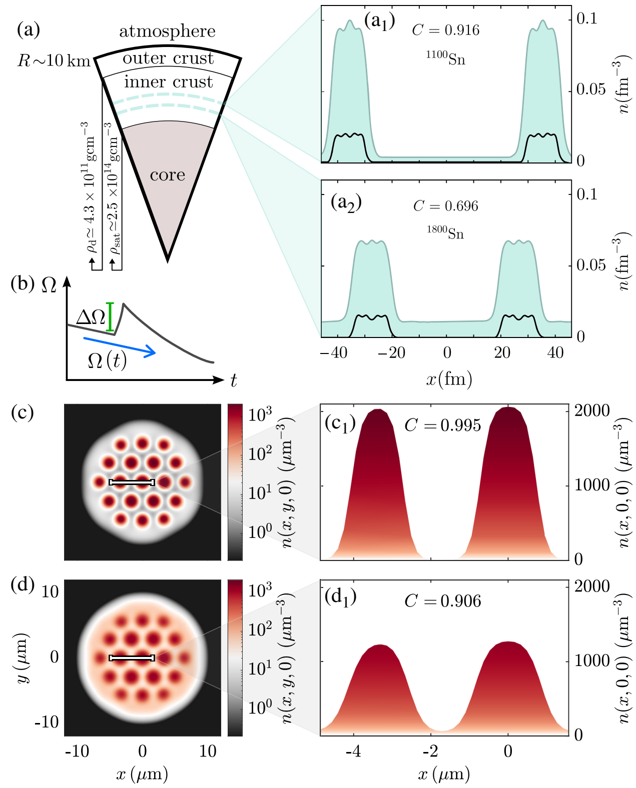

FIG. 1. Comparison between a neutron star and a dipolar supersolid. (a) Structure of a neutron star, together with the density distributions of neutrons (cyan) and protons (black) near the inner-to-outer crust, for baryonic density $n_b \simeq 5.77 \times 10^{-3}$ fm^{-3} (a$_1$) and the inner crust-to-core interface, for $n_b \simeq 2.08 \times 10^{-2}$ fm^{-3} (a$_2$) (adapted with permission from Elsevier from [27]). (b) Illustration of a glitch; see the text. (c),(d) Density distribution of a dipolar quantum gas, with the corresponding density n cut along $y = z = 0$ at (c) $a_s = 88a_0$ and (d) $a_s = 93a_0$, where a_0 is the Bohr radius. In both cases, the strength of the superfluid connection is quantified by the density contrast $C = (n_{\rm max} - n_{\rm min})/(n_{\rm max} + n_{\rm min})$.

neutron star. Figure 1(c$_1$) shows a case with weak superfluid connection, emulating the condition close to the inner-to-outer crust boundary, whereas Fig. 1(d$_1$) shows one with stronger superfluid connection, in accordance with the inner crust-to-core boundary.

The remarkable analogy between a pulsar and a dipolar supersolid can be also extended to the rotational dynamics. In both cases, the time evolution of the rotation frequency Ω can be described as [57]

$$I_s\dot{\Omega} = -N_{\rm em} - \dot{L}_{\rm vort} - \dot{I}_s\Omega, \tag{1}$$

where I_s is the moment of inertia of the solid part. For a neutron star, changes in I_s are not directly observable and can be challenging to estimate [6,70–72]. In dipolar supersolids, we have full access to the system; therefore, changes in the moment of inertia due to internal dynamics

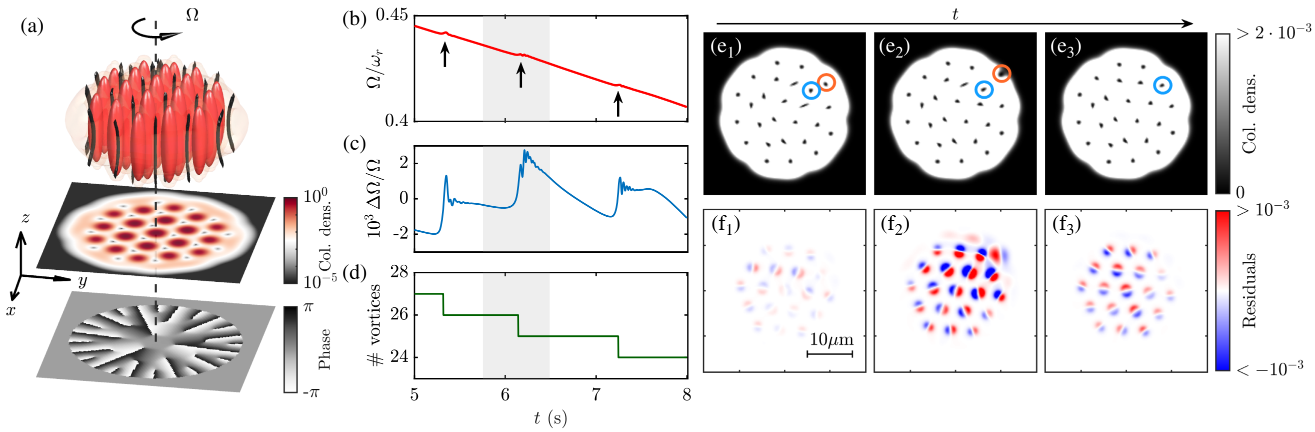

FIG. 2. Glitches in a dipolar supersolid. (a) Rotating supersolid with $\Omega = 0.41\omega_r$ and $a_s = 91a_0$. Top: dipolar supersolid showing two isosurfaces at 15% (opaque) and 0.05% (translucent) of the maximum density, and vortex lines in black. Middle: column densities normalized to the peak density. Bottom: phase profile $\arg[\Psi(x, y, z = 0)]$. (b) Rotation frequency in time, with torque $N_{\rm em} = 4.3 \times 10^{-35}$ kg m^2/s^2. Arrows indicate glitch positions. (c) Relative change in Ω, computed as $\Delta\Omega = [\Omega(t) - \Omega_{\rm lin}]/\Omega_{\rm lin}$, where $\Omega_{\rm lin}$ is the result of a linear fit of the curve in (b). (d) Vortex number. The gray shaded area in (b)–(d) highlights the time window in (e) and (f). (e) Column density saturated to highlight vortex positions and shape, with one vortex escaping (orange circle) and another taking its place (blue circle). (f) Crystal excitations, showing the column density differences between time steps, $n(t) - n(t - \Delta t)$, with $\Delta t = 2.4$ ms.

can be accurately accounted for. The quantity $N_{\rm em}$ is a spin-down torque that linearly reduces the total angular momentum of the star: This process occurs spontaneously in a pulsar due to the emission of electromagnetic radiation, whereas in a dipolar supersolid it can be controlled by slowly ramping down the rotation frequency of the trap. Finally, $L_{\rm vort}$ is the angular momentum of the superfluid part.

Despite its simplicity, Eq. (1) is able to capture very intriguing dynamics in pulsars. While the crystalline part in the inner and outer crust rigidly corotates and promptly responds to the braking torque, the superfluid component in the inner crust lags behind, storing angular momentum in the form of quantized vortices. Such vortices are mainly pinned in the interstitial regions, with a pinning force that depends on the depth of the superfluid nuclear background [4,6,27–32,73]. However, during the spin-down of the star, some vortices can stochastically unpin and escape from the inner crust, causing a sudden release of angular momentum. This is captured by the $L_{\rm vort}$ term in Eq. (1), which adds a positive contribution to $\dot{\Omega}$ whenever a vortex leaves. A glitch corresponds to a collective unpinning of vortices [74,75]. The outer crust absorbs the released macroscopic angular momentum and suddenly spins up in a steplike fashion, before relaxing and resuming its spin-down behavior; see Fig. 1(b). The glitches bring a fractional change of the rotation frequency in the range $\Delta\Omega/\Omega \sim 10^{-12} - 10^{-3}$ [76].

The question now is whether we can validate the above phenomenological description and observe glitches in a dipolar supersolid. To this end, we numerically study the spin-down of an ultracold polarized dipolar BEC in the supersolid state. The atoms with mass m are harmonically confined in a three-dimensional pancake-shaped trap, with frequencies $\boldsymbol{\omega} = (\omega_r, \omega_z) = 2\pi \times (50, 130)$ Hz. They interact via the two-body pseudopotential $U(\mathbf{r}) = (4\pi\hbar^2 a_s/m)\delta(\mathbf{r}) + (3\hbar^2 a_{\rm dd}/m)[(1 - 3\cos^2\theta)/|\mathbf{r}|^3]$, with tunable short-ranged interactions controlled by a_s, long-range anisotropic dipole-dipole interactions with effective range given by the dipolar length $a_{\rm dd}$, and θ as the angle between the polarization axis (z axis) and the vector joining two particles. We fix our study to ^{164}Dy with $a_{\rm dd} = 130.8a_0$. The evolution of the macroscopic wave function $\Psi(\mathbf{r}, t)$ is governed by the dissipative extended Gross-Pitaevskii equation (eGPE) [77–80]

$$i\hbar\frac{\partial\Psi}{\partial t} = (1 - i\gamma)\left[\mathcal{L}[\Psi; a_s, a_{\rm dd}, \boldsymbol{\omega}] - \Omega(t)\hat{L}_z\right]\Psi, \quad (2)$$

where $\mathcal{L}$ is the eGPE operator and we include dissipation through the small parameter $\gamma = 0.05$ to tune the coupling between the system and the rotating trap; see Ref. [36]. The wave function is normalized to the total atom number through $N = \int d^3\mathbf{r}|\Psi|^2 = 3 \times 10^5$. The operator $\hat{L}_z = x\hat{p}_y - y\hat{p}_x$ corresponds to rotation about the z axis and can be used to obtain the total angular momentum $L_{\rm tot} = \langle\hat{L}_z\rangle$. The superfluid angular momentum is obtained from $L_{\rm vort} = L_{\rm tot} - L_s$, with the second term L_s coming from rigid body rotation of the supersolid [81,82] (see Ref. [36]). The initial condition is found in imaginary time, at fixed $\Omega(0) = 0.5\omega_r$, giving a vortex lattice embedded within the supersolid crystal. It has been shown [81–83] that rotating supersolids host quantized vortices pinned at local minima of the supersolid density modulation, as shown in Fig. 2(a), and at saddle points between each pair of droplets [36].

The real-time spin-down of the system is obtained by simultaneously solving Eqs. (1) and (2). After generating the initial conditions, we introduce an external torque. This acts as a brake on the solid component, reducing $\Omega(t)$ over time. Our findings are shown in Fig. 2(b), where we selected an appropriate time interval to show multiple glitch events. Though at first glance the curve appears linear, dominated by $N_{\rm em}$, there are deviations from this behavior highlighted by arrows, showing the appearance of glitches in a dipolar supersolid. Visualizing instead the relative change of Ω in Fig. 2(c), we see signatures similar to pulsar glitches, with a rapid increase of Ω, followed by a slow relaxation back to linear behavior.

Unlike in pulsars, here we have unprecedented access to the internal dynamics of the dipolar supersolid. Thus, we can identify each glitch as the moment when superfluid vortices unpin and reach the trap boundary [Figs. 2(d) and 2(e)], transferring their angular momenta to the solid component by the feedback mechanism through Eq. (1). Furthermore, by tracking the unpinning and repinning of individual vortices, we are able to determine the origin of the glitch pulse shape. Here, the observed asymmetry is due to the fact that, when internal vortices are unpinned (glitch rise time), it takes some time before they repin (glitch fall time): They slowly move from one pinning site to the other; see Figs. $2(e_1)$–$2(e_3)$ [36]. Since vortex energy minima are separated by saddle points, to go from one pinning site to the other, a vortex must move across one of them [83]. In doing this, the vortex core is squeezed and then uncompressed, producing an effective friction on the movement of the vortex. Thus, the long supersolid postglitch timescale is associated with this slow percolation of vortices across the crystalline structure [36]. As far as we know, this process has never been considered in the description of the pulsar postglitch behavior.

We also have access to crystal dynamics. As a consequence of the vortex activity, the crystalline structure is deformed and excited. This is visible in the residual matter density evolution [Figs. $2(f_1)$–$2(f_3)$], where, during the glitch, each droplet is slightly deformed and vibrates. Then, during the postglitch, the droplets slowly relax toward a more uniform distribution. These excitations are due to superfluid fluxes inside the droplets and between neighboring droplets by means of the superfluid bath. Typically, we find that strong crystal excitations affect the postglitch signal of Ω, suggesting that we could infer the crystal properties through analysis of the glitch pulse shape.

The typical magnitude of a glitch is $\Delta\Omega/\Omega \sim 10^{-3}$, a giant glitch in the context of pulsars. The glitch jumps can be written as $\Delta\Omega/\Omega \simeq -\Delta L_{\rm vort}/L_{\rm vort}$, as they are dominated by the dispelling of vortices. One may naively expect to estimate $\Delta L_{\rm vort}$ as the number of vortices that unpin and reach the boundary multiplied by a quantum of angular momentum $\hbar$. Such an estimate is incorrect, because the angular momentum contribution from a vortex is reduced by the fraction of nonclassical moment of inertia $f_{\rm NCRI}$ [36], such that $L_{\rm vort}$ is at most $f_{\rm NCRI}\hbar N_v \leq \hbar N_v$ [82], for the total number of vortices N_v. Furthermore, in our finite-size system, the contribution reduces radially from the rotation axis. The combination of these phenomena is such that the effective amount of angular momentum lost by the superfluid component during a glitch is $\Delta L_{\rm vort} \simeq 10^{-2}\hbar$ [36]. This suggests that, in neutron-star glitches, the number of vortices involved in each glitch might be larger than the one estimated by assuming that each vortex carries a quantum of angular momentum.

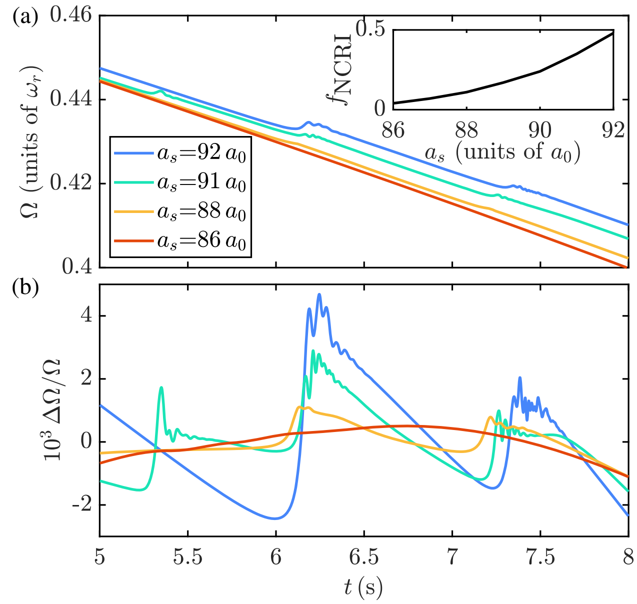

FIG. 3. Glitches originating from different radial depths. (a) Glitches as a function of the scattering length a_s. Note that $a_s = 92a_0$ emulates the conditions close to the inner crust-to-core boundary and $a_s = 86a_0$ for those in the outer crust. Inset: fraction of nonclassical rotational inertia. (b) Relative change in Ω, decreasing amplitude with scattering length. Some glitches dispel more than one vortex, increasing the amplitude.

A reduction of the vortex angular momentum due to the crystal structure also suggests that glitches in the case of vanishing superfluidity will have a small amplitude. We investigate the dependence of the glitch size on the superfluidity by varying the scattering length, as presented in Fig. 3. As the scattering length is decreased, we find that the glitch amplitude tends to decrease. When the state is in the independent droplet regime ($f_{\rm NCRI} \to 0$), glitches do not occur. The internal dynamics, though, still slightly affect the response of the system to the external torque, as indicated by the curvature of $\Delta\Omega/\Omega$. The largest glitches occur in the states with the biggest superfluid fraction and the largest pinning force between droplets. These results suggest that giant glitches in neutron stars occur from deep within the star, where the superfluid contribution to the angular momentum is largest. However, the total amplitude is also reflective of the number of unpinned vortices. The large glitch at 6.2 s with $a_s = 92a_0$ occurs when two vortices leave together. A possible identifier to discern the origin of the glitches can arise from the postglitch dynamics, which have the longest decay time at large scattering lengths.

This work represents a first step in simulating and understanding the complex dynamics of neutron stars using rotating quantum gases in the supersolid phase. We show that these systems exhibit phenomena analogous to neutron-star glitches and are primed to become a powerful tool for addressing key open questions ranging from the underlying mechanism of glitches to the system's internal dynamics. In particular, during a supersolid glitch, we observe rich dynamics: Some vortices unpin and escape toward the outer crust and, in doing so, trigger an excitation of the supersolid crystalline structure, as well as core shape deformation of the remaining migrating vortices. These dynamics, which cannot be captured in standard glitch models imposing a fixed lattice structure [70–72], could be the key for an experimental implementation of the model, where the dynamical observation of sudden changes in the droplet positions may be possible by combining optimal control methods with nondestructive imaging [84–86]. Moreover, we see that reducing the superfluidity of the supersolid leads to a reduction of the angular momentum contribution per vortex. This is a feature so far overlooked in the context of neutron stars and may explain the wide range of observed glitch amplitudes, where the smallest glitches are associated with vortex dynamics at the edge of the star.

Regarding the region of the inner crust close to the core, its investigation requires testing various lattice sizes and vortex configurations, allowing us to expand the study to nuclear vortex pinning expected to occur there [87], akin to the work in Ref. [88]. Furthermore, one could consider systems with a radially variable superfluid fraction to mimic the full structure of the neutron star. Our work opens the door for a detailed study of the droplet lattice vibration, in order to ascertain whether it is possible to extract the elastic properties of the solid from the supersolid glitch pulse shape. This would be of great astrophysical interest and would pave the way to extract the elastic properties of nuclear matter from the observed neutron-star glitch pulse shape and to test whether a glitch can trigger superfluid collective excitations [89]. Finally, future work can investigate the effects of tilting the magnetic field with respect to the rotation axis [19,90,91], as expected in pulsars, and include coupling between the supersolid and the proton type-II superconductor present in the crust, through an additional Ginzburg-Landau equation [92–94], introducing a self-consistent feedback mechanism.

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[1] E. Altman *et al.*, Quantum simulators: Architectures and opportunities, PRX Quantum **2**, 017003 (2021).
[2] S. Shapiro and S. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects*, A Wiley-Interscience Publication (Wiley, New York, 1983), 10.1002/9783527617661.
[3] N. K. Glendenning, *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity* (Springer Verlag, New York, 2000), 10.1007/978-1-4684-0491-3.
[4] P. Haensel, A. Y. Potekhin, and D. G. Yakovlev, *Neutron Stars 1: Equation of State and Structure* (Springer, New York, 2007), Vol. 326, 10.1007/978-0-387-47301-7.
[5] A. Migdal, Superfluidity and the moments of inertia of nuclei, Nucl. Phys. **13**, 655 (1959).
[6] B. Haskell and A. Sedrakian, Superfluidity and superconductivity in neutron stars, Astrophysics and Space Science Library **457**, 401 (2018).
[7] A. Hewish, S. J. Bell, J. D. Pilkington, P. Frederick Scott, and R. A. Collins, Observation of a rapidly pulsating radio source, Nature (London) **217**, 709 (1968).
[8] T. Gold, Rotating neutron stars as the origin of the pulsating radio sources, Nature (London) **218**, 731 (1968).
[9] R. N. Manchester, G. B. Hobbs, A. Teoh, and M. Hobbs, The australia telescope national facility pulsar catalogue, Astron. J. **129**, 1993 (2005).
[10] O. Hamil, J. R. Stone, M. Urbanec, and G. Urbancová, Braking index of isolated pulsars, Phys. Rev. D **91**, 063007 (2015).
[11] V. M. Kaspi and M. Kramer, Radio pulsars: The neutron star population & fundamental physics, in *Proceedings of the 26th Solvay Conference on Physics: Astrophysics and Cosmology* (2016); arXiv:1602.07738.
[12] S. Zhou, E. Gügercinoğlu, J. Yuan, M. Ge, and C. Yu, Pulsar glitches: A review, Universe **8**, 641 (2022).
[13] D. Tsakadze, S. D. Tsakadze *et al.*, Measurement of the relaxation time on acceleration of vessels with helium II and superfluidity in pulsars, Sov. Phys.-JETP **37**, 918 (1973), http://jetp.ras.ru/cgi-bin/dn/e_037_05_0918.pdf.

[14] J. Tsakadze and S. Tsakadze, Properties of slowly rotating helium II and the superfluidity of pulsars, J. Low Temp. Phys. **39**, 649 (1980).

[15] V. Graber, N. Andersson, and M. Hogg, Neutron stars in the laboratory, Int. J. Mod. Phys. D **26**, 1730015 (2017).

[16] M. A. Norcia, C. Politi, L. Klaus, E. Poli, M. Sohmen, M. J. Mark, R. N. Bisset, L. Santos, and F. Ferlaino, Two-dimensional supersolidity in a dipolar quantum gas, Nature (London) **596**, 357 (2021).

[17] T. Bland, E. Poli, C. Politi, L. Klaus, M. A. Norcia, F. Ferlaino, L. Santos, and R. N. Bisset, Two-dimensional supersolid formation in dipolar condensates, Phys. Rev. Lett. **128**, 195302 (2022).

[18] M. A. Norcia, E. Poli, C. Politi, L. Klaus, T. Bland, M. J. Mark, L. Santos, R. N. Bisset, and F. Ferlaino, Can angular oscillations probe superfluidity in dipolar supersolids?, Phys. Rev. Lett. **129**, 040403 (2022).

[19] L. Klaus, T. Bland, E. Poli, C. Politi, G. Lamporesi, E. Casotti, R. N. Bisset, M. J. Mark, and F. Ferlaino, Observation of vortices and vortex stripes in a dipolar condensate, Nat. Phys. **18**, 1453 (2022).

[20] P. Haensel and B. Pichon, Experimental nuclear masses and the ground state of cold dense matter, Astron. Astrophys. **283**, 313 (1994).

[21] J. M. Lattimer and M. Prakash, Neutron star structure and the equation of state, Astrophys. J. **550**, 426 (2001).

[22] F. Douchin and P. Haensel, A unified equation of state of dense matter and neutron star structure, Astron. Astrophys. **380**, 151 (2001).

[23] A. Y. Potekhin, A. F. Fantina, N. Chamel, J. M. Pearson, and S. Goriely, Analytical representations of unified equations of state for neutron-star matter, Astron. Astrophys. **560**, A48 (2013).

[24] B. K. Sharma, M. Centelles, X. Viñas, M. Baldo, and G. F. Burgio, Unified equation of state for neutron stars on a microscopic basis, Astron. Astrophys. **584**, A103 (2015).

[25] D. Blaschke and N. Chamel, Phases of dense matter in compact stars, Astrophysics and Space Science Library **457**, 337 (2018).

[26] G. Fiorella Burgio and A. F. Fantina, Nuclear equation of state for compact stars and supernovae, Astrophysics and Space Science Library **457**, 255 (2018).

[27] J. W. Negele and D. Vautherin, Neutron star matter at subnuclear densities, Nucl. Phys. **A207**, 298 (1973).

[28] T. Maruyama, T. Tatsumi, D. N. Voskresensky, T. Tanigawa, and S. Chiba, Nuclear, pasta structures and the charge screening effect, Phys. Rev. C **72**, 015802 (2005).

[29] P. Gögelein and H. Müther, Nuclear matter in the crust of neutron stars, Phys. Rev. C **76**, 024312 (2007).

[30] W. G. Newton and J. R. Stone, Modeling nuclear "pasta" and the transition to uniform nuclear matter with the 3d skyrme-hartree-fock method at finite temperature: Core-collapse supernovae, Phys. Rev. C **79**, 055801 (2009).

[31] F. Grill, J. Margueron, and N. Sandulescu, Cluster structure of the inner crust of neutron stars in the Hartree-Fock-Bogoliubov approach, Phys. Rev. C **84**, 065801 (2011).

[32] J. M. Pearson, N. Chamel, S. Goriely, and C. Ducoin, Inner crust of neutron stars with mass-fitted skyrme functionals, Phys. Rev. C **85**, 065803 (2012).

[33] The actual composition of the core is unknown: It is believed to be made of about 90% of neutrons and 10% of protons and electrons, but also muons or other baryons, like Δ or Σ, may be present, as well as deconfined quark matter [34,35].

[34] M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schafer, Color superconductivity in dense quark matter, Rev. Mod. Phys. **80**, 1455 (2008).

[35] R. Anglani, R. Casalbuoni, M. Ciminale, N. Ippolito, R. Gatto, M. Mannarelli, and M. Ruggieri, Crystalline color superconductors, Rev. Mod. Phys. **86**, 509 (2014).

[36] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.131.223401 for further details on supersolidity in neutron stars, the eGPE simulations, derivation of the glitch model, discussion on the pinning force, analysis on varying the model parameters N_{em}, γ, and the initial rotation frequency, and Supplemental videos for the data in Figs. 2 and 3. This includes additional Refs. [37–56].

[37] N. Chamel and P. Haensel, Physics of neutron star crusts, Living Rev. Relativity **11**, 10 (2008).

[38] J. A. Sauls, Superfluidity in the interiors of neutron stars in *Timing Neutron Stars. NATO ASI Series*, edited by H. Ögelman, E. P. J. van den Heuvel (Springer, Dordrecht, 1989), Vol. 262, 10.1007/978-94-009-2273-0_43.

[39] M. Baldo, U. Lombardo, E. E. Saperstein, and S. V. Tolokonnikov, The role of superfluidity in the structure of the neutron star inner crust, Nucl. Phys. **A750**, 409 (2005).

[40] N. Sandulescu, N. Van Giai, and R. J. Liotta, Superfluid properties of the inner crust of neutron stars, Phys. Rev. C **69**, 045802 (2004).

[41] H. S. Than, E. Khan, and N. Van Giai, Wigner-Seitz cells in neutron star crust with finite range interactions, J. Phys. G **38**, 025201 (2011).

[42] E. Khan, N. Sandulescu, and N. Van Giai, Collective excitations in the inner crust of neutron stars: Supergiant resonances, Phys. Rev. C **71**, 042801(R) (2005).

[43] T. D. Lee, K. Huang, and C. N. Yang, Eigenvalues and eigenfunctions of a Bose system of hard spheres and its low-temperature properties, Phys. Rev. **106**, 1135 (1957).

[44] S. Ronen, D. C. E. Bortolotti, D. Blume, and J. L. Bohn, Dipolar Bose-Einstein condensates with dipole-dependent scattering length, Phys. Rev. A **74**, 033611 (2006).

[45] W. Bao and H. Wang, An efficient and spectrally accurate numerical method for computing dynamics of rotating Bose–Einstein condensates, J. Comput. Phys. **217**, 612 (2006).

[46] A. M. Martin, N. G. Marchant, D. H. J. O'Dell, and N. G. Parker, Vortices and vortex lattices in quantum ferrofluids, J. Phys. Condens. Matter **29**, 103004 (2017).

[47] A. S. Bradley and B. P. Anderson, Energy spectra of vortex distributions in two-dimensional quantum turbulence, Phys. Rev. X **2**, 041001 (2012).

[48] S. M. Roccuzzo and F. Ancilotto, Supersolid behavior of a dipolar Bose-Einstein condensate confined in a tube, Phys. Rev. A **99**, 041601(R) (2019).
[49] A. Cidrim, F. E. A. dos Santos, E. A. L. Henn, and T. Macrì, Vortices in self-bound dipolar droplets, Phys. Rev. A **98**, 023618 (2018).
[50] A.-C. Lee, D. Baillie, R. N. Bisset, and P. B. Blakie, Excitations of a vortex line in an elongated dipolar condensate, Phys. Rev. A **98**, 063620 (2018).
[51] A.-C. Lee, D. Baillie, and P. B. Blakie, Numerical calculation of dipolar-quantum-droplet stationary states, Phys. Rev. Res. **3**, 013283 (2021).
[52] A. J. Leggett, Can a solid be "superfluid"?, Phys. Rev. Lett. **25**, 1543 (1970).
[53] P. B. Jones, The origin of pulsar glitches, Mon. Not. R. Astron. Soc. **296**, 217 (1998).
[54] M. Mannarelli, K. Rajagopal, and R. Sharma, The rigidity of crystalline color superconducting quark matter, Phys. Rev. D **76**, 074026 (2007).
[55] A. R. P. Lima and A. Pelster, Quantum fluctuations in dipolar Bose gases, Phys. Rev. A **84**, 041604(R) (2011).
[56] R. Schützhold, M. Uhlmann, Y. Xu, and U. R. Fischer, Mean-field expansion in Bose-Einstein condensates with finite-range interactions, Int. J. Mod. Phys. B **20**, 3555 (2006).
[57] G. Baym, C. Pethick, D. Pines, and M. Ruderman, Spin up in neutron stars: The future of the vela pulsar, Nature (London) **224**, 872 (1969).
[58] M. Ruderman, Pulsars: Structure and dynamics, Annu. Rev. Astron. Astrophys. **10**, 427 (1972).
[59] D. Pines, Neutron stars as cosmic hadron physics laboratories: What glitches teach us, in *Neutron Stars: Theory and Observation*, edited by J. Ventura and D. Pines (Springer Netherlands, Dordrecht, 1991), pp. 57–70, 10.1007/978-94-011-3536-8_5.
[60] B. Haskell and A. Melatos, Models of pulsar glitches, Int. J. Mod. Phys. D **24**, 1530008 (2015).
[61] B. Link, R. I. Epstein, and J. M. Lattimer, Pulsar constraints on neutron star structure and equation of state, Phys. Rev. Lett. **83**, 3362 (1999).
[62] J.-R. Li, J. Lee, W. Huang, S. Burchesky, B. Shteynas, F. Ç. Top, A. O. Jamison, and W. Ketterle, A stripe phase with supersolid properties in spin–orbit-coupled Bose–Einstein condensates, Nature (London) **543**, 91 (2017).
[63] J. Léonard, A. Morales, P. Zupancic, T. Esslinger, and T. Donner, Supersolid formation in a quantum gas breaking a continuous translational symmetry, Nature (London) **543**, 87 (2017).
[64] F. Böttcher, J.-N. Schmidt, M. Wenzel, J. Hertkorn, M. Guo, T. Langen, and T. Pfau, Transient supersolid properties in an array of dipolar quantum droplets, Phys. Rev. X **9**, 011051 (2019).
[65] L. Tanzi, E. Lucioni, F. Famà, J. Catani, A. Fioretti, C. Gabbanini, R. N. Bisset, L. Santos, and G. Modugno, Observation of a dipolar quantum gas with metastable supersolid properties, Phys. Rev. Lett. **122**, 130405 (2019).
[66] L. Chomaz, D. Petter, P. Ilzhöfer, G. Natale, A. Trautmann, C. Politi, G. Durastante, R. M. W. van Bijnen, A. Patscheider, M. Sohmen, M. J. Mark, and F. Ferlaino, Long-lived and transient supersolid behaviors in dipolar quantum gases, Phys. Rev. X **9**, 021012 (2019).
[67] M. Lu, N. Q. Burdick, S. H. Youn, and B. L. Lev, Strongly dipolar Bose-Einstein condensate of dysprosium, Phys. Rev. Lett. **107**, 190401 (2011).
[68] K. Aikawa, A. Frisch, M. Mark, S. Baier, A. Rietzler, R. Grimm, and F. Ferlaino, Bose-Einstein condensation of erbium, Phys. Rev. Lett. **108**, 210401 (2012).
[69] L. Chomaz, I. Ferrier-Barbut, F. Ferlaino, B. Laburthe-Tolra, B. L. Lev, and T. Pfau, Dipolar physics: A review of experiments with magnetic quantum gases, Rep. Prog. Phys. **86**, 026401 (2022).
[70] L. Warszawski and A. Melatos, Gross-Pitaevskii model of pulsar glitches, Mon. Not. R. Astron. Soc. **415**, 1611 (2011).
[71] L. Warszawski, A. Melatos, and N. G. Berloff, Unpinning triggers for superfluid vortex avalanches, Phys. Rev. B **85**, 104503 (2012).
[72] L. Warszawski and A. Melatos, Knock-on processes in superfluid vortex avalanches and pulsar glitch statistics, Mon. Not. R. Astron. Soc. **428**, 1911 (2013).
[73] P. W. Anderson and N. Itoh, Pulsar glitches and restlessness as a hard superfluidity phenomenon, Nature (London) **256**, 25 (1975).
[74] A. Melatos, C. Peralta, and J. S. B. Wyithe, Avalanche dynamics of radio pulsar glitches, Astrophys. J. **672**, 1103 (2008).
[75] P. M. Pizzochero, Angular momentum transfer in vela-like pulsar glitches, Astrophys. J. Lett. **743**, L20 (2011).
[76] C. M. Espinoza, A. G. Lyne, B. W. Stappers, and M. Kramer, A study of 315 glitches in the rotation of 102 pulsars, Mon. Not. R. Astron. Soc. **414**, 1679 (2011).
[77] F. Wächtler and L. Santos, Quantum filaments in dipolar Bose-Einstein condensates, Phys. Rev. A **93**, 061603(R) (2016).
[78] R. N. Bisset, R. M. Wilson, D. Baillie, and P. B. Blakie, Ground-state phase diagram of a dipolar condensate with quantum fluctuations, Phys. Rev. A **94**, 033619 (2016).
[79] I. Ferrier-Barbut, H. Kadau, M. Schmitt, M. Wenzel, and T. Pfau, Observation of quantum droplets in a strongly dipolar Bose gas, Phys. Rev. Lett. **116**, 215301 (2016).
[80] L. Chomaz, S. Baier, D. Petter, M. J. Mark, F. Wächtler, L. Santos, and F. Ferlaino, Quantum-fluctuation-driven crossover from a dilute Bose-Einstein condensate to a macrodroplet in a dipolar quantum fluid, Phys. Rev. X **6**, 041039 (2016).
[81] S. M. Roccuzzo, A. Gallemí, A. Recati, and S. Stringari, Rotating a supersolid dipolar gas, Phys. Rev. Lett. **124**, 045702 (2020).
[82] A. Gallemí, S. M. Roccuzzo, S. Stringari, and A. Recati, Quantized vortices in dipolar supersolid Bose-Einstein-condensed gases, Phys. Rev. A **102**, 023322 (2020).
[83] F. Ancilotto, M. Barranco, M. Pi, and L. Reatto, Vortex properties in the extended supersolid phase of dipolar Bose-Einstein condensates, Phys. Rev. A **103**, 033314 (2021).
[84] S. van Frank, M. Bonneau, J. Schmiedmayer, S. Hild, C. Gross, M. Cheneau, I. Bloch, T. Pichler, A. Negretti,

T. Calarco *et al.*, Optimal control of complex atomic quantum systems, Sci. Rep. **6**, 34187 (2016).
[85] D. Freilich, D. Bianchi, A. Kaufman, T. Langin, and D. Hall, Real-time dynamics of single vortex lines and vortex dipoles in a Bose-Einstein condensate, Science **329**, 1182 (2010).
[86] M. Gajdacz, P. L. Pedersen, T. Mørch, A. J. Hilliard, J. Arlt, and J. F. Sherson, Non-destructive faraday imaging of dynamically controlled ultracold atoms, Rev. Sci. Instrum. **84**, 083105 (2013).
[87] B. K. Link and R. I. Epstein, Mechanics and energetics of vortex unpinning in neutron stars, Astrophys. J. **373**, 592 (1991).
[88] A. Gallemí and L. Santos, Superfluid properties of a honeycomb dipolar supersolid, Phys. Rev. A **106**, 063301 (2022).
[89] N. Andersson, D. Antonopoulou, C. M. Espinoza, B. Haskell, and W. C. G. Ho, The enigmatic spin evolution of psr j0537–6910: r-modes, gravitational waves, and the case for continued timing, Astrophys. J. **864**, 137 (2018).
[90] S. B. Prasad, T. Bland, B. C. Mulkerin, N. G. Parker, and A. M. Martin, Vortex lattice formation in dipolar Bose-Einstein condensates via rotation of the polarization, Phys. Rev. A **100**, 023625 (2019).
[91] T. Bland, G. Lamporesi, M. J. Mark, and F. Ferlaino, Vortices in dipolar Bose-Einstein condensates, Comptes Rendus. Physique 1.
[92] L. Drummond and A. Melatos, Stability of interlinked neutron vortex and proton flux tube arrays in a neutron star: Equilibrium configurations, Mon. Not. R. Astron. Soc. **472**, 4851 (2017).
[93] L. Drummond and A. Melatos, Stability of interlinked neutron vortex and proton flux-tube arrays in a neutron star–II. Far-from-equilibrium dynamics, Mon. Not. R. Astron. Soc. **475**, 910 (2018).
[94] K. Thong, A. Melatos, and L. Drummond, Stability of interlinked neutron vortex and proton flux-tube arrays in a neutron star–III. Proton feedback, Mon. Not. R. Astron. Soc. **521**, 5724 (2023).