Isometric Evolution in de Sitter Quantum Gravity

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We study time evolution in a simple model of de Sitter quantum gravity, namely, Jackiw-Teitelboim with a positive cosmological constant. We find that time evolution is isometric rather than unitary. The states that are projected out under time evolution correspond to initial conditions that crunch. Our findings suggest that knowledge of bulk physics, even on arbitrarily large timescales, is insufficient to deduce the de Sitter S matrix.

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Introduction.—Do the postulates of quantum mechanics survive in quantum gravity? The main tools for studying quantum gravity, the gravitational path integral (including its Hamiltonian formulation), and string theory, naturally produce states and transition amplitudes and so start by assuming most of the postulates. However, the probabilistic interpretation of amplitudes, enforced by the unitarity of time evolution, is not guaranteed within the path integral formulation and has to be checked.

We use the gravitational path integral and find a simple mechanism whereby a sum over nonsingular geometries leads to isometric rather than unitary evolution, which we demonstrate in a simple model of de Sitter quantum gravity. The basic result is that some states evolve into singular spacetime geometries with a crunch, and others to a bounce, and the former are projected out under evolution. Evolution acts unitarily on the "code subspace" of states that do not develop a crunch, while "crunch" states are projected out under evolution. In this way the Hilbert space of bulk states is smaller than the space of asymptotic states appearing in the de Sitter S matrix. We find this to be true in Jackiw-Teitelboim (JT) gravity, a nonperturbatively soluble model of two-dimensional dilaton gravity [1–11] that has been the subject of much recent work [12]. We then discuss how our results may generalize to more realistic models of quantum gravity.

Our findings are consistent with a recent proposal [13] that time evolution is isometric for quantum gravity in expanding cosmologies. In [13], one of us gave general arguments for isometric evolution and provided examples with matter effective field theory in rigid curved spacetimes. In this Letter, we give a proof of principle for the proposal in a simple model of dynamical gravity.

Our analysis of de Sitter JT gravity builds upon previous work (including our own) [6–8]. In [8] we studied the S matrix of JT gravity to leading order in a topological expansion and to all orders in the gravitational coupling. We considered asymptotic states corresponding to large closed universes with a fixed renormalized length and on which the dilaton of JT gravity is constant. In between such states we found the infinite time evolution operator $\hat{\mathcal{U}}$ to be a projector. In this paper, we make sense of this projector, and see that it is a consequence of isometric evolution. We analyze the Hilbert space of de Sitter JT gravity at intermediate times, and find a basis in which we can cleanly identify states that correspond to bouncing and crunching cosmologies, as well as the change-of-basis matrix to the basis of asymptotic states. The sum over nonsingular geometries projects out the crunching states. We are able to write the infinite evolution operator as $\hat{\mathcal{U}} = \hat{V}\hat{V}^\dagger$, where $\hat{V}$ is the evolution operator from a bulk time to the infinite future (with $\hat{V}^\dagger$ the evolution from the infinite past to a bulk time). Crucially, we find that $\hat{V}$ is an isometry. Furthermore, while previous work on de Sitter JT gravity involved asymptotic states with constant dilaton, our analysis allows for arbitrary asymptotic states and we find isometric evolution in this richer setting. In particular, we find an infinity of null asymptotic states, so that asymptotic states with a varying dilaton differ from those with a constant dilaton by a null state.

We begin the Letter with a review of de Sitter JT gravity, followed by our analysis of isometric time evolution. We conclude a discussion, suggesting that isometric time evolution may be present in more realistic models of quantum gravity, perhaps including our own universe.

de Sitter JT gravity.—JT gravity is a model of two-dimensional gravity with a dilaton ϕ and a metric g. The action of the de Sitter version is

$$S_{\rm JT} = \frac{S_0}{4\pi}\int d^2x\sqrt{-g}\,R + \int d^2x\sqrt{-g}\,\phi(R-2) + S_{\rm bdy}.$$

The term proportional to S_0 is topological, and we take $S_0 \gg 1$ to suppress fluctuations of the spacetime topology. The second term has ϕ acting as a Lagrange multiplier (and hence it has a real contour in a Lorentzian path integral) enforcing $R = 2$. The basic solution to the field equations is global dS_2 space,

$$ds^2 = -dt^2 + \alpha^2\cosh^2(t)dx^2, \qquad \phi = \phi_0 \sinh(t), \tag{1}$$

where $x \sim x + 2\pi$ and α is a modulus labeling the space. These are bounce cosmologies where the spatial universe is a circle reaching a minimum size of $2\pi|\alpha|$ in between two asymptotically dS_2 regions reached as $t \to \pm\infty$. The general definition of an asymptotically future dS_2 region is a line element and dilaton which behave as

$$\begin{aligned} ds^2 &= -dt^2 + \left(e^{2t} + O(1)\right)dx^2, \\ \phi &= \frac{1}{2\pi}e^{t+\varphi(x)} + O(1), \end{aligned} \tag{2}$$

as $t \to \infty$, and similarly for a past asymptotic region. From this form we define asymptotically future dS_2 boundary conditions as follows. We introduce a boundary at $t = \ln\Lambda$ with Λ tending to infinity, on which the induced metric is $\sim\Lambda^2 dx^2$ and the dilaton is $\sim\Lambda e^{\varphi(x)}/2\pi$. We add the boundary term

$$S_{\rm bdy} = -\frac{S_0}{2\pi}\int_{\partial M} dx\sqrt{\gamma}K - 2\int_{\partial M} dx\sqrt{\gamma}\phi(K-1), \tag{3}$$

to the action, with γ the induced metric and K the extrinsic curvature of the boundary, and take the boundary to infinity. Through this procedure we fix a large future boundary with a renormalized boundary metric dx^2 and a renormalized dilaton $e^{\varphi(x)}$. The boundary term in the action is required so that JT gravity has a consistent variational principle with these boundary conditions.

This boundary is spacelike and therefore prepares a final quantum state labeled by the dilaton profile. We notate a final state with dilaton profile e^{φ_1} as $\langle e^{\varphi_1}|$. Similarly, asymptotically past dS_2 boundary conditions with a dilaton profile e^{φ_2} prepare an initial quantum state that we notate as $|-e^{\varphi_2}\rangle$. The relative minus sign in our labeling of past and future asymptotic states can, for now, be understood as a convention, having to do with the fact that asymptotic states are actually characterized by the renormalized dilaton times the sign of the extrinsic curvature of the boundary circle; we further comment on this in Supplemental Materials B [14]. Because the states $\langle e^{\varphi_1}|$ and $|-e^{\varphi_2}\rangle$ are prepared in the far future and past we call them asymptotic states. There are also multi-universe asymptotic states where the initial or final space is a disjoint set of n large circles, each of which is characterized by a renormalized dilaton.

Previous work [6–11] on de Sitter JT gravity focused on asymptotic states where the dilaton is constant, and so is incomplete since the most general asymptotic state has a varying dilaton. Even so, the three main quantities considered were (i) the wave function at future infinity of the no-boundary (Hartle-Hawking) state $|\varnothing\rangle$ of de Sitter JT gravity, where there is no past and the future is a large asymptotic circle; (ii) the sum over spacetimes with the topology of global dS_2, comprising the infinite-time transition amplitude between an asymptotic circle in the past and an asymptotic circle in the future; and (iii) the inner product on asymptotic states [8]. The inner product is required to obtain properly normalized transition amplitudes. (In our previous work [8] we also proposed a topological expansion for de Sitter JT gravity which we do not discuss in the present work.)

We proceed to study the more general asymptotic states. We relegate a detailed description to the Supplemental Material [14]. The main point is that JT gravity has no bulk degrees of freedom; the dilaton acts as a Lagrange multiplier enforcing the constant curvature condition, uniquely fixing the spacetime metric up to moduli that must be integrated over with the correct measure. Furthermore, as in the AdS version of JT gravity, each asymptotic boundary is equipped a single boundary degree of freedom, a "Schwarzian mode," which has to be integrated over in the quantum theory. The action for the Schwarzian mode is more complicated when the dilaton varies, but as we explain in Supplemental Materials A and B [14] its path integral can still be computed exactly. (See also the Supplemental Material [14] for a discussion of how the AdS JT matrix model of [5] can be augmented to accommodate nonconstant dilaton boundary conditions.) We note that the action is only sensible when e^{φ} is everywhere positive, or everywhere negative, and formally we must equip e^{φ} with an infinitesimal imaginary part.

Following the methods of [8] and accounting for the Schwarzian path integral with a varying dilaton, we find that the inner product of the asymptotic states $|e^{\varphi_1}\rangle$ and $|e^{\varphi_2}\rangle$ is

$$\langle e^{\varphi_1}|e^{\varphi_2}\rangle = \sqrt{\Phi_1\Phi_2}\,\delta(\Phi_1 - \Phi_2)\,e^{i\mathcal{S}[\varphi_1] - i\mathcal{S}[\varphi_2]}, \tag{4}$$

where we define $\mathcal{S}[\varphi] \coloneqq (1/2\pi)\int_0^{2\pi} dx\, e^{\varphi}\varphi'(x)^2$ and also $\Phi_i^{-1} \coloneqq (1/2\pi)\int_0^{2\pi} dx\, e^{-\varphi_i}$. A depiction of the inner product, along the lines of [8], can be seen in Fig. 1. The result (4) implies an infinite redundancy in the spectrum of asymptotic states. Consider two dilaton profiles e^{φ_1} and e^{φ_2} with the property that $\Phi_1 = \Phi_2$. Then all states of the form

$$|\Psi\rangle = e^{i\mathcal{S}[\varphi_1]}|e^{\varphi_1}\rangle - e^{i\mathcal{S}[\varphi_2]}|e^{\varphi_2}\rangle \tag{5}$$

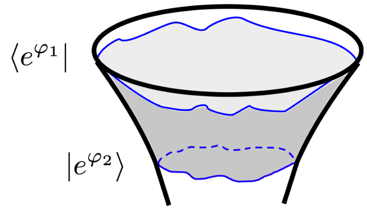

FIG. 1. A depiction of the inner product $\langle e^{\varphi_1}|e^{\varphi_2}\rangle$. Following [8], we consider boundary conditions in the future asymptotic region corresponding to a bra and a ket, and perform the path integral over those metrics that interpolate between the boundary conditions in the limit that the corresponding boundaries approach one another.

are null. We construct a physical Hilbert space $\mathcal{H}_{\rm asy}$ in the usual way by identifying any two states that differ by a null state. Under that identification the state $|e^{\varphi}\rangle$ is identified with $e^{-iS[\varphi]}|\Phi\rangle$, i.e., the state characterized by a constant dilaton with the same Fourier zero mode as $e^{-\varphi}$. The physical Hilbert space is then spanned by equivalence classes whose representatives are states with a constant dilaton $|\Phi\rangle$, with inner product $\langle\Phi_1|\Phi_2\rangle = \sqrt{\Phi_1\Phi_2}\delta(\Phi_1-\Phi_2)$. In de Sitter JT gravity the dilaton can be positive or negative and $\Phi\in\mathbb{R}$. So the Hilbert space of asymptotic states $\mathcal{H}_{\rm asy}$ is isomorphic to the Hilbert space of a quantum mechanical particle on the line. We then rescale asymptotic states as $|\Phi\rangle\to|\Phi\rangle/\sqrt{\Phi}$ so that they have the standard inner product $\langle\Phi_1|\Phi_2\rangle=\delta(\Phi_1-\Phi_2)$, and so have a completeness relation $\mathbb{1}=\int d\Phi|\Phi\rangle\langle\Phi|$.

Note that this analysis implies that the work of [8] with constant dilaton states was complete after all.

The existence of the null states is a consequence of large diffeomorphisms. Consider an asymptotically future dS_2 region (2). There is a family of diffeomorphisms that preserve the form of the line element and dilaton, but change the renormalized dilaton. These diffeomorphisms are "large," acting all the way to the boundary, and so we do not divide by them in the sum over metrics. However, they relate asymptotic states. To be more precise, the transformation

$$e^t \to \frac{e^t}{x'(y)}\left(1-\frac{e^{-2t}}{4}\frac{x''(y)^2}{x'(y)^2}+O(e^{-4t})\right),$$
$$x\to x(y)+\frac{e^{-2t}}{2}x''(y)+O(e^{-4t}), \qquad (6)$$

preserves (2) while acting on the renormalized dilaton as

$$e^{-\varphi(x)}\to x'(y)e^{-\varphi(x(y))}. \qquad (7)$$

Here $x(y)$ is a reparametrization of the spatial circle obeying $x(y+2\pi)=x(y)+2\pi$ and $x'(y)\geq 0$. This transformation preserves the Fourier zero mode of $e^{-\varphi}$, namely, Φ^{-1}. Moreover, because the dilaton has to be everywhere nonzero, the transformation can be used to relate any two dilaton profiles with the same zero mode Φ. In particular, once we know from our Schwarzian analysis that the dilaton must be either always positive or always negative, then we learn that there is a large diffeomorphism that relates the state $\langle e^{\varphi}|$ to the constant dilaton state $\langle\Phi|$.

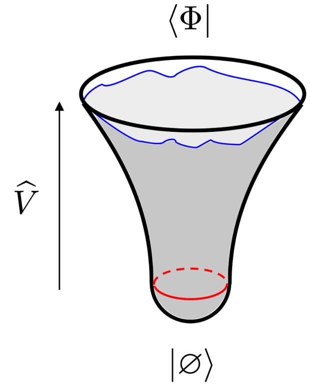

FIG. 2. The no-boundary state evolved to the infinite future to give the Hartle-Hawking state. The state $|\varnothing\rangle$ corresponding to the Euclidean cap is prepared at a finite time, and is then evolved in Lorentzian time by $\hat{V}$ to the infinite future. The wave function is naturally computed in the Φ basis by projecting onto $\langle\Phi|$ in the far future.

In the basis of properly normalized asymptotic states the wave function of the no-boundary state at future infinity is, to leading order in the topological expansion,

$$\Psi_{\rm HH}(\Phi)=\langle\Phi|{\rm HH}\rangle=\langle\Phi|\hat{V}|\varnothing\rangle\approx\frac{i^{\frac{3}{2}}\Phi}{\sqrt{2\pi}}e^{S_0+i\Phi}, \qquad (8)$$

which notably is non-normalizable: $\int d\Phi|\Psi_{\rm HH}(\Phi)|^2\propto\int dx\,x^2$ diverges. Here $\hat{V}$ is the semi-infinite evolution operator from the bulk time at which the no-boundary state is created to the infinite future. A depiction is shown in Fig. 2.

Now consider the infinite-time transition amplitudes between asymptotic states with a large universe with Φ_2 in the past and a large universe with Φ_1 in the future. The result from [8] for that amplitude, coming from the sum over cylinder geometries that smoothly connect the past and future circles, is

$$\langle\Phi_1|\hat{\mathcal{U}}|\Phi_2\rangle\approx\frac{i}{2\pi}\frac{1}{\Phi_1-\Phi_2+i\epsilon}, \qquad (9)$$

where $\approx$ means that we are neglecting higher order terms in the genus expansion, and where we have included an $i\epsilon$ prescription that renders the JT path integral convergent. The pole in this amplitude corresponds to the global dS_2 saddle (1) which, with our convention that past asymptotic states are labeled by $|-e^{\varphi}\rangle$, has $\Phi_1=\Phi_2$. Now consider a change of basis from states of definite Φ to those of its canonical conjugate, which we will call p (thinking of Φ as a position and p as a momentum). In the p basis we have

$$\langle p_1|\hat{\mathcal{U}}|p_2\rangle \approx \Theta(p_1)\delta(p_1 - p_2). \tag{10}$$

In this basis time evolution is very simple, and we see that infinite time evolution is unitary on the "code subspace" of states $|p\rangle$ with $p > 0$. In fact, there is a matrix model interpretation of this result which we found in our previous work [8].

In the next section we will see that p eigenstates are readily interpreted as bulk states, where $p > 0$ correspond to bouncing cosmologies and $p < 0$ to a crunch cosmologies.

Isometric evolution in JT.—Now we consider bulk states. We find it convenient to fix a "temporal gauge" in which the line element reads

$$ds^2 = -dt^2 + A(t,x)^2 dx^2. \tag{11}$$

Note that any metric on a cylinder can be put into this form up to a large diffeomorphism, which can be understood to act on the initial and final states. With this gauge fixing and on a finite-time cylinder, the JT action reads

$$S = 2\int dt\, dx\, \phi(\ddot{A} - A) + (\text{bdy}). \tag{12}$$

Under the field redefinition $\mathcal{Q} = \phi/\dot{A}$ and $\mathcal{P} = A^2 - \dot{A}^2$ the action is simply

$$S = -\int dt\, dx\, \mathcal{Q}\dot{\mathcal{P}} + (\text{bdy}), \tag{13}$$

an extremely simple quantum mechanics in Hamiltonian form. After this field redefinition we can adjust boundary terms so as to fix $\mathcal{Q}(x)$ on a constant time slice, which prepares a state $|\mathcal{Q}(x)\rangle$. Alternatively, we can adjust boundary terms so as to fix $\mathcal{P}(x)$ on a constant time slice, which prepares a state $|\mathcal{P}(x)\rangle$. In an asymptotically de Sitter region $\lim_{t\to\infty}\mathcal{Q}(x) = e^{\varphi(x)}/2\pi$ and so asymptotic states with fixed renormalized dilaton correspond to $\mathcal{Q}$ eigenstates. On the other hand, we can fix the initial and final states to be $\mathcal{P}$ eigenstates. Asymptotically these are Neumann-like boundary conditions, but at finite time they naturally produce bulk states.

Integrating out $\mathcal{Q}$ enforces that $\mathcal{P}$ is conserved at each x. Because $\mathcal{P}(x)$ is conserved quantum mechanically we can deduce the corresponding metric, with

$$A = c_+(x)e^t + c_-(x)e^{-t}. \tag{14}$$

Requiring that the metric is everywhere nonsingular, which implies c_+ and c_- are nonzero, there is a residual large diffeomorphism that "straightens out" $\mathcal{P}(x)$ so that it is a constant which we call $\mathcal{P}$. Initial states are equivalence classes labeled only by this constant which obey $\langle\mathcal{P}_1|\mathcal{P}_2\rangle = \delta(\mathcal{P}_1 - \mathcal{P}_2)$. This puts A^2 into the form

$$A^2 = \begin{cases} \mathcal{P}\cosh^2(t), & \mathcal{P} > 0, \\ |\mathcal{P}|\sinh^2(t), & \mathcal{P} < 0. \end{cases} \tag{15}$$

The former is simply global dS_2 with $\mathcal{P} = \alpha^2$, while the latter is singular at $t = 0$. So $\mathcal{P} > 0$ states correspond to bounce cosmologies and $\mathcal{P} < 0$ states to crunch cosmologies. The latter are projected out in the path integral formulation thanks to the sum over nonsingular geometries. That is, we build a bulk Hilbert space $\mathcal{H}_{\text{bulk}}$ out of superpositions of $|\mathcal{P}\rangle$'s with $\mathcal{P} > 0$. On that space finite time evolution $\hat{U}$ simply acts as the identity, with

$$\langle\mathcal{P}_1|\hat{U}|\mathcal{P}_2\rangle \approx \delta(\mathcal{P}_1 - \mathcal{P}_2). \tag{16}$$

Taking stock, we have two Hilbert spaces in de Sitter JT gravity: (i) a space of asymptotic states $\mathcal{H}_{\text{asy}}$ with a basis $|p\rangle$ with $p \in \mathbb{R}$ and where infinite time evolution preserves p, and (ii) a space of bulk states $\mathcal{H}_{\text{bulk}}$ with a basis $|\mathcal{P}\rangle$ with $\mathcal{P} > 0$, where $\mathcal{P}$ is conserved. So the time evolution operator $\hat{V}$ from the bulk to asymptotia is in fact a map from a smaller Hilbert space to a larger one, $\hat{V}\colon \mathcal{H}_{\text{bulk}} \to \mathcal{H}_{\text{asy}}$. The natural (and correct) guess for $\hat{V}$ is that it simply takes $\mathcal{P}$ to p. To show this consider the matrix element $\langle\Phi|\hat{V}|\mathcal{P}\rangle$. This object is the de Sitter JT version [6–8] of the "trumpet" of AdS JT gravity [5], where the initial state fixes that the geometry is $ds^2 = -dt^2 + \mathcal{P}\cosh^2(t)dx^2$ with t starting at some finite time. The JT path integral in this case reduces to a Schwarzian path integral at future infinity which depends on $\mathcal{P}$ and Φ with the result (in terms of normalized states $\langle\Phi|$ and $|\mathcal{P}\rangle$)

$$\langle\Phi|\hat{V}|\mathcal{P}\rangle \approx \frac{1}{\sqrt{2\pi}}e^{i\Phi\mathcal{P}}, \tag{17}$$

which implies $\langle p|\hat{V}|\mathcal{P}\rangle \approx \delta(p - \mathcal{P})$ as expected. To derive (17), we observe that the de Sitter trumpet amplitude is $Z_T = \sqrt{(i\Phi/2\pi)}\, e^{i\alpha^2\Phi}$, and so after accounting for the square root factor $\sqrt{\Phi}$ coming from the norm on $|\Phi\rangle$ states as well as the identification $\alpha^2 \leftrightarrow \mathcal{P}$, we obtain (17). (We have further chosen a global phase for $|\mathcal{P}\rangle$ to cancel off the $\sqrt{i}$ factor in Z_T.) More details of the trumpet amplitude are given in the Supplemental Material [14].

The operator $\hat{V}$ is therefore an isometry: the product $\hat{V}^\dagger\hat{V}$ acts as the identity on $\mathcal{H}_{\text{bulk}}$ while $\hat{V}\hat{V}^\dagger = \hat{\mathcal{U}}$ acts as a projector on $\mathcal{H}_{\text{asy}}$. This is the main result of this Letter. A depiction of the de Sitter S matrix can be seen in Fig. 3.

Using the result (17) we can reconstruct the bulk wave function of the no-boundary state. In particular, since the Hartle-Hawking state is $|\text{HH}\rangle = \hat{V}|\varnothing\rangle$, we have

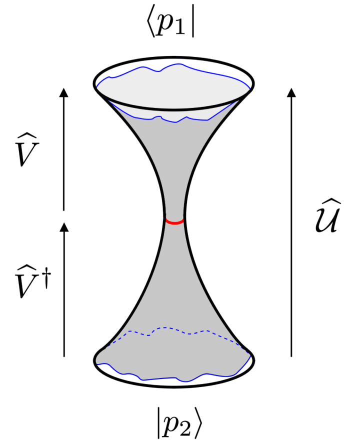

FIG. 3. The JT de Sitter S matrix, starting in the state $|p_2\rangle$ and ending in the state $\langle p_1|$. Time evolution from past infinity to the bottleneck is given by $\hat{V}^\dagger$, and time evolution from the bottleneck to future infinity is given by $\hat{V}$. Since $\hat{V}$ is an isometry, the total time evolution $\hat{\mathcal{U}} = \hat{V}\hat{V}^\dagger$ is a projector.

$\hat{V}^\dagger|\mathrm{HH}\rangle = |\varnothing\rangle$ and so

$$\langle \mathcal{P}|\varnothing\rangle \approx \delta'(\mathcal{P} - 1), \tag{18}$$

which is supported on $\mathcal{P} = 1$ and is also clearly nonnormalizable. This is consistent with the fact that the JT sphere partition function, expected on general grounds to be the norm of the no-boundary state, diverges [15].

Discussion.—Our work shows that the S matrix need not be unitary in quantum gravity. In de Sitter JT gravity this is a consequence of a mismatch between the bulk and asymptotic Hilbert spaces: initial conditions that correspond to crunching universes live in the Hilbert space of asymptotic states, but not in the “code subspace” of bulk states. As a result, complete knowledge of bulk physics, even on arbitrarily large timescales, is not enough to deduce the de Sitter S matrix. We also find that evolution is trivial within the code subspace, with finite time evolution acting as the identity. In our examples this breakdown of infinite-time unitary evolution and its replacement by a combination of projections and isometries is invisible in perturbation theory, but rather arises nonperturbatively.

There are additional contributions to the S matrix arising from higher-genus spacetimes, although these corrections are nonperturbatively suppressed in the topological expansion; as such, our conclusions in this Letter remain salient. The computation of these corrections is conceptually and technically subtle [6–8], and we provide a decisive analysis in forthcoming work [16].

Our JT example is particularly simple because it is model of pure gravity in low dimensions. It would be natural to enrich our analysis by considering JT coupled to defects, worldlines, or conformal matter. In these settings we expect the code subspace of noncrunching geometries to be much richer, with an interplay between gravity and matter. We note minimally coupling matter to JT retains the same metrics since matter does not interfere with the $R = 2$ constraint imposed by the dilaton. As such, singular metrics corresponding to crunch geometries are still excluded from the moduli space, although the precise analytic form of the S-matrix elements will be modified. Some care is required since S-matrix amplitudes in JT coupled to matter are divergent on general grounds, and so to proceed one might require an effective field theory formalism akin to [17].

An important question is if some version of our results holds in more realistic models of quantum gravity. The answer to this question is relevant for understanding the origins and ultimate fate of our Universe, as we (presumably) live in a code subspace. While we expect pure de Sitter quantum gravity in $2 + 1$ dimensions, a model with no local degrees of freedom, to be rather similar to de Sitter JT gravity, the setting of Einstein gravity in $3 + 1$ dimensions is less clear. However, [18] explains how JT analyses can in some cases be lifted to gravitational constrained saddle analyses in higher dimensions (see also [19–21]). Along these lines, some of the features of time evolution in de Sitter JT gravity generalize to the minisuperspace approximation of Einstein gravity with a positive cosmological constant where the spatial universe is a round sphere. The minisuperspace approximation can be treated quantum mechanically by recapitulating our de Sitter JT analysis (see Supplemental Material [14]), with the result that the Hilbert space of bulk states corresponds to cosmologies which bounce or crunch. The latter are projected out by evolution, consistent with isometric rather than unitary evolution. The conclusions are similar for the minisuperspace approximation of Einstein gravity coupled to a scalar field (see Supplemental Materials C [14]), and connect to the work of [22]. These analyses suggest that the basic features of our work, namely, a mismatch between bulk and asymptotic Hilbert spaces and isometric evolution, persist in more realistic settings.

A foreseeable question is whether the restriction to a sum over nonsingular geometries is realized in UV completions of de Sitter gravity, such as in string theory (if indeed a suitable stringy completion exists). For instance, maybe certain singular metrics that are sensible in string theory ought to be included in the low-energy theory. Perhaps a useful toy model to keep in mind is de Sitter JT gravity coupled to defects; here the Universe can begin or end on a defect. As such there is still a code subspace: it consists of all states which either evolve into a nonsingular geometry, or into a conical singularity that can be sourced by a defect. However not all conical singularities are allowed since the set of defects is constrained. This means that the code subspace encodes the “spectrum” of allowed singularities.

More broadly, we expect that the true Hilbert space of de Sitter quantum gravity is drastically smaller than the naïve one indicated by semiclassical gravity. In particular, holographic arguments suggest that the actual dimension is nonperturbatively finite. Our findings are a first step in this direction, where we can already see the pruning of the bulk Hilbert space in the low-energy effective description.

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[1] K. Jensen, Phys. Rev. Lett. **117**, 111601 (2016).
[2] J. Maldacena, D. Stanford, and Z. Yang, Prog. Theor. Exp. Phys. **2016**, 12C104 (2016).
[3] J. Engelsöy, T. G. Mertens, and H. Verlinde, J. High Energy Phys. 07 (2016) 139.
[4] D. Stanford and E. Witten, J. High Energy Phys. 10 (2017) 008.
[5] P. Saad, S. H. Shenker, and D. Stanford, arXiv:1903.11115.
[6] J. Maldacena, G. J. Turiaci, and Z. Yang, J. High Energy Phys. 01 (2021) 139.
[7] J. Cotler, K. Jensen, and A. Maloney, J. High Energy Phys. 06 (2020) 048.
[8] J. Cotler and K. Jensen, J. High Energy Phys. 12 (2021) 089.
[9] Y. Chen, V. Gorbenko, and J. Maldacena, J. High Energy Phys. 02 (2021) 009.
[10] U. Moitra, S. K. Sake, and S. P. Trivedi, J. High Energy Phys. 10 (2021) 204.
[11] U. Moitra, S. K. Sake, and S. P. Trivedi, J. High Energy Phys. 06 (2022) 138.
[12] T. G. Mertens and G. J. Turiaci, Living Rev. Relativity **26**, 4 (2023).
[13] J. Cotler and A. Strominger, arXiv:2201.11658.
[14] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.131.211601 for a discussion of AdS and dS JT gravity with general dilaton boundary conditions, and a higher-dimensional minisuperspace generalization of our dS JT analysis.
[15] R. Mahajan, D. Stanford, and C. Yan, J. High Energy Phys. 07 (2022) 132.
[16] J. Cotler and K. Jensen (to be published).
[17] D. L. Jafferis, D. K. Kolchmeyer, B. Mukhametzhanov, and J. Sonner, Phys. Rev. D **108**, 066015 (2023).
[18] J. Cotler and K. Jensen, Phys. Rev. D **104**, 081501 (2021).
[19] J. Cotler and K. Jensen, J. High Energy Phys. 04 (2021) 033.
[20] J. Cotler and K. Jensen, J. High Energy Phys. 09 (2021) 001.
[21] J. Cotler and K. Jensen, J. High Energy Phys. 11 (2022) 070.
[22] A. Borde, A. H. Guth, and A. Vilenkin, Phys. Rev. Lett. **90**, 151301 (2003).