Superfluid Edge Dislocation: Transverse Quantum Fluid

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Recently, it was argued [Kuklov *et al.*, Phys. Rev. Lett. **128**, 255301 (2022)] that unusual features associated with the superflow-through-solid effect observed in solid ^{4}He can be explained by unique properties of dilute distribution of superfluid edge dislocations. We demonstrate that stability of supercurrents controlled by quantum phase slips (instantons), and other exotic infrared properties of the superfluid dislocations readily follow from a one-dimensional quantum liquid distinguished by an effectively infinite compressibility (in the absence of Peierls potential) associated with the edge dislocation's ability to climb. This establishes a new class of quasi-one-dimensional superfluid states that remain stable and long-range ordered despite their dimensionality. Our theory is consistent with the existing experimental data, and we propose an experiment to test the mass-current–pressure characteristic prediction.

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Introduction.—About a decade ago the superflow-through-solid (STS) effect in a structurally imperfect crystal of ^{4}He [1–9], along with the striking companion effect of anomalous isochoric compressibility (also known as the syringe effect) [1] were attributed—by means of *ab initio* simulations—to the properties of superfluid edge dislocations (SED) [10], as envisioned by Shevchenko [11]; for a recent review, see Ref. [12]. (Specifically, the superfluidity was found only in the cores of dislocations with Burgers vector along the c-axis in hcp ^{4}He [10,13], while all other dislocations were insulating [14,15].) However, two apparently unrelated experimental features, namely (i) an exponentially strong suppression of the flow by a moderate increase in pressure and (ii) an enigmatic temperature dependence of the flow rate—hardly fitting into the Shevchenko scenario of rigid superfluid 1D channels [11,12], remained unexplained until very recently, when both dependencies were argued to be linked and accounted for by the highly unusual properties of isolated SED [16]. The arguments of Ref. [16] rest on the self-consistent assumption that a SED can support stable supercurrents despite being quasi-one-dimensional and featuring the spectrum of elementary excitations in violation of the Landau criterion (see the Supplemental Material [17] for the discussion of the Landau criterion in a non-Galilean superfluid system).

An outstanding (apparent) inconsistency between the scenario of Ref. [16] and the experimental data has been the observation of a mass-current–pressure (henceforth abbreviating as I-V, that is, current-voltage) characteristic reminiscent of a Luttinger liquid (LL) [2], while the temperature dependence of the flux was incompatible with the LL physics.

In this Letter, we put the theory of SED on a solid theoretical basis by examining the consequences of the key feature distinguishing a SED from a LL: SED is characterized by a (nearly) divergent linear compressibility imposed by the approximate translational invariance of the dislocation transverse to its Burgers vector and its axis, as illustrated in Fig. 1. The I-V characteristic that we find resolves the above-mentioned inconsistency with the experimental data and suggests a simple experiment to confirm our theory.

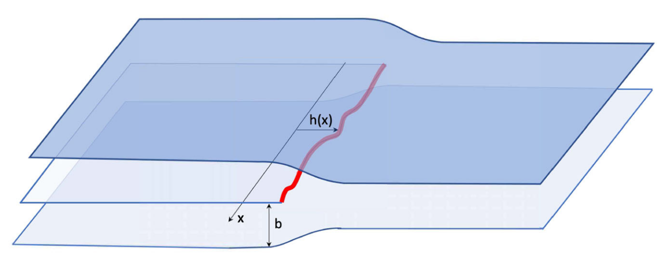

FIG. 1. Sketch of the superfluid edge dislocation marked by the bold (red) wavy line. Its Burgers vector **b** (along the c-axis) is perpendicular to the planes of atoms—two complete layers are shown as two parallel planes with straight edges and the incomplete one is limited by the dislocation line. The transverse translation invariance of $h(x)$ (continuous in the absence of the pinning Peierls potential, and discrete otherwise) is responsible for infinite compressibility of the bosonic fluid confined to the dislocation core.

Transverse quantum fluid model.—The minimal model—which we refer to as transverse quantum fluid (TQF)—proposed in Ref. [10] for SED in an ideal crystal of ^{4}He with an average superflow velocity $\sim v_0$ along its x-directed core—is given by a 1D bosonic Hamiltonian, $H[\phi, n] = \int \mathcal{H} dx$, with

$$\mathcal{H} = \frac{1}{2\kappa}(\partial_x n)^2 + \frac{n_s}{2}(v_0 + \partial_x \phi)^2. \tag{1}$$

The 1D boson density n is canonically conjugate to the superfluid phase ϕ, and the second term is the kinetic energy of the flow, with the superfluid stiffness, n_s, exponentially sensitive to the local pressure. The parameter κ is determined by the inverse shear modulus of the crystal [10]. As shown in Ref. [16], namely this feature accounts for the unusual dependence of the critical current on temperature.

The key feature distinguishing TQF from LL is the absence of the compressibility term $\chi^{-1}n^2$, namely, TQF is characterized by a divergent compressibility χ. The condition $\chi^{-1} = 0$ is enforced by the translation invariance of the dislocation motion transverse to its core axis and its Burgers vector (Peierls potential effects that violate this symmetry and lead to finite compressibility are discussed below). This feature is illustrated in Fig. 1: The dislocation climb displacement $\delta h(x)$ corresponds to the 1D boson density change,

$$\delta n(x) = \delta h(x)/a^2, \tag{2}$$

along the dislocation, where a is a lattice constant that we set to unity hereafter. The associated spectrum ω_k of elementary excitations (at $v_0 = 0$) is straightforwardly found to be quadratic in the wave vector k along the dislocation,

$$\omega_k = Dk^2, \qquad D = \sqrt{n_s/\kappa}, \tag{3}$$

as a direct consequence of the aforementioned translation invariance, that leads to $n \to n + \text{const}$ invariance of (1).

We note that the *quadratic* dispersion (3) appears to violate the Landau criterion for the critical velocity, $v_0 < \{\omega_k/k\}_{\min}$, for arbitrary small v_0. However, as we discuss in the Supplemental Material (SM) [17], the Landau criterion for instability relies on the existence of the term $\delta n(v_0 + \partial_x \phi)^2 = 2v_0 \delta n \partial_x \phi + \cdots$, which is forbidden for SED by the climb translation invariance, allowing only density derivatives, $\partial_x n$, in the TQF Hamiltonian. Thus, at sufficiently small v_0, SED (described by TQF) does not develop the Landau instability, and we must consider superflow relaxation via quantum phase slips, i.e., instantons (space-time vortices) in the phase field $\phi(x, \tau)$ configuration [20].

The Euclidean Lagrangian density of TQF, corresponding to (1), can be obtained after eliminating the density n,

$$\mathcal{L} = \frac{\kappa}{2}(\partial_x^{-1}\partial_\tau \phi)^2 + \frac{n_s}{2}(\partial_x \phi)^2 + n_s v_0 \partial_x \phi, \tag{4}$$

where the long-range operator ∂_x^{-1} is defined by its Fourier transform, namely, $\partial_x^{-1} \to -ik^{-1}$. The last (boundary) term in (4) is important only in the presence of instantons, and we have omitted an irrelevant constant.

Goldstone modes and long-range order.—Mean-squared fluctuations of the superfluid phase ϕ, computable within the Gaussian approximation,

$$\langle \phi^2 \rangle = \frac{1}{\kappa} \int^{2\pi/a} \frac{d\omega dk}{(2\pi)^2} \frac{k^2}{\omega^2 + D^2 k^4} \approx \frac{1}{a\sqrt{\kappa n_s}}, \tag{5}$$

are finite, and TQF thus features a long-range order in the superfluid field $\psi = e^{i\phi}$ at zero temperature (T), despite being one dimensional [21]. Given the exact mathematical symmetry between ϕ and n in (1), the field $\eta = e^{i2\pi n}$ also exhibits long-range order at zero temperature.

At this point we recall that SED is subject to a Peierls potential, $U = U_0 \cos(2\pi n)$ [utilizing the density n—displacement h relation, (2)], omitted in the formulation (1). This sets certain limitations on the applicability of the TQF as an asymptotic model for SED. Long-range order in η implies that U is a relevant perturbation, which suppresses the dislocation climb motion and induces a crossover of the excitation spectrum to a conventional (LL-type) linear form at small momenta, $k/(2\pi) < \xi^{-1}$, with the length scale ξ diverging with the crystal's vanishing shear modulus $\sim \kappa^{-1}$ [22]. (In the opposite limit, the LL superfluid undergoes a phase transition to a Mott insulator [22].) Concomitantly, at length scales longer than ξ, the zero-temperature long-range order of SED changes to the algebraic quasi-long-range one, typical for LL. Such a crossover was observed in model simulations of Refs. [21,22] through a finite-size scaling of compressibility χ for a pinned dislocation with $h(0) = h(L) = 0$: from TQF's $\chi \sim L^2$ to a constant in the $L \to \infty$ limit, as $T \sim 1/L \to 0$.

In what follows, we will focus on the exotic TQF regime on scales below ξ, where SED is quantum rough and the associated 1D superfluid is long-range ordered. Equivalently, one can view TQF as the asymptotic limit ($\xi \to \infty$) of SED. We demonstrate that such a simplification is sufficient for explaining the experimental data with high accuracy.

Confinement of instantons.—We now analyze the stability of this novel one-dimensional superfluidity to superflow-induced instantons [20]. To account for these, we consider non-single-valued configurations of the superfluid phase $\phi(x, \tau)$, corresponding to its space-time vortex configurations. To this end, we introduce the velocity field $v_\mu \equiv \partial_\mu \phi$ in the $(1+1)$-dimensional space-time $x_\mu = (x, \tau)$. The instantons have the form of point-vortex singularities in the otherwise regular field v_μ:

$$\partial \times v = q(x_\mu) = \sum_j q_j \delta^2(x_\mu - x_{\mu,j}), \tag{6}$$

where $\partial \times v \equiv \epsilon_{\mu\nu}\partial_\mu v_\nu$ is shorthand notation for $(1+1)$ space-time curl of v_μ and q_j and $x_{\mu,j}$ are, respectively, the "charge" (an integer multiple of 2π) and space-time position of the jth instanton.

With these ingredients we can now straightforwardly derive the instanton action $S[q(x_\mu)]$ from the Lagrangian density (4) for $\phi(x_\mu)$. The most direct way to obtain this (with alternative derivations presented in Ref. [17]) is to enforce the topological constraint (6) using a functional delta function in a path integral for a partition function over v_μ,

$$Z = \int [dv_\mu][dq][d\lambda] e^{-\int dx d\tau \mathcal{L}[v_\mu, \lambda, q]}, \tag{7}$$

with the Lagrangian density (for $v_0 = 0$)

$$\mathcal{L} = \frac{\kappa}{2}(\partial_x^{-1} v_\tau)^2 + \frac{n_s}{2}(v_x)^2 + i\lambda(\partial \times v - q). \tag{8}$$

In $\mathcal{L}$ we implemented the vorticity constraint (6) via a functional delta function as an integral over the auxiliary field $\lambda(x_\mu)$. Performing the Gaussian integration over fields $v_\mu(x_\mu)$ and $\lambda(x_\mu)$ gives the instanton action

$$S = \frac{n_s}{2} \int \frac{d\omega dk}{(2\pi)^2} \frac{|q_{\omega,k}|^2}{\omega^2 + D^2 k^4} \tag{9}$$

$$= \frac{1}{2} \int_{x_\mu, x'_\mu} q(x_\mu) V(x_\mu - x'_\mu) q(x'_\mu), \tag{10}$$

where $V(x,\tau)$ is the space-time instanton-instanton interaction, which after subtracting the self action $V(0,0)$, becomes

$$V(x,\tau) = n_s \int \frac{d\omega dk}{(2\pi)^2} \frac{e^{i(\omega\tau + kx)} - 1}{\omega^2 + D^2 k^4} \approx \begin{cases} -\frac{n_s\sqrt{|\tau|}}{2\sqrt{\pi D}}, & \text{for } x^2 \ll D\tau, \\ -\frac{n_s|x|}{4D}, & \text{for } x^2 \gg D\tau. \end{cases} \tag{11}$$

This result can be complementarily obtained (see Ref. [17]) through the saddle-point equation for v_μ together with the Fourier-transformed instanton constraint, Eq. (6), finding

$$v_\tau = \frac{-iD^2 k^3 q}{D^2 k^4 + \omega^2}, \qquad v_x = \frac{i\omega q}{D^2 k^4 + \omega^2}. \tag{12}$$

The interaction kernel in (11) is reminiscent (but misses a factor of $-\partial_x^2$ in the numerator) of interaction between dislocations in the 2D classical smectic [23]. This difference is expected as the 2D smectic elasticity is "softer" than in the XY model, while here it is "stiffer" than in the XY model because of the extra ∂_x^{-1} factor in $\mathcal{L}$ of (4). The latter effect is directly related to divergent compressibility, which allows stronger density fluctuations and thereby "stiffens" the canonically conjugate superfluid phase ϕ. As a result, in contrast to the 2D smectic (where interaction is weak and dislocations are deconfined) [23,24], the TQF instantons with opposite "charges" are *always confined* by the interaction $V(\tau, x)$ in (11), featuring a power-law growth with spatial and temporal separation. Note that this behavior is consistent with the 1D long-range superfluid order at $T = 0$ discussed above [21].

Metastability of the superflow and nonlinear I-V response.—We now demonstrate that instanton confinement, (11), leads to the exponential in $1/v_0$ metastability of the superflow. This is readily seen by examining the contribution of the v_0-dependent term to the Langragian density (4). Since this is a boundary term, its presence does not change the conditions (6) and action in (11), leaving the solution (12) intact. Further simplification comes from the fact that the metastable regime corresponds to appropriately small values of v_0, when the destabilization channel is associated with a well-isolated instanton–anti-instanton pair characterized by space-time coordinates (x_+, τ_+) and (x_-, τ_-). Straightforward integration in the v_0-dependent term in (4), $\int dx \partial_x \phi(x,\tau) = \phi(+\infty,\tau) - \phi(-\infty,\tau)$, gives its contribution to the instanton pair action (for the pair with "charges" $\pm 2\pi$)

$$S_{v_0} = v_0 n_s \iint d\tau dx v_x = 2\pi v_0 n_s \int_{\tau_+}^{\tau_-} d\tau = 2\pi v_0 n_s (\tau_+ - \tau_-). \tag{13}$$

We see that, in a close similarity with the vortex–anti-vortex pair in a 2D superfluid, the superflow generates a transverse (in space-time) "force" pulling the pair apart along the imaginary-time direction.

Combining this with the competing v_0-independent part of the pair action at short distances, $S_{\rm inst}^{(0)} \approx V(x=0,\tau)$, given by (10) and (11), we find the total instanton-pair action,

$$S_{\rm inst} \approx \frac{2\pi^{3/2} n_s}{\sqrt{D}} \sqrt{|\tau|} - 2\pi n_s |v_0 \tau|, \tag{14}$$

where $(x,\tau) \equiv (x_+ - x_-, \tau_+ - \tau_-)$. The maximum-action instanton-pair configuration controlling the dissipation is reached at $\sqrt{\tau^*(v_0)} = \sqrt{\pi}/(2\sqrt{D} v_0)$, and the corresponding action is given by

$$S_{\rm inst}^* \equiv S_{\rm inst}(\tau^*) \approx \frac{\pi^2 n_s}{2D v_0}. \tag{15}$$

In the exponential approximation [20], this defines the probability of the instanton nucleation:

$$\mathcal{P} \sim e^{-v_c/v_0}, \qquad v_c = \frac{\pi^2 n_s}{2D} = \frac{\pi^2 \sqrt{n_s \kappa}}{2}, \tag{16}$$

in sharp contrast with the LL's power-law dependence $\mathcal{P} \sim (v_0)^g$, with $g > 0$.

Another qualitative difference with the LL physics comes from kinematic considerations regarding the ultimate decay of the superflow. In the translation-invariant LL, the decay of the supercurrent is kinematically forbidden because it is impossible to simultaneously satisfy the conservation of energy and momentum under generic conditions. One needs either impurities and/or commensurate external potential to absorb the momentum released by the supercurrent when the phase winding number changes by ± 1 due to a quantum phase slip. In TQF, the decay of the supercurrent into elementary excitations is kinematically allowed due to the quadratic dispersion, Eq. (3). However, at small values of v_0, this involves a large number of elementary excitations. Indeed, the energy and momentum released by a supercurrent in a single phase slip are (respectively) $\Delta E = 2\pi n_s v_0$ and $\Delta P = 2\pi n_s$. Suppose these are absorbed by $\mathcal{N}$ quasiparticles with momentum $k_* = \Delta P/\mathcal{N}$. From the energy conservation, $\Delta E = \mathcal{N} D \Delta P^2/\mathcal{N}^2 = D\Delta P^2/\mathcal{N}$, we then readily obtain $\mathcal{N} = D\Delta P^2/\Delta E = 2\pi n_s D/v_0$ and $k_* = v_0/D$. In what follows we will not be pursuing further the analysis of specific details of the decay and assume that the exponential factor in Eq. (16) controls the rate of the phase slips.

Experimental implications.—The results obtained above allow us to resolve an apparent disagreement between the scenario advocated in Ref. [16] and the experimental data on the I-V characteristic of the STS effect. Experimentally, the flow rate F (the mass current, proportional to v_0) as a function of the chemical potential bias (the "voltage") $\Delta\mu$ was found to be consistent with the sublinear power law [2,7]:

$$F = A(T)(\Delta\mu)^{\alpha} \qquad (\alpha < 1). \tag{17}$$

The authors of Ref. [2] reported $\alpha \approx 0.3 \pm 0.1$, independent of T up to $T \approx 0.5$ K. This value agrees well with $\alpha \approx 0.24$, observed at $T < 0.2$ K in Ref. [7]. However, as $T \to 1$ K, the value of $\alpha(T)$ was found to cross over to $\alpha \approx 0.5$ [7]. If it were not for the drastic temperature dependence of the flux amplitude, $A(T)$, as well as the less shocking but still unexpected temperature dependence of $\alpha(T)$, it would be natural to interpret (17) as the manifestation of the LL behavior [2].

By suggesting a solution for the temperature dependence of the amplitude $A(T)$, Ref. [16] questioned the LL origin of the dependence (17), without providing a detailed mechanism. Present analysis offers a concrete alternative to the LL interpretation based on the TQF instanton mechanism of the phase slips described above by Eqs. (15) and (16). If the probability of phase slips is controlled by the TQF instanton action (15) rather than by the matrix element of the transition from the initial to the final state of the system, the I-V characteristic of the

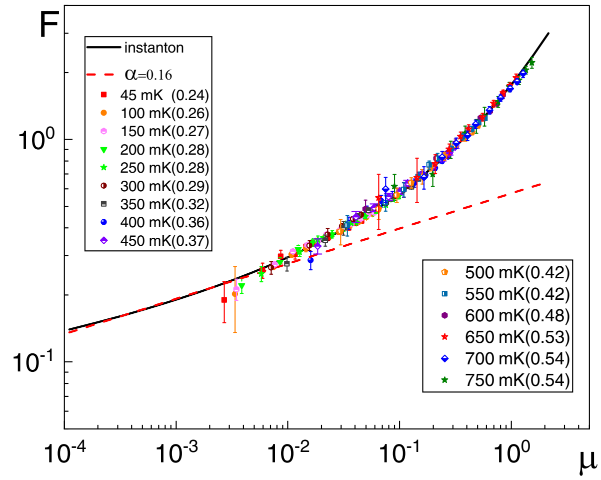

FIG. 2. The I-V characteristic master curve (solid line) of the TQF, Eq. (18) (units are arbitrary). Symbols are the experimental data from Fig. 5(a) of Ref. [7] [presented also in Fig. 1 of Ref. [17]] collected at different T and interpreted within the sub-Ohmic dependence (17). The corresponding values of T and α (in parentheses) are shown in the legend. The data points from a set at a given T are shifted without changing their log-log slopes to achieve the collapse onto the master curve (see the text below).The dashed line, as an example, corresponds to the power law (17) with $\alpha = 0.16$.

superfluid edge dislocation can be obtained within an elementary hydrodynamics approach (see Supplemental Material [17]). Accordingly, the resulting I-V curve can be described by the relation $\mathcal{P}_0 v_0 \mathcal{P} = \Delta\mu$ where $\mathcal{P}$ is given in Eq. (16) and $\mathcal{P}_0$ is a constant that includes the details of the mechanisms for the transfer of the total momentum and energy from the current to the excitations. Then, introducing dimensionless variables $\hat{v} = v_0/v_c$, $\mu = \Delta\mu/(\mathcal{P}_0 v_c)$, the normalized flow velocity $\hat{v}$ (in units of critical velocity, v_c) obeys the relation

$$\hat{v} e^{-1/\hat{v}} = \mu, \tag{18}$$

with the flux F through a sample containing many dislocations being $F \propto \hat{v}$. While the overall curve $\hat{v}$ vs μ is obviously inconsistent with the dependence (17), its significant parts can be well fit by this dependence, provided the interval of μ variation does not exceed 2 orders of magnitude—as demonstrated by the dashed line in Fig. 2.

In accordance with the large-fluctuation scenario of Ref. [16], the crossover between different parts of the curve takes place with increasing temperature when at fixed $\Delta\mu$, the dimensionless bias $\mu \sim \Delta\mu/\sqrt{n_s(T)}$ increases exponentially, shifting the I-V characteristic from $\alpha \approx 0.24$ at low T towards $\hat{v} \sim 1$, where $\alpha \approx 0.5 \div 0.6$. This behavior is demonstrated by the good collapse of the data F vs μ [7] on the master curve in Fig. 2. (See more details in Ref. [17].)

Summary and outlook.—Motivated by a recent suggestion that phase slips in a superfluid edge dislocation are expected to be qualitatively distinct from those in a LL and allow for a metastable superflow in 1D, we presented and analyzed the TQF model as the natural asymptotic limit of SED. We demonstrated that the quantum Lagrangian of TQF predicts confinement of quantum phase slips, implying the *exponential* nonlinear I-V characteristic, along with other special properties that have no analogs in known one-dimensional systems.

This confinement of instantons, accompanied by the quadratic dispersion of elementary excitations—the hallmark of the TQF—follows directly from its divergent compressibility in the absence of the Peierls potential. Despite superficial similarity with the ideal Bose gas, the physics here is qualitatively distinct because (i) SED lacks Galilean invariance and (ii) translational invariance in the climb direction prohibits terms in the TQF Hamiltonian that are responsible for the Landau-type instability—a vanishing critical current—for a parabolic dispersion in the ideal Bose gas.

Our theory is consistent with the existing experimental data on the superflow-through-solid effect thus offering a resolution of a vexing controversy in the data interpretation. The exponential I-V characteristic that we predict is in stark contrast with the LL power law, that requires a variable exponent to fit the experimental data. This naturally brings up a proposal for the compelling experimental test: detailed measurements of the I-V characteristic at low temperature and different external pressures and then collapsing the data on the master curve predicted by Eq. (18) and illustrated in Fig. 2. Specifically, (i) extending the range of the biases $\Delta\mu$ should demonstrate the deviation from the LL dependence (17), (ii) applying external pressure at small T should decrease n_s and, accordingly, shift the effective α from 0.24 to higher values, (iii) decreasing temperature below 45 mK (the lowest studied in Ref. [7]) should either decrease α to even smaller values—as demonstrated by the dashed line in Fig. 2—or eventually bring the dislocation to the LL regime, where α is determined by the emerging finite compressibility.

The role of ^{3}He impurities in modifying the I-V characteristic calls for special attention. An intriguing possibility is that pinning of the superclimbing dislocations (alike to the basal dislocations [25]) can occur while not completely suppressing the superflow through them. This would result in the suppression of the TQF regime in favor of the LL behavior. The existing discrepancy between the amount of ^{3}He needed to affect the superflow [5] (see more details in [17], Sec. VII A) requires additional experiments in the geometry of the inverse syringe effect [5] aimed at studying the I-V characteristic at different concentrations of the impurities.

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