Colorful Mirror Solution to the Strong *CP* Problem

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We propose theories of a complete mirror world with parity (P) solving the strong CP problem. P exchanges the entire standard model with its mirror copy. We derive bounds on the two new mass scales that arise: v' where parity and mirror electroweak symmetry are spontaneously broken, and v_3 where the color groups break to the diagonal strong interactions. The strong CP problem is solved even if $v_3 \ll v'$, when heavy colored states at the scale v_3 may be accessible at LHC and future colliders. Furthermore, we argue that the breaking of P introduces negligible contributions to $\bar{\theta}_{\rm QCD}$, starting at three-loop order. The symmetry breaking at v_3 can be made dynamical, without introducing an additional hierarchy problem.

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Introduction.—The QCD Lagrangian contains a CP-odd term,

$$\bar{\theta}_{\rm QCD}\frac{g_s^2}{32\pi^2}G_a^{\mu\nu}\tilde{G}_{a,\mu\nu}, \tag{1}$$

where $G_a^{\mu\nu}$ is the gluon field strength tensor, g_s the strong coupling constant, $\tilde{G}_{a,\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G_a^{\alpha\beta}$, and $\bar{\theta}_{\rm QCD}$ is an angle which quantifies the breaking of CP in strong interactions. It is a free parameter of QCD, however, experimental constraints on the electric dipole moment of the neutron imply $\bar{\theta}_{\rm QCD} \lesssim 10^{-10}$ [1]. The lack of understanding of the smallness of $\bar{\theta}_{\rm QCD}$ has been dubbed the strong CP problem. The puzzle is made even sharper by the presence of weak interactions, which are such that $\bar{\theta}_{\rm QCD}$ receives a contribution through the chiral transformation needed to diagonalize the quark mass matrix M,

$$\bar{\theta}_{\rm QCD} = \theta_{\rm QCD} + \arg\det(M), \tag{2}$$

where $\theta_{\rm QCD}$ is the bare theta angle. The two contributions to $\bar{\theta}_{\rm QCD}$ arise from very different physics and have no reason to cancel in the standard model.

Three approaches to this problem have received considerable attention in the literature: a massless quark [2–5], spontaneously broken P or CP symmetries [6–16], or a spontaneously broken global chiral symmetry à la Peccei-Quinn [17–20].

Although it was recognized in the 1970s that parity might solve the strong CP problem [21,22], early attempts to construct such theories, based on the left-right extension of the electroweak group to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$, were unsuccessful, until Babu and Mohapatra discovered a solution in a simple model with a separate Higgs doublet for each SU(2) group [6]. In this model, P forces $\theta_{\rm QCD}$ to zero and the fermion Yukawa matrices to be Hermitian, hence $\bar{\theta}_{\rm QCD} = 0$ at tree level. Shortly after, the same authors UV completed their construction through a see-saw mechanism involving heavy vectorlike fermions [7]. A realistic vacuum can occur at tree level, via a soft breaking of P, or can arise radiatively [14]. In both cases, the resulting radiative corrections to $\bar{\theta}_{\rm QCD}$ occur at two-loop order and can be small enough to solve the strong CP problem, while offering the prospect of an observable neutron electric dipole moment [14,23,24]. An alternative model was constructed in Ref. [8] by doubling the SM electroweak group to $[\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y] \times [\mathrm{SU}(2)' \times \mathrm{U}(1)']$. A single SM-like Higgs doublet and three generations of SM-like fermions were introduced for each of the SM and mirror electroweak sectors. Both sectors share a common strong interaction. P forces $\theta_{\rm QCD}$ to zero, while the Yukawa matrices for SM fermions are arbitrary but Hermitian conjugates of those of mirror fermions, so that the quark contributions to $\bar{\theta}_{\rm QCD}$ are canceled by the mirror contributions. A hierarchy of Higgs vacuum expectation values (VEVs), $v' \gg v$, can again be obtained either by soft P breaking or by radiative contributions to the Higgs potential [15]; in both cases the contributions to $\bar{\theta}_{\rm QCD}$ arise at three loops, as with radiative contributions in the SM [25], and are small. In a final comment of Ref. [8], it was suggested that this theory could be unified, into an $\mathrm{SU}(5) \times \mathrm{SU}(5)'$ or $\mathrm{SO}(10) \times \mathrm{SO}(10)'$ theory. The resulting $\mathrm{SU}(3) \times \mathrm{SU}(3)'$ group of strong interactions would be reduced to QCD by breaking to the diagonal $\mathrm{SU}(3)_{\mathrm{QCD}}$ combination, with P then fixing the $\bar{\theta}$ parameter of QCD to zero. Such unified theories have not been constructed, and we are not aware of any discussions of this mechanism in the literature.

In this Letter, we propose the simplest theory of a complete mirror world with P solving the strong CP problem, and discuss its phenomenology. Parity exchanges the entire SM for its mirror copy and there are only two relevant free parameters beyond those of the SM. One is the mass scale v', where parity and mirror electroweak symmetry are spontaneously broken, and the other is the mass scale v_3, where $\mathrm{SU}(3)\times\mathrm{SU}(3)'$ breaks to $\mathrm{SU}(3)_{\mathrm{QCD}}$. Importantly, the strong CP problem is solved even if $v_3 \ll v'$.

Solution to the strong CP problem.—We mirror the full SM gauge group and therefore start with $\mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{SU}(3)'\times\mathrm{SU}(2)'\times\mathrm{U}(1)'$. In left-right theories, one leaves the fermion content of the SM unchanged, and the right-handed (RH) fermions are grouped into prime electroweak doublets. Here, however, we cannot simply assign prime color to such a doublet, as that would generate $\mathrm{SU}(3)^{(\prime)3}$ gauge anomalies. Therefore, we are led to a complete mirror theory, where one also doubles the fermionic spectrum. We denote the mirror partners of the SM fields with a prime. In particular, we have two Higgs bosons, each charged under only one of the two worlds and responsible for the breaking of its electroweak sector. Our mirror world Lagrangian thus reads

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{SM}'} + \tilde{\lambda}|H|^2|H'|^2, \tag{3}$$

where $\mathcal{L}_{\mathrm{SM}'}$ has the same form as the SM Lagrangian, but all fields and couplings are primed. For simplicity, the kinetic mixing [26] and dimension-5 neutrino mass operators are not shown, as we do not need them for our analysis of the strong CP problem.

The gauge and field content of the model is now such that one can pair the fields via spacetime parity. More precisely, we compose the usual action of P with a $\mathbb{Z}_2$ symmetry which exchanges the SM and mirror fields. At the level of the gauge bosons, one has

$$A^a_\mu(t,\vec{r}) \xrightarrow{\mathbb{Z}_2} A'^a_\mu(t,\vec{r}) \xrightarrow{P} A'^{a\mu}(t,-\vec{r}). \tag{4}$$

(Because of the independent C invariance of each Yang-Mills theory, even with θ terms, it is actually equivalent to impose either P or CP. See Supplemental Material [27]. For simplicity, in the following we impose P.) Parity interchanges left-handed (LH) and RH fermions, and hence sends

$$Q(t,\vec{r}) \xrightarrow{P\circ\mathbb{Z}_2} \gamma^0 Q'^c(t,-\vec{r}), \tag{5}$$

with Q' in the $\bar{\mathbf{3}}$ of $\mathrm{SU}(3)'$, and similarly for the other fermions. It also exchanges $H \leftrightarrow \tilde{H}'$. The θ angles are odd under P, hence $\theta = -\theta'$ and $\mathcal{L}$ only contains

$$\frac{\theta}{16\pi^2}\left[g_3^2\mathrm{Tr}(G\tilde{G}) - g_3'^2\mathrm{Tr}(G'\tilde{G}')\right]. \tag{6}$$

Finally, parity imposes that the SM and mirror gauge couplings are equal, in particular $g_3 = g_3'$, but we keep the two couplings explicit in (6) since they will run differently below the scale of P breaking. From (6), it appears clearly that breaking $\mathrm{SU}(3)\times\mathrm{SU}(3)'$ to its diagonal subgroup [then identified with $\mathrm{SU}(3)_{\mathrm{QCD}}$] provides a perfect cancellation of $\overline{\theta}_{\mathrm{QCD}}$, as long as no new phases are introduced by the sector that generates this breaking. This discussion accounts also for the electroweak contributions: P sends $\bar{Q}u\tilde{H}$ to $(\bar{Q}'u'\tilde{H}')^\dagger$, hence the Yukawa matrices in the SM and mirror sectors are Hermitian conjugates of one another. Thus, the $\overline{\theta}^{(\prime)}$ angles also appear as in Eq. (6). We therefore extend the Lagrangian of our model in order to accommodate the color breaking:

$$\mathcal{L} \to \mathcal{L} + \mathcal{L}_{\mathrm{breaking}}. \tag{7}$$

We discuss $\mathcal{L}_{\mathrm{breaking}}$ in more detail below.

$\mathrm{SU}(3)\times\mathrm{SU}(3)'$ *breaking.*—The mechanism presented in Sec. II does not depend on a specific symmetry breaking sector, $\mathcal{L}_{\mathrm{breaking}}$, as long as it provides the breaking pattern $\mathrm{SU}(3)\times\mathrm{SU}(3)'\to\mathrm{SU}(3)_{\mathrm{QCD}}$. We present here a simple realization of this symmetry breaking by the VEV of a bifundamental scalar field $\Sigma_{ii'}$. We nevertheless stress that this realization is by no means unique and different scenarios have different particle spectra and phenomenological signatures. For instance, models that dynamically break the color groups to their diagonal subgroup are discussed in Supplemental Material [27].

Modulo conjugation, we can consider two cases for the charges of Σ: $(\mathbf{3},\mathbf{3})$ or $(\mathbf{3},\bar{\mathbf{3}})$. When Σ acquires a VEV proportional to the identity matrix, $\langle\Sigma\rangle = (v_3/\sqrt{6})\mathbf{1}$, the vacuum preserves the diagonal $\mathrm{SU}(3)_{\mathrm{QCD}}$. Projecting onto the massless gluons, one finds $g_3^2 G\tilde{G} = g_3'^2 G'\tilde{G}'$, confirming the cancellation of θ_{QCD} from Eq. (6) (see Supplemental Material [27] for details). Anticipating the discussion in Sec. IV, we stress that this cancellation holds even when $g_3 \neq g_3'$, as long as $\theta = -\theta'$. Such a VEV is easy to achieve via the most general potential of Σ, which reads

$$\begin{aligned} V(\Sigma) = &-m^2\mathrm{Tr}(\Sigma\Sigma^\dagger) + c\,\mathrm{Tr}^2(\Sigma\Sigma^\dagger) \\ &+ \tilde{c}\,\mathrm{Tr}(\Sigma\Sigma^\dagger)^2 + [\tilde{m}\det(\Sigma) + \mathrm{H.c.}]. \end{aligned} \tag{8}$$

In addition to this potential, $\mathcal{L}_{\mathrm{breaking}}$ contains mixing terms between Σ and the Higgs fields. Since $v_3 \gg v$, the couplings to H are irrelevant for our discussion, while the VEV of H' simply shifts the couplings shown in Eq. (8). The vacua of this model have been thoroughly studied in Ref. [36]: there are parameter ranges where the unbroken gauge symmetry in the global minimum is $\mathrm{U}(1)^2$, $\mathrm{SU}(2)^2\times\mathrm{U}(1)$, or $\mathrm{SU}(3)$. Only the latter is of interest for us, which is, for instance, the only (global and local) minimum when $m^2 \geq 0$ and $c, \tilde{c} \geq 0$ (see Ref. [36] for the complete set of conditions). Finally, we stress that with our choice of charges, Σ does not couple to fermions at the renormalizable level and its VEV does not reintroduce CP phases in their mass matrices.

We discuss the need for parity breaking in the next section, however here we note that if v_3 is larger than the parity breaking scale, the low energy description of our model coincides with models where only electroweak forces are mirrored [6–8,14–16]. On the other hand, experimental bounds are much stricter on the scale of parity breaking than they are on v_3, hence the scenario where v_3 is at the lowest possible scale is the most phenomenologically interesting and novel. Further details on the associated spectrum of physical scalars in the infrared are given in Supplemental Material [27].

Parity breaking and energy scales.—We showed in the previous sections how to obtain a perfect cancellation of $\bar{\theta}_{\mathrm{QCD}}$ when P connects the SM to its mirror copy. However, if P is unbroken, this possibility is ruled out by experiment. Collider and cosmological probes require the mirror sector to decouple at low energies, and therefore P must be broken at some high energy scale. This is most easily achieved by making the VEV of the mirror Higgs much larger than that of its SM companion: $v' \gg v$. Such a hierarchical vacuum can be obtained at tree level, via explicit soft breaking of parity [6,7,16], or through loop-induced corrections, as in Higgs parity [15]. These mechanisms for spontaneous breaking of parity in the electroweak sector can occur even when the Higgs doublets have quartic couplings to the colored Σ field, regardless of whether v_3 is larger or smaller than v'. In any case, since the parity breaking is spontaneous or soft, the mirror Yukawa matrices remain the Hermitian conjugates of those of the SM at the scale v'. The present solution to the strong CP problem is therefore completely defined by two energy scales: v' and v_3. The additional parameters associated with the specific $\mathrm{SU}(3) \times \mathrm{SU}(3)'$ breaking mechanism are not relevant for the strong CP problem, as long as they provide the right breaking pattern (but they are relevant for studying the phenomenology of any precise model).

Independently of the breaking mechanism, there is the requirement that the model solves the strong CP problem, despite parity being broken. Contributions to $\bar{\theta}_{\mathrm{QCD}}$ beyond those of Sec. II, which cancel, can be classical or quantum. Quantum contributions to $\bar{\theta}_{\mathrm{QCD}}$ are discussed in the next section. Classical ones yield an upper bound on v' due to expected new physics at most at the Planck scale. More precisely, there are dangerous dimension-six operators of the form

$$\frac{g_3^2 \mathrm{Tr} G\tilde{G}}{16\pi^2 M_P^2}\left(\lambda |H|^2 + \lambda' |H'|^2\right) - (g_3, G, H \leftrightarrow g_3', G', H'), \tag{9}$$

where the pattern of couplings is chosen so as to respect parity [37]. There can also be corrections to the Yukawa couplings of the form

$$\bar{Q}\frac{Y_{u,1}|H|^2 + Y_{u,2}|H'|^2}{M_P^2} u\tilde{H} + \begin{pmatrix} Y_{u,i}, Q, u, H \\ \leftrightarrow Y_{u,i}^{\dagger}, Q', u', H' \end{pmatrix}, \tag{10}$$

and similarly in the down sector. When the two Higgses receive different VEVs, such operators reintroduce $\bar{\theta}_{\mathrm{QCD}} \neq 0$. For order one Wilson coefficients, the presence of the first kind of operators imposes $v' \lesssim 10^{14}$ GeV, as shown in Fig. 1. The contribution of the second kind is enhanced by the inverse of the small quark Yukawas, strengthening the bound by roughly 2.5 orders of magnitude, unless the flavor structure of the matrices $Y_{u,i}$ is similar to that of Y_u in a full model of flavor. In that case, the bound is unchanged.

A second set of model-independent constraints comes from collider bounds on the mirror quarks, which become charged under $\mathrm{SU}(3)_{\mathrm{QCD}}$ below v_3. The lightest mirror quarks, in particular the mirror up-quark, are stable and, once pair-produced in p-p collisions, they form fractionally charged colorless bound states with SM quarks or gluons produced by their own color field [41]. Therefore, the best constraints come from LHC searches for heavy stable electrically charged particles [42,43]. We recasted the ATLAS search for stable gluinos and charginos of Ref. [43], finding a lower bound of $m_{u'} \gtrsim 1.3$ TeV. The Yukawas of the two worlds being related by parity, this translates into $v' \approx \sqrt{2} m_{u'}/Y_u \gtrsim 1.5 \times 10^8$ GeV, as shown in Fig. 1.

Further constraints come from the different runnings of the gauge couplings and the presence of additional fields. While the former are qualitatively independent of the $\mathrm{SU}(3) \times \mathrm{SU}(3)'$ breaking mechanism, the latter are

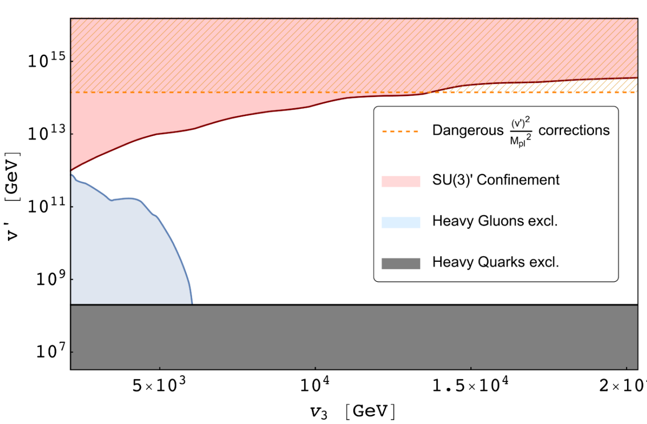

FIG. 1. Allowed parameter space in the v_3-v' plane. The region excluded by collider searches for heavy gluons (blue) and heavy quarks (gray), by the requirement that $\mathrm{SU}(3)'$ does not confine before v_3 (solid red line) and that higher dimensional operators do not reintroduce a sizable θ_{QCD} (dashed orange line) are shown.

strongly model dependent. The running of g_3 differs from that of g_3' below v', since the quarks are much heavier in the mirror sector [44]. As we anticipated in Sec. 3, this does not affect the cancellation of $\bar{\theta}_{\rm QCD}$. Nevertheless, we ought to require that $\mathrm{SU}(3)'$ does not confine before v_3, as it would modify the potential of the order parameter breaking $\mathrm{SU}(3)\times\mathrm{SU}(3)'$. The confinement of one of the two gauge groups does not ensure anymore that the two groups are broken to the diagonal subgroup and that the strong *CP* problem is solved. Since the gauge coupling running depends on the precise particle content, we consider for concreteness the case of the bifundamental scalar discussed in Sec. 3. We then obtain another upper bound for v' as a function of v_3, as shown in Fig. 1. Note that the gauge couplings must match to the QCD coupling constant at v_3 and be equal at v', hence their values and runnings are known for given v' and v_3.

The spectrum of the theory contains a massive color-octet vector coupled to (mirror) quarks and various scalar states, some of which are charged under $\mathrm{SU}(3)_{\rm QCD}$ (see Supplemental Material [27] for further details). Bounds from collider searches of the former have been extensively discussed in Refs. [28,45] and are shown in Fig. 1. Accounting for the requirement that $\mathrm{SU}(3)'$ does not confine, we find $v_3 \gtrsim 0.85$ TeV. For these values, the collider bounds on colored scalars can always be avoided by an appropriate choice of the parameters in the Σ potential [28], so we do not discuss them further. In other realizations of the color breaking mechanism, there may be scalars or fermions significantly lighter than the heavy gluons, resulting in the blue region extending toward the right side of the plot.

Quantum corrections to $\bar{\theta}$.—The fact that all classical sources of strong *CP* violation cancel to sufficient accuracy does not yet ensure that the model solves the strong *CP* problem: one also needs to check that the spontaneous breaking of parity does not reintroduce $\bar{\theta}_{\rm QCD}$ at the quantum level [23,25,46–48]. We find that, in our model, no contributions to $\bar{\theta}_{\rm QCD}$ exist before three-loop order.

To see this, it is useful to keep in mind that one needs both *P* and *CP* violation to generate a nonzero $\bar{\theta}_{\rm QCD}$. To begin with, the VEV of Σ does not spontaneously break $(C)P$ and does not introduce any new *CP* phase. Indeed, for real $\tilde{m}$ (which can always be achieved upon rephasing Σ), $\langle\Sigma\rangle$ can be chosen to be diagonal and real [36]. Therefore, the gauge, self-interactions, and VEV of Σ respect the various discrete symmetries which act as follows when Σ transforms as a $(\mathbf{3},\bar{\mathbf{3}})$ (see Supplemental Material [27] for further details),

$$\Sigma \xrightarrow{P\circ\mathbb{Z}_2} \Sigma^\dagger \xrightarrow{C} \Sigma^T. \tag{11}$$

In particular, a real $\tilde{m}$ is compatible with *P*. Similarly, the VEVs of the two Higgses can both be chosen real via gauge transformations, hence neither the scalar potential nor the scalar VEVs break *CP*. Therefore, the physical sources of *CP* violation are fully contained in the Yukawa matrices and are constrained by the large $\mathrm{U}(3)^6$ quark flavor symmetry: they reduce to two copies of the Jarlskog invariant of the SM [49,50]. At energies larger than v', parity equates them while below v', parity is broken and they run differently. Nevertheless, there are no diagrams contributing to $\bar{\theta}_{\rm QCD}$ before the three-loop order. The argument goes as follows: corrections to $\bar{\theta}_{\rm QCD}$ are associated with loop corrections to the two-point functions of fermions, while Jarlskog-like structures only arise in diagrams with at least four Cabibbo-Kobayashi-Maskawa insertions. It is quite simple to see that the simplest diagrams arise at two loops, with two *W* boson propagators closing onto a single quark line. However, those involve a single SM copy at a time and it has been shown in Ref. [25] that they vanish [51]. In our model, it also turns out that there are very few three-loop diagrams beyond those already considered by Ref. [25]. New diagrams would either mix the two SM copies, or involve the new boson Σ. Above the scale v_3, both kinds of diagrams require four *W* bosons, and at least two gluon lines in addition to a line of either Σ or a mirror fermion, or a vertex mixing H, H'. They are all at least four-loop suppressed. Below v_3, a single kind of new three-loop diagrams exists, namely the exact copies of the leading diagrams considered in Ref. [25], upon replacing massless gluon propagators by massive ones and $g_s \to (g_3/g_3')^{\pm 1} g_s$ for each of the two copies. Since our massive gluons are heavy and $g_3 \sim g_3'$, their contributions are at most comparable to that of massless gluons. We therefore conclude that loop contributions to $\bar{\theta}_{\rm QCD}$ in our model are comparable to those in the SM, and totally negligible. Let us stress that, although we explicitly referred to Σ in the previous discussion, the conclusion also holds for composite models, discussed in Supplemental Material [27]. Finally, there are nonperturbative contributions to $\bar{\theta}_{\rm QCD}$ which are sensitive to the nonzero *CP*-odd θ angles above v_3. Despite being nonperturbative, those effects can be sizable due to the fact that the gauge couplings of $\mathrm{SU}(3)\times\mathrm{SU}(3)'$ can be much larger than that of $\mathrm{SU}(3)_{\rm QCD}$ at v_3 [5,52–55]. The small instantons generate fermionic 't Hooft determinants [56] which lead to corrections to the fermion masses. As said above, the absence of free parameters beyond v_3 and v' in our model allows us to compute those terms unambiguously. Those involving the SM fermions are sufficiently suppressed by the product of all Yukawa couplings, while those involving the mirror fermions are suppressed by the small gauge couplings when v' is large. They can become sizable when the mirror up quark mass is close to v_3, but such situations also correspond to gauge couplings which remain small at v_3, as can be seen from Fig. 1. We have checked that the induced shift of $\bar{\theta}_{\rm QCD}$ is compatible with the current bounds, and does not exclude more regions of parameter space in Fig. 1.

Conclusion.—We have proposed simple theories of a complete mirror world where parity composed with the mirror exchange symmetry solves the strong *CP* problem. The new feature of our construction is the presence of a mirror strong interaction, and therefore of two nonvanishing θ angles. The solution to the strong *CP* problem and the experimental viability of the model rely on two symmetry breakings: the breaking of the color groups to their diagonal subgroup at the scale v_3 makes the effective low-energy $\bar{\theta}_{\rm QCD}$ angle vanish through destructive interference, while the colored mirror fermions are made heavy through the breaking of parity by a large mirror electroweak scale $v' \gg v$. We focus on the scenario where $v_3 \ll v'$; if $v_3 \geq v'$, the effective theory below v_3 is the model of Refs. [8,15]. In addition, saturating the experimental constraints allows v_3 to be much below v', leading to the richest phenomenology; new colored states may be accessible at colliders. Because of the symmetry structure of the model, the loop corrections to $\bar{\theta}_{\rm QCD}$ are shown to be under control everywhere in parameter space. We stress the high predictive power of this mirror world, with only two scales characterizing its qualitative features: v_3 and v'. All the other scales in the mirror world are related by parity to those in the SM. It is also worth noting the rich cosmology of our models due to the presence of heavy fermions, scalars, and vectors at different energy scales, as well as various phase transitions. These topics are currently under investigation.

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[26] Note that the kinetic mixing can be forbidden at tree level by embedding the U(1) gauge groups into a larger non-Abelian

group. Moreover, the loop contributions below the breaking scale of that new group are naturally suppressed in our model. Finally, collider bounds on the model given an $\mathcal{O}(1)$ kinetic mixing are weaker than the limits considered in the following and we do not consider cosmological ones, as the latter depend on assumptions on the cosmological history which we remain agnostic about in the present Letter and leave for future work.

[27] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.131.221802, which includes Refs. [28–35], for a discussion of how the symmetries (in particular, P and CP) are realized in the model, for a presentation of the states of masses $\sim v_3$, and for models which achieve the breaking $\mathrm{SU}(3) \times \mathrm{SU}(3)' \to \mathrm{SU}(3)_{\mathrm{QCD}}$ via new confining gauge groups.

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