# Nature of Optical Thermodynamic Pressure Exerted in Highly Multimoded Nonlinear Systems

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The theory of optical thermodynamics provides a comprehensive framework that enables a self-consistent description of the intricate dynamics of nonlinear multimoded photonic systems. This theory, among others, predicts a pressurelike intensive quantity ($\hat{p}$) that is conjugate to the system's total number of modes ($M$)—its corresponding extensive variable. Yet at this point, the nature of this intensive quantity is still nebulous. In this Letter, we elucidate the physical origin of the optical thermodynamic pressure and demonstrate its dual essence. In this context, we rigorously derive an expression that splits $\hat{p}$ into two distinct components, a term that is explicitly tied to the electrodynamic radiation pressure and a second entropic part that is responsible for the entropy change. We utilize this result to establish a formalism that simplifies the quantification of radiation pressure under nonlinear equilibrium conditions, thus eliminating the need for a tedious evaluation of the Maxwell stress tensor. Our theoretical analysis is corroborated by numerical simulations carried out in highly multimoded nonlinear optical structures. These results may provide a novel way in predicting and controlling radiation pressure processes in a variety of nonlinear electromagnetic settings.

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*Introduction.*—The presence of nonlinearity in highly multimoded photonic systems can give rise to a broad array of novel phenomena [1–16] but at the same time introduces several new fundamental theoretical challenges in understanding and predicting the emerging intricate spatiotemporal dynamics [17–19]. To address many of these issues, quite recently, an optical thermodynamic theory was put forward [20–22], capable of describing the collective behavior of the "photon gas" in heavily multimoded systems under weak nonlinearity [Fig. 1(a)]. In this formalism, an extensive optical entropy was self-consistently introduced $S = S(U, M, \mathcal{P})$ that directly involves the two invariants of the system, i.e., the "internal energy" $U$ and total optical power $\mathcal{P}$ as well as the total number of modes $M$. In general, it was found that at equilibrium, the power distribution among the various modes obeys a Rayleigh-Jeans law [14–16]—a response that in turn maximizes the optical entropy [20,21]. In this regard, the theory of optical thermodynamics [20–22] has provided a clear pathway in harvesting notions from statistical mechanics when studying various nonlinear multimoded optical configurations, like optical fibers, waveguide lattices, and cavity systems [14–16,20–29].

A fundamental tenet of statistical mechanics states that, for each extensive variable, there should exist a conjugate intensive quantity that acts as a thermodynamic force [30]. In the context of optical thermodynamics, the optical temperature and chemical potential can be defined through

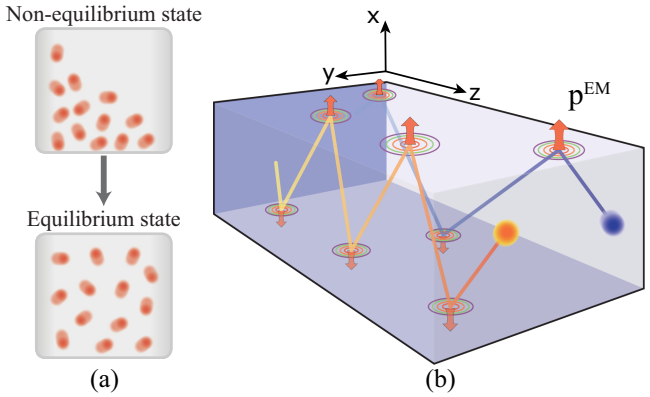


FIG. 1. (a) Photons in a nonlinear multimoded optical system. An optical arrangement can be excited with a random initial state. After the modes are thermalized, an equilibrium state is attained by maximizing the optical entropy. (b) Different photon modes can bounce at the waveguide boundaries with distinct transverse wave vectors, resulting in unique radiation pressure contributions. At equilibrium, the total pressure can be deduced by summing incoherently the contributions from all modes once weighted with respect to a Rayleigh-Jeans distribution.

 

the fundamental thermodynamic equation, according to $1/T = \partial S/\partial U$ and $\mu/T = -\partial S/\partial \mathcal{P}$, respectively [20]. Most importantly, these two intensive quantities $(T, \mu)$ directly govern the energy and power exchange $(\Delta U, \Delta \mathcal{P})$ between optical subsystems in thermal contact with each other [20]. Meanwhile, the fundamental equation of thermodynamics predicts an additional intensive quantity, the so called "optical thermodynamic pressure," i.e., $\hat{p}/T = \partial S/\partial M$ that, in principle, results from a change in the total number of modes $M$. However, the physical manifestations of this thermodynamic quantity $\hat{p}$ have so far remained elusive. If indeed it represents a thermodynamic force, could a possible interpretation of $\hat{p}$ be tied to actual pressure effects, such as those resulting from radiation pressure forces at the boundaries of dielectric materials?

Thus far, radiation pressure forces have found significant applications in laser cooling [31–33], optical trapping, and manipulation of microscopic objects [34–38]. At the same time, optomechanical arrangements based on strong phonon-photon coupling effects are nowadays actively pursued [39–46]. One way to understand these phenomena is through the perspective of momentum exchange between an object or medium and the electromagnetic field. For example, as Fig. 1(b) shows, in a waveguide system, the bouncing photons conveyed in each guided mode continuously exert an electromagnetic radiation pressure perpendicular to the boundaries, which can be determined via the Maxwell stress tensor [47–49]. In general, the Maxwell stress tensor can be employed when the field distribution is known *a priori*. However, standard electromagnetic techniques fail in chaotic systems (i.e., multimoded nonlinear configurations) where, at any given instance of time, the system exists in a random and unknown state. To tackle such a problem, it would be necessary to employ the toolset of thermodynamics and statistical mechanics. In a statistical mechanics context, historically, electromagnetic pressure has been mainly studied in black-body systems [50–55]. An interesting question to ask is whether a connection between radiation pressure and statistical mechanics can be constructed for purely optical environments where thermalization ensues because of "photon-photon collisions."

In this Letter, we elucidate the physical nature of the optical thermodynamic pressure, as defined within the context of optical thermodynamics, and formally derive an equation that unifies it with the electromagnetic radiation pressure. We show that radiation pressure in large chaotic arrangements can be directly correlated with the system's intensive thermodynamic quantities, such as the optical temperature and chemical potential and can thus be effortlessly calculated under thermal equilibrium conditions. Our results are found in agreement with the Maxwell stress tensor formalism that is evaluated through extensive numerical simulations. Consequently, we demonstrated that optical thermodynamics [20–22] can directly provide the radiation pressure resulting from thousands of modes (once optical thermal equilibrium is reached) without any knowledge of the vectorial structure of the electromagnetic fields involved.

*Background.*—We begin by considering an arbitrary nonlinear multimoded optical waveguide system supporting a finite number of $M$ bound modes, propagating along the axial direction $z$. Each mode $|\psi_k\rangle$ and its corresponding propagation constant $\beta_k$ can be obtained from the pertinent eigenvalue problem associated with the evolution equation $id|\Psi\rangle/dz = -\widehat{H_L}|\Psi\rangle$ where $\widehat{H_L}$ is the linear Hamiltonian operator of the optical system. Under weakly nonlinear conditions, the total Hamiltonian of this arrangement is primarily dominated by its linear component, i.e., $H \approx H_L$ [20,21]. In this regime, the role of nonlinearity is merely to ergodically and chaotically promote power exchange among the various modes—in a way akin to particle collisions in ideal gases responsible for thermalization.

In conservative systems, one can directly identify two invariants which are completely specified by the initial excitation conditions [22]: (i) the "internal energy" of the system $U = -\sum_{k=1}^{M} \beta_k n_k$ that physically represents the Minkowski longitudinal electrodynamic momentum flow density [56] and (ii) the total optical power $\mathcal{P} = \sum_{k=1}^{M} n_k$ transported in this guiding configuration. In the above expressions, $n_k$ represents a quantity that is proportional to the power $|c_k|^2 = \mathcal{P}_0 n_k$ conveyed by mode $|\psi_k\rangle$ where the arbitrary level $\mathcal{P}_0$ denotes the power of each discrete power packet (see Supplemental Material [57]). Note that $c_k(z) = \langle \psi_k | \Psi(z) \rangle$. As indicated in previous studies, upon thermalization, the expectation values of the power occupancies among the modes eventually settle into a Rayleigh-Jeans (RJ) distribution, that is, $n_k = -T/(\beta_k + \mu)$ where $T$ represents the optical temperature and $\mu$ is the corresponding chemical potential [20–22,66]. Interestingly, these systems display the following global equation of state: $U - \mu\mathcal{P} = MT$ (see Supplemental Material [57]).

*Optical thermodynamic pressure.*—Of importance is the classical entropy associated with these photonic multimode systems [20–26,28,29],

$$S = \sum_{k=1}^{M} \ln n_k. \tag{1}$$

In the microcanonical ensemble, the entropy is expressed as a function of the other three extensive variables, $S = S(U, M, \mathcal{P})$, a relation that directly leads to the fundamental thermodynamic equation $TdS = dU - \mu d\mathcal{P} + \hat{p} dM$ where $1/T = \partial S/\partial U$, $\mu/T = -\partial S/\partial \mathcal{P}$ and

$$\frac{\hat{p}}{T} = \frac{\partial S}{\partial M}. \tag{2}$$

Here $\hat{p}$ is what we call "optical thermodynamic pressure." Like the other two intensive quantities $T$ and $\mu$ that are conjugate to their corresponding variables $U$ and $\mathcal{P}$, the

optical thermodynamic pressure $\hat{p}$ mathematically emerges from its complementarity with respect to the number of modes $M$. While $T$ and $\mu$ act in every respect as thermodynamic forces that govern the exchange of $U$ and $\mathcal{P}$ [20], at this point it is unknown if indeed a similar physical significance can be ascribed to $\hat{p}$—the topic of this Letter. To address this fundamental issue we consider the optical thermodynamic pressure as obtained from the entropy at equilibrium

$$\frac{\hat{p}}{T} = \frac{\partial}{\partial M}\bigg|_{\mathcal{P},U} \left\{ \sum_{k=1}^{M} \ln\left[ -\frac{T}{\beta_k(M) + \frac{1}{\mathcal{P}}(U - MT)} \right] \right\}, \quad (3)$$

where in obtaining Eq. (3) we used Eqs. (1) and (2) in conjunction with the RJ distribution and the global equation of state (see Supplemental Material [57]). In Eq. (3), $\beta_k(M)$ denotes the propagation eigenvalue constant of mode $k$ which is also a function of $M$. These eigenvalues are arranged according to $\beta_1 \geq \beta_2 \geq \ldots \geq \beta_M$, where $\beta_1$ stands for the eigenvalue of the ground state while $\beta_M$ corresponds to that associated with the highest-order mode [58]. The following relation proves useful in our analysis:

$$\frac{\partial S(U, M, \mathcal{P}; T)}{\partial M}\bigg|_{\mathcal{P},U} = \frac{\partial S}{\partial M} + \frac{\partial S}{\partial T}\frac{\partial T}{\partial M} = \frac{\partial S}{\partial M}. \quad (4)$$

In obtaining Eq. (4) we used the fact that at thermal equilibrium, $\partial S/\partial T = 0$ as shown in the Supplemental Material [57]. Given that the system is highly multimoded, the summation in Eq. (3) can be written as an integral (a first order approximation to the Euler-Maclaurin formula [59]),

$$\frac{\hat{p}}{T} = \frac{\partial}{\partial M}\bigg|_{\mathcal{P},U} \left\{ \int_1^M \ln\left[ -\frac{T}{\beta(M,k) + \frac{1}{\mathcal{P}}(U - MT)} \right] dk \right\}. \quad (5)$$

The partial derivative in Eq. (5) can now be performed by employing the Leibniz integral rule (see Supplemental Material [57]), and yields

$$\frac{\hat{p}}{T} = \frac{1}{T}\int_1^M n_k \frac{\partial \beta(M,k)}{\partial M} dk + \left[ \ln\left( -\frac{T}{\beta_M + \mu} \right) - 1 \right]. \quad (6)$$

Equation (6) provides a general expression for the optical thermodynamic pressure. To understand the physical ramifications of the terms involved in Eq. (6), let us first recall that the Minkowski longitudinal electrodynamic momentum flow density $\mathcal{M}$ [56] is given by $\mathcal{M} \propto -U = \sum_{k=1}^{M} \beta_k n_k$. In this respect, the first term in Eq. (6) corresponds to a change in the longitudinal momentum flow when the supported number of modes $M$ varies. Typically, for a waveguide arrangement, a change in the number of modes can be achieved by increasing the structure size, in other words, "expanding" the waveguide. From previous studies [42,60–64,67–71], one finds that the local pressure $p_k^{EM}$ exerted by mode $k$ (carrying power $P_k = \mathcal{P}_0 n_k$) can be directly related to the total work performed during a virtual expansion through $\int_{\mathcal{C}} p_k^{EM} dl = \omega^{-1} P_k d\beta_k/d\xi$ where $\omega$ is the angular frequency of the optical field, $\xi$ represents the dimension relevant to the virtual expansion, and the line integral is performed around the waveguide boundary ($\mathcal{C}$) in the $x$-$y$ plane. In a waveguide arrangement, the local pressure $p_k^{EM}$ depends on the electromagnetic field profile and hence it may not be uniform around the circumference. In this respect, one can define a spatially averaged electrodynamic pressure two $\overline{p}_k^{EM}$ representations are different, according to $\bar{p}_k^{EM} = W^{-1}\int_{\mathcal{C}} p_k^{EM} dl$ where $W$ is the relevant boundary length and $p_k^{EM}$ is the local pressure at each point at the boundary. For example, in a regular optical fiber with radius $a$, the electromagnetic force tends to expand the fiber along the radial direction. In this case, the line integral will be performed around the fiber circumference and hence $W = 2\pi a$. By considering that the total number of supported modes is related to $\xi$ through $M = M(\xi)$, the average electrodynamic pressure produced by each mode can be rewritten as $\bar{p}_k^{EM} = (\omega W)^{-1} P_k (dM/d\xi)(d\beta_k/dM)$. Given that upon optical thermalization, the modal fields happen to be mutually incoherent, one can then obtain the total average electrodynamic pressure $p_T^{EM}$ by summing the contributions from all the modes, i.e., $p_T^{EM} = \sum_k \bar{p}_k^{EM}$. From all the above considerations, it is straightforward to show that the first term in the rhs of Eq. (6) represents the total average electrodynamic pressure and hence Eq. (6) can be recast as

$$\hat{p} = Q \cdot p_T^{EM} + T \ln\left( -\frac{T}{\beta_M + \mu} \right) - T, \quad (7)$$

where the prefactor $Q = \omega W \mathcal{P}_0^{-1}/(dM/d\xi)$ is completely determined by the waveguide system itself (see Supplemental Material [57]).

Equation (7) indicates that the thermodynamic pressure in highly multimoded systems is directly related to the electrodynamic pressure once a universal entropic term that solely depends on $T$ and $\mu$ is taken into account. To elucidate the physical significance of this additional universal term $T \ln[-T/(\beta_M + \mu)] - T$, we make use of the fundamental thermodynamic equation $TdS = dU - \mu d\mathcal{P} + \hat{p} dM$. If the total number of modes $M$ is increased by $dM$ through a virtual expansion, both the entropy and the electromagnetic momentum change by an amount $(dS, dU)$ while the power flowing in this system $\mathcal{P}$ remains invariant, i.e., $d\mathcal{P} = 0$. In this regard, the fundamental thermodynamic equation is reduced to $TdS = dU + \hat{p} dM$. From our previous analysis one can readily show that $-dU/dM = \int_1^M n_k[\partial\beta(M,k)/\partial M]dk = Q \cdot p_T^{EM}$. Consequently we find that the following relations will hold true for such an irreversible expansion:

$dS/dM = \ln[-T/(\beta_M + \mu)] - 1, \quad -dU/dM = Q \cdot p_T^{EM}$. These last two expressions imply that if the waveguide expands slowly, the universal term $T \ln[-T/(\beta_M + \mu)] - T$ in Eq. (7) will be responsible for the entropy increase while the electrodynamic pressure $p_T^{EM}$ will account for the resulting longitudinal momentum flow change. We must emphasize that the optical thermodynamic pressure is always present in a thermalized electromagnetic system and does not require such an expansion process to manifest itself (hence we consider the expansion as virtual). Moreover, to illustrate how a lateral force can lead to a change in the longitudinal momentum flow we include a rigorous proof in the Supplemental Material [57].

Of interest is to consider optical multimoded systems where the density of states is self-similar, a condition that is upheld by the majority of optical arrangements when dealing with a large number of modes. In this case, the thermodynamic Euler equation leads to the following simple expression [20,21]:

$$\frac{\hat{p}}{T} = \frac{S}{M} - 1. \tag{8}$$

From here, one quickly obtains the following relation:

$$p_T^{EM} = \frac{T}{Q}\left[\frac{S}{M} - \ln\left(-\frac{T}{\beta_M + \mu}\right)\right]. \tag{9}$$

Equation (9) represents an important result: it allows one to obtain in an effortless manner the electrodynamic pressure once a system attains a RJ thermal equilibrium at a temperature $T$ and chemical potential $\mu$. This can be accomplished by only knowing the linear spectrum of the system $\beta_k$ and the initial excitation conditions that specify the invariants $U, \mathcal{P}$. From this minimal information, one can directly calculate $S$, $T$, $\mu$ and therefore $p_T^{EM}$ from Eq. (9). This is quite impressive given that it is possible to obtain the electrodynamic pressure without having any detailed knowledge as to the electromagnetic vectorial structure of the modes themselves—an ingredient traditionally required to compute the corresponding Maxwell stress tensor elements. These results can be further extended for non-self-similar systems, as shown in the Supplemental Material [57].

*Numerical results.*—To exemplify the validity of our theoretical results, we investigate electromagnetic pressure effects in a weakly guided step-index fiber with a radius $a = 15$ μm, as shown in Fig. 2. The core refractive index of this fiber is $n_1 = 1.46$ while in the cladding it is $n_2 = 1.4454$. This fiber supports $M = 86$ modes [58,65] in each polarization and conveys in total 2 MW of power at a wavelength of 1064 nm. Our quasi-continuous-wave beam propagation simulations show that once the fiber is excited with an arbitrary spatial beam profile, it gradually undergoes thermalization into a RJ distribution, resulting in maximum

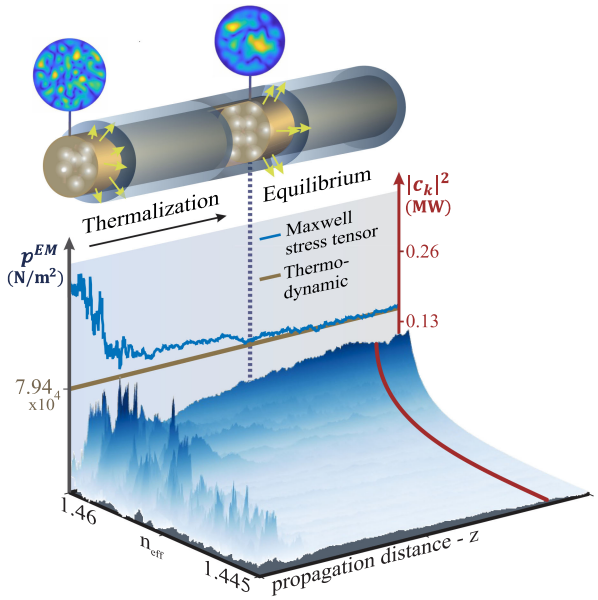


FIG. 2. A beam with an irregular spatial profile is injected into a nonlinear multimode weakly guided step-index fiber, thermalizing into a RJ distribution at the output. Here, the power occupancies $|c_k|^2$ of all guided modes are shown as a two-dimensional surface plot, whose axes involve the effective refractive index of each mode and the propagation distance $z$. The electromagnetic pressure $p^{EM}$ is calculated at each $z$ point via the Maxwell stress tensor (blue curve), displaying a gradual relaxation towards a stable equilibrium value. The brown line corresponds to the electromagnetic pressure calculated via optical thermodynamics [Eq. (9)].

decoherence between the modes. The electromagnetic pressure at each point $z$ can be monitored by performing a spatial average of the local forces per unit area along the circumference of the fiber, i.e., $p^{EM} = (2\pi)^{-1} \int_0^{2\pi} f(\theta) d\theta$, where $f(\theta)$ can be computed using the Maxwell stress tensor or the Minkowski-Helmholtz formula. During thermalization, the total electromagnetic pressure relaxes into an equilibrium value. Here, we show that this equilibrium value can be instead evaluated effortlessly using Eq. (9) where for step-index fibers $Q = 4\pi c \mathcal{P}_0^{-1}/[k_0(n_1^2 - n_2^2)]$ (see Supplemental Material [57]) and $\beta_M \simeq k_0 n_2$ with $k_0 = \omega/c$. The two results, i.e., the theoretical value predicted by Eq. (9) (brown line in Fig. 2) and the equilibrium value of the electromagnetic pressure as obtained from numerical simulations, are in excellent agreement with each other, providing compelling evidence as to the power of the thermodynamic approach. Notably, Eq. (9) involves solely thermodynamic variables that can be directly extracted from the initial condition, in contrast to conventional approaches that require a full simulation of the nonlinear dynamics.

To further demonstrate the universality of our methodology, we next evaluate the electrodynamic pressure for a high contrast step-index fiber with a radius $a = 20$ μm. This is done across a continuous range of optical energies $U$, covering almost the entirety of accessible optical

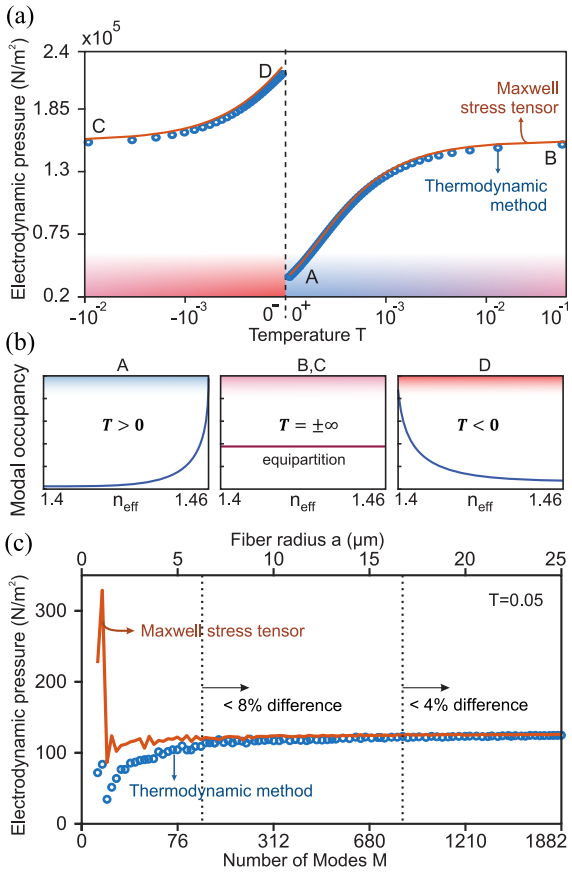


FIG. 3. (a) Electrodynamic pressure in a high contrast step-index fiber under different initial excitation conditions. The blue dots represent the pressure calculated via the optical thermodynamic method [Eq. (9)], while the red line depicts the pressure as obtained from the Maxwell stress tensor. (b) Modal occupancies corresponding to different temperature regimes. (c) Electrodynamic pressure in a high contrast step-index fiber as the number of modes $M$ varies.

temperatures $(-\infty, +\infty)$. This fiber ($n_1 = 1.46$ and $n_2 = 1.4$) supports in total $M = 1210$ modes (in both polarizations) at $\lambda_0 = 1064$ nm [58], that include vectorial transverse electric (TE) modes, transverse magnetic (TM) modes and hybrid (EH and HE) modes [58]. In all cases, the total optical power is taken to be $P = 2$ MW. The results in Fig. 3(a) show that indeed these two methods are in excellent agreement across the entire temperature range. In Fig. 3(b), we plot the corresponding Rayleigh-Jeans distributions under positive and negative temperature conditions [points $A$ and $D$ in Fig. 3(a)] where the lower- and higher- order modes are favored, respectively. Meanwhile, at infinite temperatures, $T \to \pm\infty$ [points $B$ and $C$ in Fig. 3(a)], power equipartition ensues among modes. Interestingly, as $T \to 0^-$, the electrodynamic pressure increases. Intuitively, this can be understood given that the predominant higher-order modes tend to bounce more frequently at the core-cladding interface —thus boosting up the pressure. The converse is true as $T \to 0^+$ in which case the lowest order modes (bouncing at very small angles) are occupied. In this case, the electrodynamic pressure is at its lowest, in accordance with Fig. 3(a). Finally, we investigate the electrodynamic pressure for varying fiber radii $a$ when the initial $(U, \mathcal{P})$ are such that the temperature and chemical potential $(T, \mu)$ in the RJ distribution remain the same. As illustrated in Fig. 3(c), the discrepancy between the two methods diminishes as the number of modes increases, an expected outcome considering that the derivation of Eq. (9) relies on the assumption that the optical system is highly multimoded.

*Conclusion.*—In conclusion, we have investigated the physical essence of the optical thermodynamic pressure, as defined under the framework of optical thermodynamics. We formally demonstrated that this optical thermodynamic quantity is directly tied to the actual electromagnetic pressure once a universal entropic component is taken into account. This result provides an altogether new approach to compute radiation pressure effects in highly multimoded nonlinear arrangements effortlessly, without requiring any knowledge of the complex electromagnetic modal field distributions. We must emphasize that the theoretical analysis presented herein is general and can be applied to any electromagnetic system that can display thermalization. Of interest would be to experimentally observe these processes. In this regard, radiation pressure induced liquid core fibers with high nonlinearities could be a promising direction [72,73]. Finally, we note that in Ref. [74], the electrodynamic pressure was computed in optical lattice settings using an alternative approach. In those arrangements, the pressure was defined as the conjugate variable of the system volume.

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