Scrambling Transition in a Radiative Random Unitary Circuit

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We study quantum information scrambling in a random unitary circuit that exchanges qubits with an environment at a rate p. As a result, initially localized quantum information not only spreads within the system, but also spills into the environment. Using the out-of-time-order correlator (OTOC) to characterize scrambling, we find a nonequilibrium phase transition in the directed percolation universality class at a critical swap rate p_c: for $p < p_c$ the ensemble-averaged OTOC exhibits ballistic growth with a tunable light cone velocity, while for $p > p_c$ the OTOC fails to percolate within the system and vanishes uniformly within a finite timescale, indicating that all local operators are rapidly swapped into the environment. To elucidate its information-theoretic consequences, we demonstrate that the transition in operator spreading coincides with a transition in an observer's ability to decode the system's initial quantum information from the swapped-out, or "radiated," qubits. We present a simple decoding scheme which recovers the system's initial information with perfect fidelity in the nonpercolating phase, and with continuously decreasing fidelity with decreasing swap rate in the percolating phase. Depending on the initial state of the swapped-in qubits, we further observe a corresponding entanglement transition in the coherent information from the system into the radiated qubits.

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Understanding the complexity of quantum states and operators undergoing time evolution is a key challenge with potential implications across fields, from condensed matter physics, through quantum gravity, to quantum computation. In condensed matter physics, insights on the growth of operator complexity have inspired new ways of computing the dynamics of thermalizing systems [1–8]. Operator growth, as measured for example by out-of-time-order correlations (OTOCs) [9–12], is also considered as key to relating boundary to bulk dynamics in the conjectured AdS/CFT correspondence [13–19]. Furthermore, understanding complexity growth in terms of scrambling of quantum information has revealed connections between the dynamics of black holes and the capacity of artificial quantum circuits to encode and process quantum information [20–23].

Physical observables evolved by simple models of unstructured unitary circuits or of thermalizing many-body Hamiltonians are expected to scramble and grow in complexity indefinitely, or at least to astronomical timescales [19,24]. But these models may be too simplified in some cases. For example, as information is scrambled in a black hole some of it is lost to Hawking radiation [25]. Similarly decoherence in quantum circuits implies that some of the information is ultimately lost, or shared with external degrees of freedom [8,26,27]. Can there exist sharp thresholds or phase transitions in scrambling, or in the flow of quantum information, tuned by the rate of such loss processes?

Recently, it has been discovered that random unitary circuits (RUCs) interspersed with local projective measurements can exhibit two distinct dynamical phases, characterized respectively by the partial protection or rapid destruction of initially encoded quantum information, which are separated by a continuous phase transition at a nonzero critical measurement rate [26,28–34]. However, measurements are highly nonrepresentative of generic errors, and moreover, such measurement-induced phase transitions (MIPTs) typically face exponentially large postselection barriers to experimental observation [35–37]. It is natural to ask if a phase transition in scrambling and information flow can occur in a RUC without measurements, thereby avoiding the postselection problem altogether.

In this Letter, we present a simple model of a RUC that exhibits a phase transition from scrambling to nonscrambling dynamics. We extend previous works [38–43] exploring operator growth in closed-system RUCs by allowing the system to exchange qubits with an environment [44–47]; as a consequence, initially localized quantum information not only spreads within the system, but also spills into the environment. Using a mapping to a classical nonequilibrium statistical mechanics model of population dynamics, we show that the circuit exhibits tunable scrambling: increasing the rate of qubit swaps reduces the OTOC light cone velocity within the system until it vanishes at a critical swap rate. At this point, the

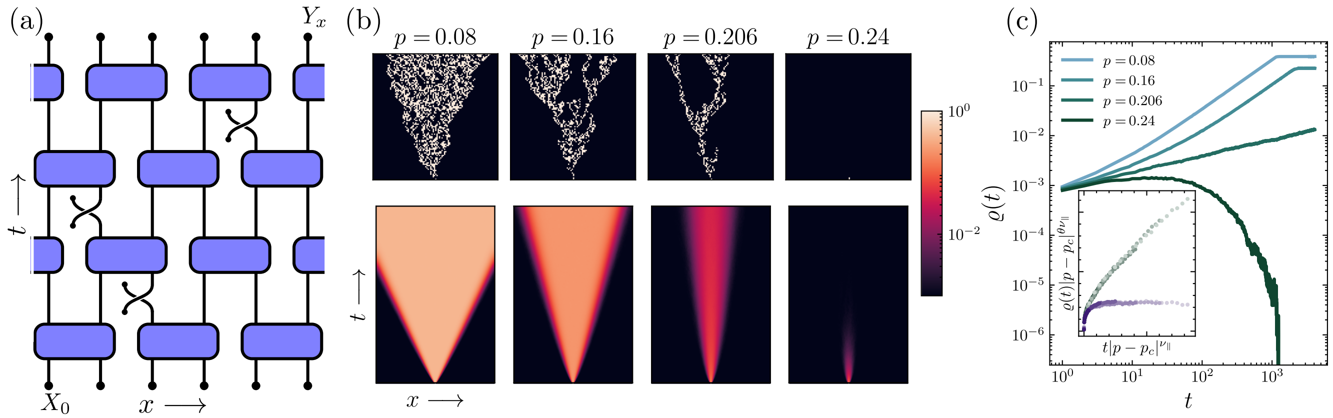

FIG. 1. (a) Circuit diagram for the model studied. An initial operator X_0 is Heisenberg evolved via a random unitary circuit. In between layers of unitary gates, swaps with ancilla qubits occur with probability p. The OTOC $C(x, t)$ is obtained from the commutator $[X_0(t), Y_x]$. (b) Top: OTOCs for typical Clifford circuit realizations, for four swap rates p, for a system of size $N = 100$. White denotes $C(x, t) = 1$, while black denotes $C(x, t) = 0$. Bottom: averaged OTOC for the same swap rates in a system of size $N = 1024$, depicting the narrowing and eventual vanishing of the light cone. Colors are plotted on a log scale for increased contrast. (c) Integrated OTOC $\varrho(t) = (1/N) \sum_x \overline{C(x, t)}$ for several swap rates p in a system of size $N = 1024$. $\varrho(t)$ exhibits linear growth and saturates at a finite value for $p < p_c$, exhibits power-law growth with exponent $\theta \simeq 0.3175$ at the critical point $p_c \simeq 0.206$, and rapidly decays to zero for $p > p_c$. Inset: scaling collapse for several swap rates below p_c (green) and above p_c (purple), using DP exponents $\theta_{\mathrm{DP}} \approx 0.3136$ and $\nu_{\parallel,\mathrm{DP}} \approx 1.734$ [51,60].

model exhibits a phase transition in the directed percolation (DP) universality class [48–51] to a nonscrambling phase. For swap rates above this threshold, all local operators initially within the system are rapidly swapped to the environment.

In contrast to previous works on operator growth in open systems [8,52–59], the transition in the OTOC described here requires that an observer has access to the swapped-out, or “radiated,” environment qubits. To determine the implications of the transition in operator spreading on the flow of quantum information, we consider a thought experiment in which an observer attempts to recover quantum information stored in the initial state of the system from the radiated qubits alone. Motivated by an analogy to previous studies of quantum information scrambling in black holes [21,23,45,46], we provide a simple algorithm by which an observer with access to the radiated qubits can decode this quantum information with perfect fidelity in the nonpercolating phase of the circuit, but with an imperfect fidelity set by the DP survival probability in the percolating phase. We numerically demonstrate a corresponding transition in the coherent information into the radiated qubits, which depends nontrivially on the observer's knowledge of the qubits swapped into the system.

Model.—We consider a one-dimensional system of N qubits with periodic boundary conditions undergoing brick-wall random unitary evolution [Fig. 1(a)]. Each two-qubit unitary gate is independently drawn from either the Haar or Clifford ensemble. Between layers of unitary gates, each system qubit is swapped with an environmental ancilla qubit with probability p; crucially, we use a fresh ancilla qubit for each such interaction, and we do not trace out the ancilla following the system-ancilla interaction. For now, we leave the initial state ρ_0^{SE} on the system S and environment E unspecified.

We study operator spreading in the RUC U_t by computing the out-of-time-order correlator (OTOC) [9,10,12,39,40], defined here as

$$C(x, t) = \frac{1}{4} \mathrm{tr}\Big\{\rho_0^{SE} [X_0(t), Y_x]^\dagger [X_0(t), Y_x]\Big\}, \qquad (1)$$

where Y_x is the Pauli-Y operator for the xth qubit, and $X_0(t) = U_t^\dagger X_0 U_t$ is the Heisenberg-evolved Pauli-X operator for the zeroth qubit after t layers of unitary gates.

References [39,40] previously studied operator spreading in closed-system RUCs using the OTOC. Upon introducing swap gates with an environment, a new physical feature emerges: the operator $X_0(t)$ not only spreads within the system, but also spills into the environment. By tuning the swap rate p, the rate of operator growth within the system can be slowed or even halted altogether.

To see this concretely, it is illuminating to consider the evolution of the OTOC in random Clifford circuits [61,62]; see the Supplemental Material [63] for a corresponding calculation for Haar-random circuits. Since Eq. (1) is second-order in $U_t \otimes U_t^*$ and the Clifford ensemble forms a unitary 3-design, the ensemble-averaged OTOC $\overline{C(x, t)}$ behaves identically in the Haar and Clifford circuits [72,73]. Whereas $X_0(t)$ evolves into a superposition of many Pauli strings in generic circuits, in a Clifford circuit $X_0(t)$ remains a single Pauli string at all times, and $C(x, t) = 1$ whenever $X_0(t)$ has Pauli content X or Z at site

x and vanishes otherwise. Noting that each of the three Pauli operators appear within $X_0(t)$ at a given site with equal probability, we can express $\overline{C(x,t)} = \frac{2}{3}\overline{n_x(t)}$ in terms of an occupation number $n_x(t)$ which equals one whenever $X_0(t)$ has nontrivial (i.e., nonidentity) Pauli content on site x in a given Clifford circuit.

We interpret the evolution of the ensemble $\{n_x(t)\}$ as a stochastic evolution of particles on a tilted square lattice. Vertices of the lattice sit at the center of unitary gates, and the occupation numbers $\{n_x(t)\}$ at each integer time step define a distribution of particles along the edges of the lattice. The rules for obtaining $\{n_x(t+1)\}$ from $\{n_x(t)\}$ are determined by noting that a two-qubit Clifford gate can evolve a nontrivial two-qubit Pauli string to any of 15 nontrivial two-qubit Pauli strings (up to phase) with equal probability, while the trivial Pauli string always evolves to the trivial Pauli string. Upon adding system-ancilla swap gates, the Pauli content of each site x has an additional probability p of swapping onto an environment qubit: effectively, the particle hops from the system to the environment, and the Pauli content of site x is set to the identity. We therefore obtain the following probabilities for the propagation of particles from an occupied vertex [74]:

$$t \quad \tfrac{3}{5}(1-p)^2, \quad \tfrac{1}{5}(1-p) + \tfrac{3}{5}p(1-p), \quad \tfrac{3}{5}p^2 + \tfrac{2}{5}p, \qquad (2)$$

where a dark line indicates the propagation of a particle, while a dotted line indicates no propagation of a particle. The case $p = 0$ returns the results of Refs. [39,40], which can be interpreted as the stochastic evolution of particles which can diffuse, spread, or coalesce, but never annihilate; the average behavior of the OTOC can be computed exactly in this case, and one finds a ballistic growth of the OTOC with a light cone front that broadens as $\sqrt{t}$.

In contrast, introducing $p > 0$ allows particles to annihilate, and yields the phenomenology of a DP problem [48,49,51]. The OTOC's light cone velocity continuously decreases with increasing p and vanishes at critical swap rate p_c, at which point there is a phase transition in the DP universality class [63]. For swap rates $p < p_c$, $X_0(t)$ percolates throughout the system qubits with a finite probability $P(t)$ corresponding to the survival probability of the associated DP process. On the other hand, for $p > p_c$ the particle distribution is rapidly driven to the absorbing state $n_x(t) = 0$; the entire operator $X_0(t)$ is swapped into the environment within a finite timescale, and $C(x,t)$ vanishes uniformly at all times thereafter. These qualitative features are observed in Clifford numerical simulations, as demonstrated in Fig. 1(b).

Note that the stochastic rules (2) are not microscopically equivalent to standard bond DP, due to correlations between the two edges leaving a vertex. This feature is not expected to affect the universal behavior of the two phases or the transition, in accordance with the DP hypothesis [75,76]. To confirm that the phase transition lies within the DP universality class, we use Clifford simulations to numerically compute the integrated OTOC $\varrho(t) = (1/N)\sum_x \overline{C(x,t)}$ for several swap rates [Fig. 1(c)]. We find that $\varrho(t)$ grows linearly and saturates at a nonzero value for $p < p_c$, while it rapidly decays to zero for $p > p_c$. At $p_c \simeq 0.206$, $\varrho(t) \sim t^{\theta}$ grows as a power law with exponent $\theta \simeq 0.3175$, in good quantitative agreement with the critical exponent $\theta_{\rm DP} \approx 0.3136$ governing the analogous growth of particle density in bond DP [60]. In the Supplemental Material [63] we present additional numerical evidence for DP universality at the phase transition.

Decoding transition.—The growth of the OTOC can be associated with the spreading of quantum information [12,23,77]. However, this interpretation has important subtleties in our model. Suppose that an observer Alice stores a k-qubit message in the initial state of the system, which then undergoes time evolution by the circuit U_t. It may be tempting to associate the percolating OTOC to a capacity for another observer, Bob, to recover Alice's message from the qubits remaining in the system. However, closer examination shows that even when local operators develop large support on the system, they have substantially larger support on the qubits swapped out to the environment. As a result, Bob can never recover any fraction of Alice's message at long times.

To explicate the connection between operator spreading and the flow of quantum information in our model, we instead ask whether an eavesdropper Eve, who collects the qubits swapped out of the system, can recover Alice's message. We demonstrate here that the transition in operator spreading coincides with a transition in the fidelity with which Eve can decode Alice's quantum information from the radiated qubits using a simple decoding protocol, shown in Fig. 2(a) and discussed below. In the discussion, we comment on an analogy between our model and that of the Hayden-Preskill thought experiment [21], and the relation between our decoding protocol and a similar protocol proposed for Hayden and Preskill's problem [23].

Alice's state is stored initially on the first k qubits of S, denoted S_1. The remaining $N-k$ qubits of S are denoted S_2. After evolving SE by the circuit U_t, Eve collects the radiated qubits E and attempts to decode Alice's state without access to S. To construct the decoder, Eve first introduces an extra set of N qubits $S' = S_1' \cup S_2'$ initialized in an arbitrary state, then applies the reverse unitary circuit $U_t^{\dagger}$ on $S'E$ [Fig. 2(a)]. Deep in the nonpercolating phase, where Alice's state is always swapped entirely into the environment, such a decoding protocol will perfectly reproduce Alice's state on S_1'.

To quantify the success of Eve's decoding for general p, we encode Alice's state using a reference system A initialized in a maximally entangled state $|\Phi^{+}_{AS_1}\rangle$ with S_1,

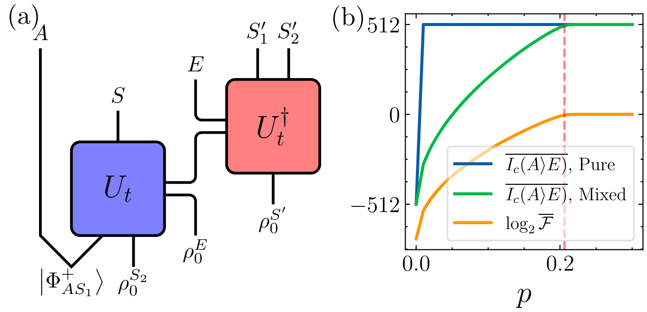

FIG. 2. (a) Simple decoding protocol for recovering quantum information encoded initially within the system. Following random unitary evolution with swap gates on SE via the circuit U_t [Fig. 1(a)], swapped-out or "radiated" qubits are swapped back into a second system S' via the reverse circuit $U_t^\dagger$. (b) Late-time log-averaged fidelity $\log_2 \overline{\mathcal{F}}$ (green) for the simple decoding protocol, and late-time average coherent information $\overline{I_c(A\rangle E)}$ for an initially maximally mixed environment (orange) and an initially pure environment (blue), as a function of swap rate p in a system of size $N = 512$. Averages are taken over 400 Clifford circuit realizations. The dotted red line denotes $p_c \simeq 0.206$, as estimated from the OTOC.

and compute the fidelity with the same state on AS_1' following the decoding protocol:

$$\mathcal{F}(t) = \mathrm{tr}\left\{\left|\Phi^+_{AS_1'}\right\rangle\left\langle\Phi^+_{AS_1'}\right|U_t'^{\dagger}U_t\rho_0 U_t^\dagger U_t'\right\}, \qquad (3)$$

where $\rho_0 = |\Phi^+_{AS_1}\rangle\langle\Phi^+_{AS_1}| \otimes \rho_0^{S_2S'E}$ is the initial state on $ASS'E$, which consists of the entangled state $|\Phi^+_{AS_1}\rangle\langle\Phi^+_{AS_1}|$ on AS_1 and an arbitrary product state $\rho_0^{S_2S'E}$ on the remaining qubits, and U_t' denotes the unitary circuit acting on $S'E$. A maximal fidelity $\mathcal{F} = 1$ implies that an arbitrary initial state on S_1 will be exactly reproduced on S_1' at the end of the protocol, while a fidelity $\mathcal{F} = 2^{-2k}$ indicates that the final state on S_1' is uncorrelated with the initial state of S_1.

For simplicity, we first assume that Alice's message contains $k = 1$ qubit. Then, it is straightforward to show that in a given Clifford circuit, $\mathcal{F}(t)$ simply counts which of the operators $X_0(t)$, $Y_0(t)$, and $Z_0(t)$ are swapped entirely into the environment by time t. Through the mapping to the stochastic model, this occurs for each such operator with probability $1 - P_1(t)$, where $P_1(t)$ is the survival probability of the associated DP process initialized with a single particle at $t = 0$. In the Supplemental Material [63], we show that this observation results in an average decoding fidelity $\overline{\mathcal{F}(t)} = 1 - \frac{3}{4}P_1(t)$ for $k = 1$ encoded qubit. More generally, for arbitrary k we obtain the bounds

$$1 - P_k(t)[1 - 2^{-2k}] \leq \overline{\mathcal{F}(t)} \leq 1 - P_1(t)[1 - 2^{-2k}], \qquad (4)$$

where $P_k(t)$ is the survival probability of k initial particles arranged side-by-side. Notably, since $\mathcal{F}(t)$ is second-order in $U_t \otimes U_t^*$, this result holds identically in both the Haar and Clifford ensembles. In the nonpercolating phase $p > p_c$ each survival probability falls to zero exponentially quickly, resulting in a perfect decoding fidelity at late times as expected. On the other hand, for small $p < p_c$ the survival probability is large, and the decoding fidelity is close to that of a random final state on AS_1'. The numerical late-time behavior of $\log_2 \overline{\mathcal{F}}$ for $k = N$ is shown in Fig. 2(b).

Information transition.—While we have shown that the decoding fidelity for the simple decoder of Fig. 2(a) undergoes the same transition as the OTOC, we are more generally interested in the maximal amount of quantum information Eve can recover from the radiated qubits E using *any* decoding protocol. In other words, we would like to characterize the quantum channel capacity of the circuit U_t, regarded as a noisy quantum channel from S_1 to E [78–83]. Towards this end we consider the coherent information $I_c(A\rangle E) = H_E - H_{AE}$ from A to E, where $H_R = -\mathrm{tr}\rho_t^R \log \rho_t^R$ is the von Neumann entropy of subsystem R. The coherent information can then be used to lower-bound the single-shot quantum channel capacity of the circuit [78,80,83,84]. In this section it is useful to focus on the case $k = N$, although the generalization to arbitrary k is straightforward [63].

Unlike the OTOC and the decoding fidelity given above, the behavior of $I_c(A\rangle E)$ depends strongly on the initial state of the qubits swapped into the system. First suppose that the swapped-in qubits are initialized in the maximally mixed state, $\rho_0^E = (1/2^{N_E})\mathbb{1}$, where $N_E \sim pNt$ is the total number of swaps; physically, this corresponds to the case in which Eve has no prior knowledge of the qubits swapped into the circuit. Then, one can show diagrammatically [63] that the subsystem purity $\mathrm{tr}[(\rho_t^{AE})^2] = 2^{N-N_E}\mathcal{F}$ is proportional to the decoding fidelity. This in turn implies that $I_c(A\rangle E) = N + \log_2\mathcal{F}$ in individual Clifford circuit realizations. Although the logarithm prevents a simple statistical mechanics interpretation for the *average* coherent information in either Clifford or Haar random circuits, we observe numerically [Fig. 2(b)] that $\overline{I_c(A\rangle E)}$ in the Clifford ensemble exhibits the same qualitative features as $N + \log_2 \overline{\mathcal{F}}$ and undergoes a transition at the same critical swap rate as the fidelity.

In contrast, suppose now that the swapped-in qubits are initialized in a definite pure state, $\rho_0^E = |0\rangle\langle 0|^{\otimes N_E}$. Physically, this case occurs when Eve has perfect knowledge of the initial state of each swapped-in qubit. Since the global state is now pure, we can compute $I_c(A\rangle E) = -I_c(A\rangle S)$ by tracing over the swapped-out qubits in E, upon which the swap operation becomes equivalent to an amplitude-damping channel [85]. The system density matrix ρ_t^S therefore evolves via a strictly contractive quantum channel with a unique fixed point, and rapidly forgets its initial conditions and approaches a unique steady state. As a result, we expect $\rho_t^{AS} \simeq \rho_t^A \otimes \rho_t^S$ to rapidly factorize into a product state. This implies that $I_c(A\rangle S) = -N$ after a finite timescale, indicating that no information can be transmitted from Alice to Bob through the system as expected. But for a globally pure state, this immediately suggests $I_c(A\rangle E) = N$ is maximal. We confirm numerically [Fig. 2(b)] that in this case the coherent information is indeed maximal for any $p > 0$. Physically, this result implies that Eve can in principle use her knowledge of the swapped-in qubits to decode Alice's information from the radiated qubits for any $p > 0$ [63].

Discussion.—We have demonstrated a DP phase transition in the operator dynamics and the flow of quantum information in a RUC which exchanges qubits with an environment. If an observer Alice stores a quantum message in the initial state of the system, another observer Eve can utilize a simple decoding protocol [Fig. 2(a)] to recover Alice's message from the radiated qubits with perfect fidelity in the nonpercolating phase $p > p_c$ and with imperfect fidelity in the percolating phase $p < p_c$.

The highly scrambling intrasystem unitary dynamics in our model plays an essential role in determining the critical swap rate p_c and in obtaining a transition at a nonzero p_c. Indeed, in an alternate model consisting of non-scrambling dynamics such as free fermion evolution, it is straightforward to show that the non-percolating phase occurs for all $p > 0$ [63]. In contrast, the use of swap gates for system-environment interactions is not a physically crucial feature. For example, generic Haar-random gates between the system and environment can also drive a DP transition if the intrasystem dynamics is less scrambling, or if multiple rounds of system-environment interactions are allowed between layers of intrasystem gates.

We can draw an analogy between our model and the Hayden-Preskill thought experiment [21] by imagining our RUC as a black hole emitting Hawking radiation via qubit swaps with the environment. However, there are crucial differences: first, instead of assuming that the unitary circuit scrambles completely before emitting Hawking radiation, the radiation here is emitted dynamically throughout the scrambling process. Second, Eve does not have access to early radiation maximally entangled with the black hole prior to scrambling; as a result, Eve must collect a large amount (at least of order N) of radiation before she can decode Alice's message. Despite these differences, our model is a close analog to previous RUC models of evaporating black holes [45,46], and suggests the possibility of analogous phase transitions in the recoverability of quantum information in these models. Furthermore, while our decoding scheme [Fig. 2(a)] is closely analogous to the decoder proposed for Hayden and Preskill's problem [23], the decoding scheme in the present work does not require Grover search or postselection on exponentially rare measurement outcomes.

It is fruitful to compare the observed transition with the MIPT in monitored RUCs [26,29,30,86]. In this setting, there is also a phase transition in the dynamics of quantum information: for small measurement rates below a threshold there is a finite capacity for Alice to transmit quantum information through the system, while above the threshold measurements are capable of destroying Alice's information [31,32]. However, unlike the MIPT, the transition discussed in the present work requires no mid-circuit measurements and therefore does not suffer from a postselection problem. As a result, our transition does not face the same *fundamental* barriers to experimental observation as the MIPT in monitored RUCs.

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