Comment on "Absence versus Presence of Dissipative Quantum Phase Transition in Josephson Junctions"

In Ref. [1], a Josephson junction shunted by an Ohmic transmission line is studied. The authors present a phase diagram with features not anticipated in the established literature [2]. We show that their numerical renormalization group (NRG) calculation suffers from several flaws and cannot be trusted to substantiate their claims.

NRG captures low energy physics by building recursive Hamiltonians, $H_{N+1} = H_N + \Delta H_{N+1}$, that are iteratively diagonalized. Scale separation is required for NRG to work, i.e., ΔH_{N+1} should decrease exponentially with N [3]. For the NRG scheme in Ref. [1], ΔH_{N+1} is of the same order as H_0 [see Eqs. (S51) and (S52) in the Supplemental Material to [1].] This is a known problem that can only be cured by introducing an infrared cutoff [4]. As a result, the NRG fails to flow to the correct infrared fixed point. To demonstrate this, we considered large conductance α and large E_J/E_C, where the system studied in [1] is nearly harmonic, allowing us to expand $-E_J \cos(\Xi) \simeq E_J(\Xi^2/2 - 1)$. We compared low energy spectra obtained with the NRG scheme of [1] for the cosine and quadratic potentials, to the exact spectrum obtained for the latter. As the top panel of Fig. 1 shows, the NRG results diverge from the exact spectrum after the seventh RG step. Thus the NRG scheme proposed in [1] is unreliable and cannot be trusted to predict the phase diagram. (See appendix of [5] for discussion of the RG flow of mobility μ_{10}.)

The phase diagram in [1] is flawed in another way. Even if one trusted the employed NRG scheme, the reentrant superconductivity seen at small α and small E_J/E_C is a numerical artifact. The blue dots in the bottom panel of Fig. 1 reproduce the result for $\langle\cos(\varphi)\rangle$ vs α at $E_J/E_C = 0.15$ in the upper panel of Fig. 4 of [1], obtained with the truncation parameter $n_B = 15$ in each mode for $N > 0$. For this result to be correct, it must not change when n_B is increased. Instead we see that the region, where $\langle\cos(\varphi)\rangle$ vanishes, grows to include the interval $\alpha \in [0, 0.2]$ when n_B is increased. Thus, the apparent reentrant superconductivity in the phase diagram in [1] stems from unconverged data. In [1] it is argued that superconductivity makes common sense when the junction is shunted by a sufficiently large impedance. We stress that taking the thermodynamic limit $N \to \infty$ before $\alpha \to 0$ couples the junction to divergent φ fluctuations that render the junction's zero-frequency response nontrivial. The object Letter also contains a brief functional renormalization group (fRG) argument in support of superconductivity at $\alpha < 1$ and large E_J/E_C. The approximations involved are not controlled by any obvious small parameter. It is still not known whether fRG can reproduce infrared Luttinger exponents for $1 < \alpha < 2$ [4], where phase slips affect results nontrivially. Until this is settled, fRG's validity in the more challenging $\alpha < 1$ regime remains unclear.

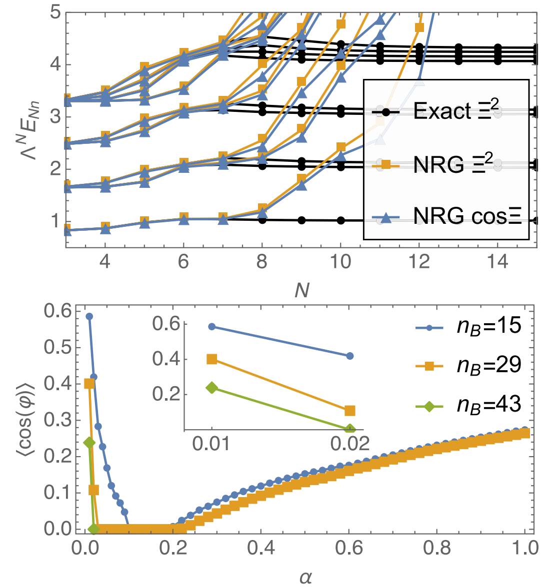

FIG. 1. Top: low energy spectrum vs NRG step N, scaled with Λ^N. Results of the NRG scheme in [1] for the cosine and quadratic potential are compared to exact results for the quadratic potential. We took $n_S = 50$ kept states, $n_B = 300$ bosonic states for $N = 0$ and $n_B = 15$ for $N > 0$, $\Lambda = 2.0$, $\alpha = 10$, $E_C = 0.01W$, $E_J/E_C = 10$. Bottom: $\langle\cos(\varphi)\rangle$ vs α, for $E_J/E_C = 0.15$, like the triangles in the top panel of Fig. 4 of [1]. The blue dots reproduce the result of [1] with the same truncation parameter $n_B = 15$ for $N > 0$. The yellow squares and green diamonds were obtained by increasing n_B to 29 and 43, respectively. The inset closeups on the two smallest values of α, which are still unconverged at $n = 43$, show a downward trend.

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