Quartet Superfluid in Two-Dimensional Mass-Imbalanced Fermi Mixtures

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Quartet superfluid (QSF) is a distinct type of fermion superfluidity that exhibits high-order correlation beyond the conventional BCS pairing paradigm. In this Letter, we report the emergent QSF in 2D mass-imbalanced Fermi mixtures with two-body contact interactions. This is facilitated by the formation of a quartet bound state in vacuum that consists of a light atom and three heavy fermions. For an optimized heavy-light number ratio 3 : 1, we identify QSF as the ground state in a considerable parameter regime of mass imbalance and 2D coupling strength. Its unique high-order correlation can be manifested in the momentum-space crystallization of a pairing field and density distribution of heavy fermions. Our results can be readily detected in Fermi-Fermi mixtures nowadays realized in cold atoms laboratories, and meanwhile shed light on exotic superfluidity in a broad context of mass-imbalanced fermion mixtures.

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A basic idea to achieve the superfluidity of fermions is to bind an even number of fermions into a composite boson and then let bosons condense. A classic example is the Bardeen-Cooper-Schrieffer (BCS) superfluid in terms of the condensation of Cooper pairs [1], which reflects the two-body correlation among spin-1/2 fermions and has achieved great success not only in solid states but also in cold atoms [2,3] and nuclear matters [4,5]. Going beyond the BCS framework, a fascinating yet challenging direction is to engineer superfluids with higher-order correlations. Along this direction, a leading case is the quartet superfluid (QSF), a distinct type of fermion superfluidity based on the condensation of four-fermion clusters. Previous studies have revealed QSF in spin-3/2 fermions [6], nuclei with α-particle condensation [7–11], biexciton condensates [12], and various systems hosting charge-4e superconductivity [13–23]. However, stringent conditions are required therein such as multicomponents, multibody interactions, or pair fluctuations under particular symmetries, which make the experimental exploration of QSF rather rare and difficult in practice.

Recently, mass-imbalanced Fermi mixtures realized in ultracold gases, such as ^{40}K-^{6}Li [24–26], ^{161}Dy-^{40}K [27,28], and ^{53}Cr-^{6}Li [29–31], offer a much easier platform for achieving QSF. The predominant few-body correlation in these systems can be inferred from the formation of universal few-body clusters that consist of a light atom and several heavy fermions. Each cluster bound state requires the heavy-light mass ratio beyond certain critical value [32–40] but still small enough to avoid any Efimovian binding [34,41,42]. These clusters are therefore believed to be elastically stable under collision. Physically, their formation is due to a long-range heavy-heavy attraction mediated by the light atom, which competes with a repulsive centrifugal barrier in p-wave channel [32,43]. As such, the critical mass ratio to support a tetramer bound state (a quartet) is found to be quite high in 3D [33,34], but is sufficiently low in 2D [37] so as to be accessible by all Fermi mixtures listed above. This quartet formation has been shown to fundamentally change the destiny of Fermi polaron compared to equal mass case, when increasing the attraction between light impurity and heavy majorities [44]. Then the ultimate question is how would the quartet affect the many-body property of heavy-light mixtures? In particular, can QSF emerge as a ground state? If so, this will be the *simplest* fermion system so far to support QSF, i.e., with only two components and under two-body contact interactions. Such a system would be much more convenient to manipulate experimentally.

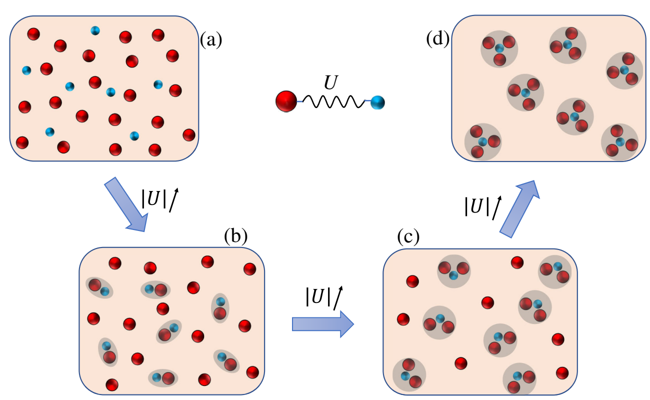

FIG. 1. Schematics for different phases in heavy(red)-light (blue) fermion mixtures with number ratio 3 : 1. When increasing the attraction strength $|U|$, the system undergoes a sequence of phase transitions from the normal mixture (a) to mixed pairing superfluid and normal state (b), mixed trimer liquid and normal state (c), and finally to quartet superfluid (d).

In this Letter, we unveil the emergent QSF in 2D mass-imbalanced Fermi mixtures. A variational ansatz is constructed to describe QSF with optimal heavy-light number ratio 3 : 1, which well incorporates the essential four-body correlations in a many-body setting. In contrast to previous studies of mass-imbalanced fermions that focused on pairing superfluids [45–52], our Letter demonstrates that QSF is always the ground state under sufficient mass imbalance and heavy-light attraction strength. This suggests a sequence of phase transitions, namely, from a normal Fermi mixture to eventually a QSF, when increasing the heavy-light attractions (see Fig. 1). We have mapped out a phase diagram for QSF and other competing states as tuning the 2D coupling strength and mass imbalance (Fig. 2). Furthermore, we show that the unique high-order correlation of QSF manifests itself in the momentum-space crystallization of a pairing field and density distribution of heavy fermions (Fig. 3). The QSF, which emerges in the seemingly simple framework of mass-imbalanced two-component fermions, represents a qualitatively new kind of high-order superfluidity in strongly correlated fermionic matter. Our results can be readily probed in a number of Fermi mixtures realized in ultracold atoms, and meanwhile shed light on exotic superfluidity in a broad context of fermion systems with mass imbalance, such as the semiconducting transition metal dichalcogenides.

We start from the following Hamiltonian ($\hbar = 1$):

$$H = \sum_{\mathbf{k}} \left(\epsilon^l_{\mathbf{k}} l^\dagger_{\mathbf{k}} l_{\mathbf{k}} + \epsilon^h_{\mathbf{k}} h^\dagger_{\mathbf{k}} h_{\mathbf{k}}\right) + \frac{g}{S} \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} l^\dagger_{\mathbf{q}-\mathbf{k}} h^\dagger_{\mathbf{k}} h_{\mathbf{k}'} l_{\mathbf{q}-\mathbf{k}'}. \quad (1)$$

Here $h^\dagger_{\mathbf{k}}$ and $l^\dagger_{\mathbf{k}}$ respectively create a heavy and a light fermion at momentum $\mathbf{k}$ with energy $\epsilon^{h,l}_{\mathbf{k}} = \mathbf{k}^2/(2m_{h,l})$, and their mass ratio is $\eta \equiv m_h/m_l (> 1)$; the 2D bare coupling g is renormalized through $1/g = -1/S \sum_{\mathbf{k}} 1/(\epsilon^l_{\mathbf{k}} + \epsilon^h_{\mathbf{k}} + E_{2b})$, where S is the system area and $E_{2b} = (2m_r a^2)^{-1}$ is the two-body binding energy given by scattering length a and reduced mass $m_r = m_l m_h/(m_l + m_h)$. In this Letter, we consider the most favorable heavy-light number ratio for QSF, i.e., $N_h : N_l = 3 : 1$. Accordingly we introduce a momentum unit as $k_F = \sqrt{4\pi N_Q/S}$, with $N_Q = N_l = N_h/3$ the number of quartets.

For a microscopic description of QSF, we utilize the exact wave function of a zero-momentum quartet (tetramer bound state) in vacuum that is created by

$$Q^\dagger = \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} \phi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} l^\dagger_{-\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3} h^\dagger_{\mathbf{k}_1} h^\dagger_{\mathbf{k}_2} h^\dagger_{\mathbf{k}_3}. \quad (2)$$

Treating the quartet as a composite boson and recalling the intrinsic relation between fermion superfluidity and Bose condensation, we write down a quartet-condensed coherent state, $e^{\lambda Q^\dagger}$, to describe the QSF state of fermions, which can be further simplified as

$$\Psi_{\mathrm{QSF}} = \prod_{\langle \mathbf{k}_1\mathbf{k}_2\mathbf{k}_3 \rangle} (1 + \psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} l^\dagger_{-\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3} h^\dagger_{\mathbf{k}_1} h^\dagger_{\mathbf{k}_2} h^\dagger_{\mathbf{k}_3}) |0\rangle. \quad (3)$$

Here the variational coefficients $\psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}$ are antisymmetric with respect to the exchange $\mathbf{k}_i \leftrightarrow \mathbf{k}_j$ $(i \neq j)$, and $\langle\rangle$ denotes that we avoid any double counting of $\mathbf{k}$ triples $\{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\}$.

Different from the well-known BCS ansatz $[\sim \prod_{\mathbf{k}} (1 + \psi_{\mathbf{k}} l^\dagger_{-\mathbf{k}} h^\dagger_{\mathbf{k}})]$ where only one $\mathbf{k}$ index is used to characterize each Cooper pair, the QSF wave function (3) displays a much higher degree of freedom given three momenta (i.e., a $\mathbf{k}$ triple) in each bracket to label a quartet. This implies a higher degree of complexity in treating the many-body problem, especially when considering the Pauli effect and the nonuniqueness of fermion occupation from multibrackets. In this Letter, as the first attempt to include quartet correlation in tackling the fermion superfluid problem, we shall neglect the contribution from these complexities. It can be proved that their induced corrections are of the order $\sim\psi^2$, which can be well controlled as long as all $\psi \ll 1$ (valid especially in the strong coupling regime) [53].

Under the above treatment, we can expand the thermodynamic potential $\Omega \equiv \langle H - \mu N_Q \rangle_{\mathrm{QSF}}$ as a function of ψ, with μ introduced as the quartet chemical potential:

$$\Omega = \sum_{\langle \mathbf{k}_1\mathbf{k}_2\mathbf{k}_3 \rangle} \frac{|\psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}|^2}{1 + |\psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}|^2} E_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} + \sum_{\langle \mathbf{k}_1\mathbf{k}_2\mathbf{k}_3 \rangle} \frac{\psi^*_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}}{1 + |\psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}|^2} (\Delta_{\mathbf{k}_2\mathbf{k}_3} - \Delta_{\mathbf{k}_1\mathbf{k}_3} + \Delta_{\mathbf{k}_1\mathbf{k}_2});$$

here $E_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} = \epsilon^l_{-\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3} + \epsilon^h_{\mathbf{k}_1} + \epsilon^h_{\mathbf{k}_2} + \epsilon^h_{\mathbf{k}_3} - \mu$, and the auxiliary function $\Delta_{\mathbf{k}_2\mathbf{k}_3}$ is defined as

$$\Delta_{\mathbf{k}_2\mathbf{k}_3} = \frac{g}{S} \sum_{\mathbf{k}_1} \frac{\psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}}{1 + |\psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}|^2}. \quad (4)$$

Minimizing Ω via $\partial\Omega/\partial\psi^*_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} = 0$, we obtain

$$\psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} = \frac{E_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} - \sqrt{E^2_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} + 4(\Delta_{\mathbf{k}_2\mathbf{k}_3} - \Delta_{\mathbf{k}_1\mathbf{k}_3} + \Delta_{\mathbf{k}_1\mathbf{k}_2})^2}}{2(\Delta_{\mathbf{k}_2\mathbf{k}_3} - \Delta_{\mathbf{k}_1\mathbf{k}_3} + \Delta_{\mathbf{k}_1\mathbf{k}_2})}. \quad (5)$$

Further utilizing (4) we arrive at the self-consistent equation for $\{\Delta_{\mathbf{k}\mathbf{k}'}\}$:

$$-\frac{S}{g} \Delta_{\mathbf{k}_2\mathbf{k}_3} = \sum_{\mathbf{k}_1} \frac{\Delta_{\mathbf{k}_2\mathbf{k}_3} - \Delta_{\mathbf{k}_1\mathbf{k}_3} + \Delta_{\mathbf{k}_1\mathbf{k}_2}}{\sqrt{E^2_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} + 4(\Delta_{\mathbf{k}_2\mathbf{k}_3} - \Delta_{\mathbf{k}_1\mathbf{k}_3} + \Delta_{\mathbf{k}_1\mathbf{k}_2})^2}}. \quad (6)$$

The number equation $N_Q = -\partial\Omega/\partial\mu$ is written as

$$2N_Q = \sum_{\langle \mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\rangle}\left(1-\frac{E_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}}{\sqrt{E^2_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}+4(\Delta_{\mathbf{k}_2\mathbf{k}_3}-\Delta_{\mathbf{k}_1\mathbf{k}_3}+\Delta_{\mathbf{k}_1\mathbf{k}_2})^2}}\right). \tag{7}$$

Physically, $\Delta_{\mathbf{k}_2\mathbf{k}_3}$ in Eq. (4) can be viewed as the pairing field of QSF, since it is obtained by contracting the internal degree of one heavy-light pair in the quartet while leaving the two additional heavy fermions free at $\mathbf{k}_2$, $\mathbf{k}_3$. This is dramatically different from the pairing field in usual BCS theory which is a constant rather than $\mathbf{k}$ dependent. In fact, the right sides of Eqs. (5)–(7) suggest a superposed pairing field, $\tilde{\Delta}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} \equiv \Delta_{\mathbf{k}_2\mathbf{k}_3} - \Delta_{\mathbf{k}_1\mathbf{k}_3} + \Delta_{\mathbf{k}_1\mathbf{k}_2}$, to uniquely identify a quartet in $\mathbf{k}$-space. In the strong coupling limit with deep quartet binding, we have $|\tilde{\Delta}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}| \ll |\mu|$ and therefore Eq. (6) well reproduces the exact equation for a quartet bound state in vacuum [34,36,37]. This guarantees the picture of quartet condensation in this limit with $\mu \to E_Q$, where E_Q is the binding energy of a vacuum quartet.

We have numerically solved Eqs. (6) and (7) to obtain μ and $\{\Delta_{\mathbf{k}\mathbf{k}'}\}$ for given interaction strength $\ln(k_F a)$ and mass ratio η. The total energy $E \equiv \langle H\rangle_{\rm QSF}$ can then be computed straightforwardly. Note that in Eq. (7), the summation on $\mathbf{k}$ triples brings another relevant parameter S/a^2 to the problem, which is taken as 100 throughout the Letter. We have checked that different S/a^2 will not qualitatively change our results [53].

In Figs. 2(a1) and 2(a2), we take the Dy-K and K-Li mixtures as two experimentally relevant examples and plot their corresponding energy per quartet (E/N_Q) and chemical potential (μ) as functions of $\ln(k_F a)$. As expected, in the strong coupling limit $\ln(k_F a) \to -\infty$, both E/N_Q and μ approach E_Q (dashed horizontal lines). When moving to weaker couplings, both quantities increase up to a critical coupling strength where $\mu \sim -E_{2b}$, see circles in Figs. 2(a1) and 2(a2), beyond which Eqs. (6) and (7) fail to produce a convergent solution. Before reaching this point, however, the solutions can become unphysical due to the violation of Pauli principle, i.e., the $\mathbf{k}$-space number of heavy fermions exceeds beyond 1 ($N^h_{\mathbf{k}} > 1$), as bounded by triangles in Fig. 2(a2). Such violation can be attributed to the neglect of Pauli effect in treating $\Psi_{\rm QSF}$ [53], which results in a quick accumulation of $N^h_{\mathbf{k}}$ beyond 1 as the system departs from the strong coupling regime. In the following, we shall only take the physical solutions of QSF with all $N^h_{\mathbf{k}} < 1$, i.e., for $\ln(k_F a)$ ranging from $-\infty$ to the circles in Fig. 2(a1) and to the triangles in Fig. 2(a2).

We have compared the energy of QSF with all other competing states including a normal mixture, various pairing superfluids studied for mass-imbalanced fermions [45–52], a trimer liquid proposed in 3D [54], as well as a pentamer liquid [53]. Finally a phase diagram is mapped

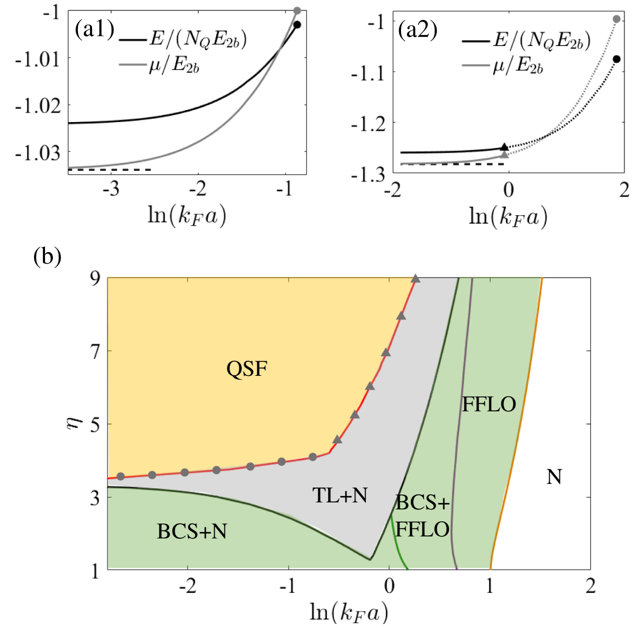

FIG. 2. Emergence of quartet superfluid in 2D heavy-light fermion mixtures with number ratio 3:1. (a1),(a2) Energy per quartet E/N_Q and chemical potential μ of the quartet superfluid as functions of coupling strength $\ln(k_F a)$ for mass ratio $\eta = 161/40$ (a1) and $40/6$ (a2). Here the energy unit is the two-body binding energy E_{2b}, and the horizontal dashed line shows the quartet binding energy in vacuum [37]. The circles denote the termination of self-consistent solutions from Eqs. (6) and (7). The triangles in (a2) mark the location when the Pauli principle starts to be violated, and thus the solutions from triangles to circles are unphysical. (b) Phase diagram in $[\ln(k_F a), \eta]$ parameter plane. The phases (from left to right) are the quartet superfluid ("QSF," yellow area), mixed trimer liquid and normal state ("TL + N," gray), states involving pairing superfluids ("BCS + N," "BCS + FFLO," "FFLO," green area), and the normal state ("N," white). The QSF boundaries with circles or triangles are in accordance with notations in (a1),(a2).

out in $[\ln(k_F a), \eta]$ plane for a fixed number ratio $N_h/N_l = 3$; see Fig. 2(b). The relevant phases appearing on the diagram are QSF (yellow area), mixed trimer liquid and normal state ("TL + N," gray), states involving pairing superfluids (green), and the normal mixture ("N," white). The pairing superfluids include the Fulde-Ferrell-Larkin-Ovchinnikov superfluid ("FFLO") and two phase-separated states [between BCS and FFLO ("BCS + FFLO") and between BCS and normal ("BCS + N")]. For TL + N, we have approximated it as two homogeneous Fermi seas of trimers and of excess heavy atoms, each comprising N_l particles. In justifying this, we require the trimer on top of a heavy Fermi sea to be a true bound state ($E_t < 0$), with energy lower than that of the corresponding atom-dimer threshold ($E_t < E_d + E^h_F$). This leads to the phase boundaries between TL + N and other pairing superfluids in Fig. 2(b), and we have confirmed that within the gray area TL + N is indeed more energetically favorable than all other available states. Similarly, we have also considered a pentamer liquid in coexistence with a light Fermi sea, and found that such state always has a higher energy than QSF and thus is not relevant (see more details in the Supplemental Material [53]).

Importantly, Fig. 2(b) shows that QSF represents the ground state in a considerably broad parameter region with $\ln(k_F a) \lesssim 0$ and $\eta > \eta_Q \sim 3.4$ (η_Q is the critical mass ratio to support a 2D quartet in vacuum [37]). Moreover, it tells us that under sufficient $\eta (> \eta_Q)$, the system undergoes a sequence of phase transitions as increasing the heavy-light attractions, i.e., from a normal mixture at weak coupling to states involving pairing superfluid or trimer liquid, and finally ending up at QSF at strong coupling (see also Fig. 1). We remark here that the occurrence of these transitions is physically robust, because QSF cannot adiabatically connect to a normal Fermi sea as interactions are reduced. This can be clearly seen from its wave function (3), which involves a fundamental restructuring of heavy-light distributions in $\mathbf{k}$ space and thus cannot reproduce two uncorrelated Fermi seas by sending ψ to ∞. It is very different from the BCS ansatz [1], which in a weak coupling limit is just a slight modification of a normal Fermi sea. Therefore the BCS ansatz can well describe a smooth BCS-BEC crossover for balanced spin-1/2 fermions, but here a sequence of transitions are produced for mass- or spin-imbalanced systems. Such difference is intrinsically due to the high-order correlation hidden in QSF, as revealed below.

Different from all other phases in Fig. 2(b), QSF exhibits unique high-order correlation in $\mathbf{k}$ space. Such correlation originates from the internal structure of quartet wave function $\psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}$, as shown in Fig. 3(a), which has the largest weight if the $\mathbf{k}$ triple $(\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3)$ forms a regular triangle. This is consistent with the crystalline structure of a quartet in vacuum [37]. Physically, the triangular structure emerges because it is highly symmetric and thus provides the largest phase space for three fermions scattering within a quartet unit. Similar triangular distribution also appears in the pairing field $\Delta_{\mathbf{k}_0\mathbf{k}}$, as shown in Fig. 3(b), when fixing $\mathbf{k}_0$ at the largest $|\psi|$ (red point). Interestingly, $\Delta_{\mathbf{k}_0\mathbf{k}} \sim \mathbf{k}$ exhibits a chiral distribution, i.e., its sign switches depending on whether $\mathbf{k}$ moves clockwise or anticlockwise from $\mathbf{k}_0$. This can be attributed to the antisymmetry of $\Delta_{\mathbf{k}_0\mathbf{k}}$ with respect to $\mathbf{k}_0 \leftrightarrow \mathbf{k}$, as required by its definition in Eq. (4).

To experimentally detect the above correlation, we propose measuring the density-density correlation function of heavy fermions in $\mathbf{k}$ space [53]:

$$D_h(\mathbf{k}_0, \mathbf{k}) \equiv \langle n_h(\mathbf{k}_0) n_h(\mathbf{k})\rangle - \langle n_h(\mathbf{k}_0)\rangle\langle n_h(\mathbf{k})\rangle. \quad (8)$$

Here $\langle n_h(\mathbf{k})\rangle$ is the mean density distribution of heavy fermions. In Figs. 3(c) and 3(d), we show $\langle n_h(\mathbf{k})\rangle$ and $D_h(\mathbf{k}_0, \mathbf{k})$ for a typical QSF state of K-Li mixture. We can see that $\langle n_h(\mathbf{k})\rangle$ is peaked at a finite $|\mathbf{k}|$ and shows a dip at

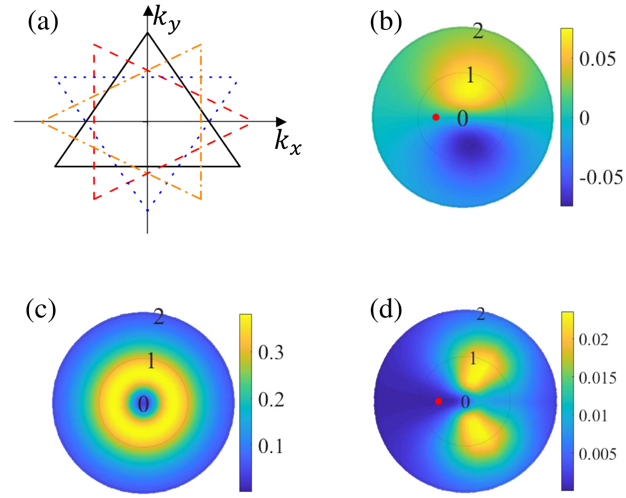

FIG. 3. Momentum-space correlation of quartet superfluid. (a) The largest $|\psi_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}|$ with $(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ forming a regular triangle. Several degenerate triangles are shown, which have the same value of $|\psi|$. (b) $\Delta_{\mathbf{k}_0\mathbf{k}}$ (in units of E_{2b}) in $\mathbf{k}$ space, with $\mathbf{k}_0$ pinned at one momentum for the largest $|\psi|$ (red point). (c) Mean density distribution of heavy fermions, $\langle n_h(\mathbf{k})\rangle$ (in units of S^{-1}). (d) Density-density correlation of heavy fermions, $D_h(\mathbf{k}_0, \mathbf{k})$ (in units of S^{-2}), with $\mathbf{k}_0$ pinned at one maximum of $\langle n_h\rangle$ (red point). In (b)–(d), we take $\eta = 40/6$ and $\ln(k_F a) = -0.58$.

$\mathbf{k} = 0$, dramatically different from the distributions of a normal Fermi sea or a pairing superfluid. Fixing $\mathbf{k}_0$ at the peak of $\langle n_h\rangle$, $D_h(\mathbf{k}_0, \mathbf{k})$ shows two visible peaks in $\mathbf{k}$ space, which form a regular triangle together with $\mathbf{k}_0$. This visualizes the unique high-order correlation in QSF that is absent in all other states. For instance, one has $D_h(\mathbf{k}_0, \mathbf{k}) = 0$ for normal state and any type of pairing superfluids. In cold atoms experiment, the mean density and density-density distributions can be measured, respectively, using the time of flight technique and atom noise in absorption images [57–62] or single atom resolved image [63].

At finite low temperatures, the 2D QSF is expected to survive with quasi-long-range order and a Berezinskii-Kosterlitz-Thouless(BKT)-type transition can occur at a critical $T_{\rm BKT}$. In the strong coupling regime with deep quartet binding, we estimate $T_{\rm BKT}$ through the 2D quasicondensation of bosons [64] as $T_{\rm BKT}/T_F^Q = \ln^{-1}[-\xi/(2\pi)\ln(\sqrt{4\pi}k_F a)]$, with $\xi = 380$ and $T_F^Q = k_F^2/[2(m_l + 3m_h)]$. Here we have assumed the quartets interacting via a repulsive potential with range $\sim a$ [65]. For $\ln(k_F a) \in [-5, -2]$, we obtain a slowing varying $T_{\rm BKT}/T_F^Q = 0.18 \sim 0.26$. Given successful measurements of $T_{\rm BKT}$ in pairing superfluids of spin-1/2 Fermi gases [66–69], we expect the BKT transition of QSF can also be explored in mass-imbalanced Fermi mixtures [24–31], for instance, by measuring the quartet momentum distribution similar to Ref. [66].

In the future, the present theory could be further improved by incorporating the Pauli effect while treating

Eq. (3), which becomes more important when departing from the strong coupling regime. Moreover, it is interesting to consider a general number ratio ($N_h/N_l \neq 3$), where QSF may coexist with other states and result in an even richer phase diagram. In addition, to be more relevant to ultracold experiments, it is desirable to address the effects of finite T and finite effective range over the whole interaction regime in a quasi-2D geometry. For a pure 3D system, QSF is expected to appear at higher η that can support a quartet in vacuum [33,34]. In this case, the true long-range order of QSF can extend to finite T, and at deep bindings the critical T_c approaches the transition temperature for quartet condensation, i.e., $T_c/T_F^Q = 0.44$.

Finally, it is worth pointing out that the many-body phenomenon of QSF is deeply rooted in the highly nontrivial few-body physics, namely, the exact quartet formation of heavy-light fermions in vacuum. Based on this, we expect similar high-order superfluids to exist in a broad class of fermion systems with imbalanced (effective) masses. For instance, in a spin-orbit coupled atomic gas [70] the lower helicity branch can possess a large effective mass [71], which may serve as the heavy component to interact with other (light) fermion species. Another promising system is the monolayer transition metal dichalcogenides with charged excitons (trions) [72–74], where the mass-imbalanced mixture can consist of trions and electrons (or holes). Indeed, recent theories have revealed the existence of few-body clusters therein [75,76]. For potentially exotic superfluids in these systems, our Letter suggests the few-to-many perspective as always a reliable route to approach them.

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