Masuki *et al.* Reply: In our Letter [1], we determined the ground-state phase diagram of the resistively shunted Josephson junction (RSJ) based on two independent nonperturbative renormalization group (RG) analyses, namely, numerical RG (NRG) and functional RG (fRG). Our main finding is that the insulating phase is strongly suppressed to deep charging regimes due to the dangerously irrelevant term ν. We benchmarked our NRG scheme by making comparisons with the known results of the boundary sine-Gordon model corresponding to the case of $\nu \to 0$.

In the preceding Comment [2], Sépulcre, Florens, and Snyman (SFS) claim that our NRG scheme is unreliable because the resulting excitation spectrum is different from what would be expected when approximating the cosine potential by the quadratic one. SFS also argue that the phase mobility can converge to a tiny but nonzero value, which they claim contradicts the fact that the phase mobility should be zero in the superconducting regime. Lastly, SFS points out the need to include more bosonic modes when $\alpha \ll 1$. After carefully studying the Comment, we conclude that it does not affect the main finding of our Letter. We also note that the Comment is solely concerned with NRG analysis, and fRG analyses in the Letter (which unambiguously support the NRG results) remain intact.

First of all, we recall that the excitation spectrum of the NRG shows the qualitatively different behaviors between $\alpha > \alpha_c$ and $\alpha < \alpha_c$. As detailed in Ref. [1], these behaviors are consistent with the interpretation that the cosine potential is relevant (irrelevant) in the superconducting (insulating) regime, which has been also checked in our benchmark calculations of the boundary sine-Gordon model. The upward shift of the excitation spectrum reflects the relevance of the potential and consistently accompanies the suppression of the phase mobility as expected in the superconducting phase. Thus, the suppression of the phase mobility can be used as an indicator of the superconducting phase.

In Fig. 1(a), we plot the NRG flow of the phase mobility μ_{10} obtained for different cases as in the Comment [2]. When plotted in the linear scale, all the results agree well with each other and converge to zero. While in practice the phase mobility might converge to a tiny nonzero value [2], one can easily see an abrupt change of the converged value as E_J/E_C is increased [Fig. 1(b)]. This sharp transition can be used to accurately locate the transition point. We speculate that a minuscule residual phase mobility results from accumulated numerical errors due to the relevant potential term.

Regarding the last comment, at least in the current implementation of the NRG, it is technically challenging to accurately analyze the regime $\alpha \ll 1$, as we have already explained in the Supplemental Material of Ref. [1]. This manifests itself in the relatively large error bars and a dashed extrapolated phase boundary in Fig. 1(a) in Ref. [1]. In this sense, we agree that there is room for further improving quantitative accuracy of NRG scheme at $\alpha \ll 1$.

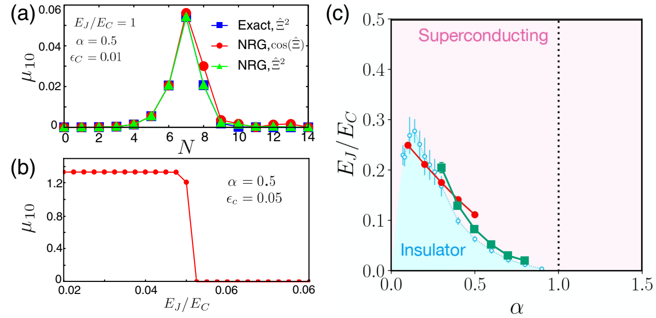

FIG. 1. (a) Phase mobility μ_{10} plotted against the number of NRG steps N. (b) Converged values of μ_{10} at $\alpha = 0.5$. (c) Ground-state phase diagram of RSJ reproduced from Ref. [3]. The red points (blue circles) are the phase boundaries obtained from the fRG analysis presented in Ref. [3] (the NRG [1]). The green circles represent the phase boundary obtained after fixing the typo pointed out in Ref. [2].

Lastly, we note that all the qualitative features found in NRG analysis precisely agreed with the ones revealed by the independent fRG analysis. In a recent work [3], this agreement has been further confirmed at the quantitative level by employing an advanced fRG analysis, which gives a further support of our findings [Fig. 1(c)].

Y. A. acknowledges support from the Japan Society for the Promotion of Science through Grant No. JP19K23424 and from JST FOREST Program (Grant No. JPMJFR222U, Japan). M. O. is supported in part by MEXT/JSPS KAKENHI Grants No. JP17H06462 and No. JP19H01808, JST CREST Grant No. JPMJCR19T2.

Kanta Masuki[1,*], Hiroyuki Sudo[1], Masaki Oshikawa[2,3], and Yuto Ashida[1,4,†]

[1]Department of Physics, University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
[2]Institute for Solid State Physics, University of Tokyo
Kashiwa, Chiba 277-8581, Japan
[3]Kavli Institute for the Physics and Mathematics of the Universe (WPI), University of Tokyo
Kashiwa, Chiba 277-8583, Japan
[4]Institute for Physics of Intelligence, University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

Received 10 July 2023; accepted 9 October 2023; published 8 November 2023

DOI: 10.1103/PhysRevLett.131.199702

[*]masuki@g.ecc.u-tokyo.ac.jp
[†]ashida@phys.s.u-tokyo.ac.jp

[1] K. Masuki, H. Sudo, M. Oshikawa, and Y. Ashida, Phys. Rev. Lett. **129**, 087001 (2022).
[2] T. Sépulcre, S. Florens, and I. Snyman, preceding comment, Phys. Rev. Lett. **131**, 199701 (2023).
[3] T. Yokota, K. Masuki, and Y. Ashida, Phys. Rev. A **107**, 043709 (2023).

