

## Classical Drift in the Arnold Web Induces Quantum Delocalization Transition

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We demonstrate that quantum dynamical localization in the Arnold web of higher-dimensional Hamiltonian systems is destroyed by an intrinsic classical drift. Thus quantum wave packets and eigenstates may explore more of the intricate Arnold web than previously expected. Such a drift typically occurs, as resonance channels widen toward a large chaotic region or toward a junction with other resonance channels. If this drift is strong enough, we find that dynamical localization is destroyed. We establish that this drift-induced delocalization transition is universal and is described by a single transition parameter. Numerical verification is given using a time-periodically kicked Hamiltonian with a four-dimensional phase space.

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*Introduction.*—Chaotic dynamics in higher-dimensional Hamiltonian systems and its quantum mechanical consequences play an important role in many fields of physics. This is well established for describing atoms, molecules, and chemical reactions [1–4]. More recently the dynamics in phase space has also been exploited in the context of quantum many-body systems [5,6]. For example for Bose-Hubbard systems semiclassical methods allow one to explain many phenomena [7–9], including spectral statistics, entanglement, and many-body quantum scars. For current implementations of quantum computers the destructive role of classical chaos has been demonstrated [10,11].

The classical phase space of such systems is typically governed by the presence of regions with regular dynamics and regions with chaotic dynamics. In higher-dimensional systems all chaotic regions are connected and form a single network, the so-called Arnold web [12]. The structure of this network is governed by resonance channels, characterized by regular dynamics fulfilling resonant frequency conditions. The chaotic layers of the resonance channels allow for transport in the Arnold web by so-called Arnold diffusion [12–17]. Thus, even in predominantly regular phase-space regions, the Arnold web makes it possible to get arbitrarily close to any point in phase space.

In the context of quantum mechanics an essential question is how eigenstates and wave packets explore the intricate Arnold web. Obviously, the chaotic layer of a resonance channel must be wide enough to accommodate a wave packet with minimal uncertainty. But there is another restriction caused by the important phenomenon of dynamical localization [18,19]. It leads to quantum localization in the presence of classical chaotic diffusion. For a resonance channel this was demonstrated in a 4D map [20,21]. So one expects that a quantum wave packet cannot explore a resonance channel of the Arnold web.

In this Letter, however, we demonstrate that dynamical localization is destroyed by a drift in the Arnold web, if the drift is strong enough. Such a drift generically occurs when resonance channels widen toward the chaotic region or toward junctions of resonance channels. We give a universal description of this drift-induced delocalization transition, see Fig. 1. To this end we introduce a transition parameter, which depends on the classical drift velocity, the available chaotic phase-space volume, and the size of a Planck cell. We demonstrate the drift-induced delocalization transition using a 4D symplectic map. Thus quantum wave packets may explore more of the Arnold web than previously expected.

*Delocalization transition.*—Dynamical localization is a fundamental phenomenon in quantum chaos if the corresponding classical systems shows diffusion along a one-dimensional coordinate [18,19]. One finds that the localization length is proportional to the classical diffusion coefficient. The time evolution of a quantum mechanical wave packet mimics classical diffusion up to the break time  $t^*$  at which it localizes with localization length  $\lambda$ .

Dynamical localization can be destroyed by various mechanisms, e.g. (i) noise induced by coupling to an environment [22–27], (ii) diffusion in higher dimensions [28,29], or (iii) many-body interactions [30,31], which recently has been studied experimentally in ultracold gases [32,33].

Here we address the general question of dynamical localization in the presence of an intrinsic classical drift, which occurs in addition to classical chaotic diffusion. One intuitively expects that quantum localization might be destroyed and quantum transport made possible again. But how strong has the drift to be?

We propose that there is a universal transition from localization to delocalization in the presence of a classical

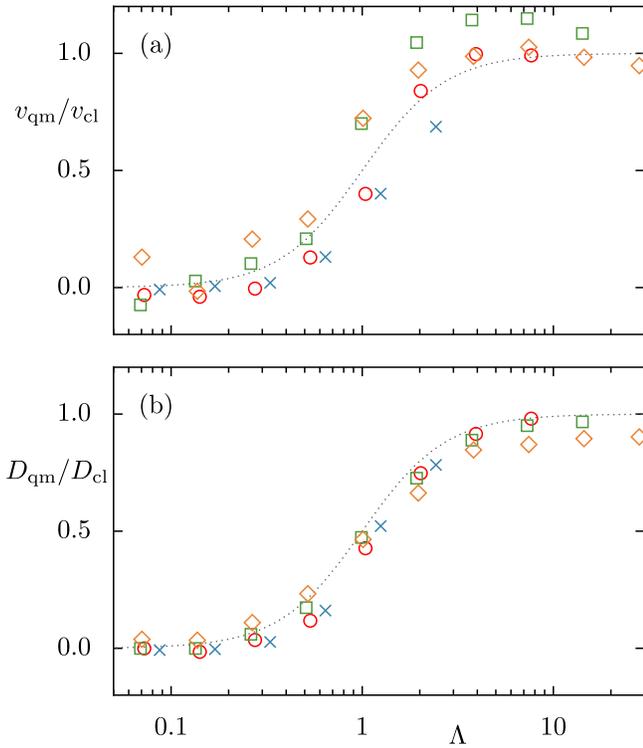


FIG. 1. Universal transition from dynamical localization to (a) quantum drift with velocity  $v_{qm}$  and (b) quantum diffusion with diffusion coefficient  $D_{qm}$  scaled by classical values  $v_{cl}$  and  $D_{cl}$ , respectively. The transition is characterized by the universal transition parameter  $\Lambda$ , Eqs. (1) and (2). The function  $\Lambda^2/(1+\Lambda^2)$  serves as a guide to the eye (dotted line). The wave packet dynamics is studied for the kicked Hamiltonian, Eq. (5), with parameters and symbols described in the text.

drift with velocity  $v_{cl}$ . Without drift, a wave packet localizes at the break time  $t^*$  with localization length  $\lambda$ . With drift, a wave packet has at this time  $t^*$  shifted by a distance  $|v_{cl}|t^*$ . If this distance is larger than  $\lambda$ , no localization with  $\lambda$  is possible. This motivates the definition of a transition parameter  $\Lambda$ , which is given by the ratio of drift distance to localization length in the case of no drift,

$$\Lambda = \frac{|v_{cl}|t^*}{\lambda}. \quad (1)$$

We expect that localization is destroyed if  $\Lambda \gg 1$ , while it should still occur if  $\Lambda \ll 1$ .

Before we provide details of the delocalization transition let us give a visual overview. The universal transition at  $\Lambda = 1$  is demonstrated in Fig. 1 for an example system, a kicked Hamiltonian, Eq. (5), leading to a 4D symplectic map. It has a dominant resonance channel, see Fig. 2, along which classical chaotic diffusion and drift takes place, see Fig. 3. The quantum dynamics of a wave packet in this resonance channel is characterized by a quantum drift velocity and a quantum diffusion coefficient, see Fig. 4.

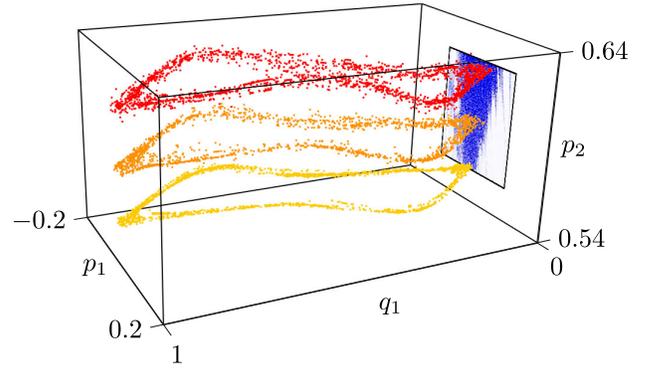


FIG. 2. 3D phase-space slice of the 4D map, Eq. (6), at  $q_2 = 0.5$ . Three ensembles with initial conditions started at different positions  $p_2$  near  $(q_1, p_1) = (0, 0)$  in the resonance channel are shown for 200 iterations. They spread out uniformly in the chaotic region around the resonance and slightly diffuse along  $p_2$ . For larger  $p_2$  the chaotic region is wider, as can be also seen on the  $q_1 = 0$  plane in which the fast Lyapunov indicator from Fig. 3(a) is shown.

*Transition parameter.*—We show for a  $2f$ -dimensional symplectic map that the transition parameter  $\Lambda$ , Eq. (1), can be more conveniently expressed in terms of properties of the classical system and the size  $h^f$  of a Planck cell by

$$\Lambda(x) = \frac{|v_{cl}(x)|\tau w_{ch}(x)}{h^f}. \quad (2)$$

Here  $x$  is a local phase-space coordinate along the resonance channel,  $w_{ch}(x)$  is the cross-sectional volume of the chaotic region of the resonance channel at  $x$ , i.e. a  $(2f - 1)$ -dimensional phase-space volume, and  $\tau$  is the time period of the map. Typically, the drift velocity  $v_{cl}$  and the volume  $w_{ch}$  will depend on the coordinate  $x$  along the resonance channel and thus so will the transition parameter  $\Lambda$ . Note that for  $f \geq 2$  there are resonance channels with 1 up to  $f - 1$  resonance conditions and we study the maximal case, where the resonance channel is extended in one dimension. We expect that these one-dimensional resonance channels have the largest impact on transport.

Equation (2) gives  $\Lambda$  as the chaotic  $2f$ -dimensional phase-space volume explored due to the drift within one time period of the map compared to a Planck cell. Let us remark, that such a ratio of a phase-space volume to the size of the Planck cell, also appears in the transport across partial barriers in phase space [34–41].

We can get from Eq. (1) to Eq. (2) by using the Siberian argument [42] generalized to a one-dimensional resonance channel of a  $2f$ -dimensional symplectic map: We assume that all eigenfunctions in the chaotic region of the resonance channel localize with the same localization length  $\lambda$ . Then the number of states excited by a wave packet in the resonance channel is given by

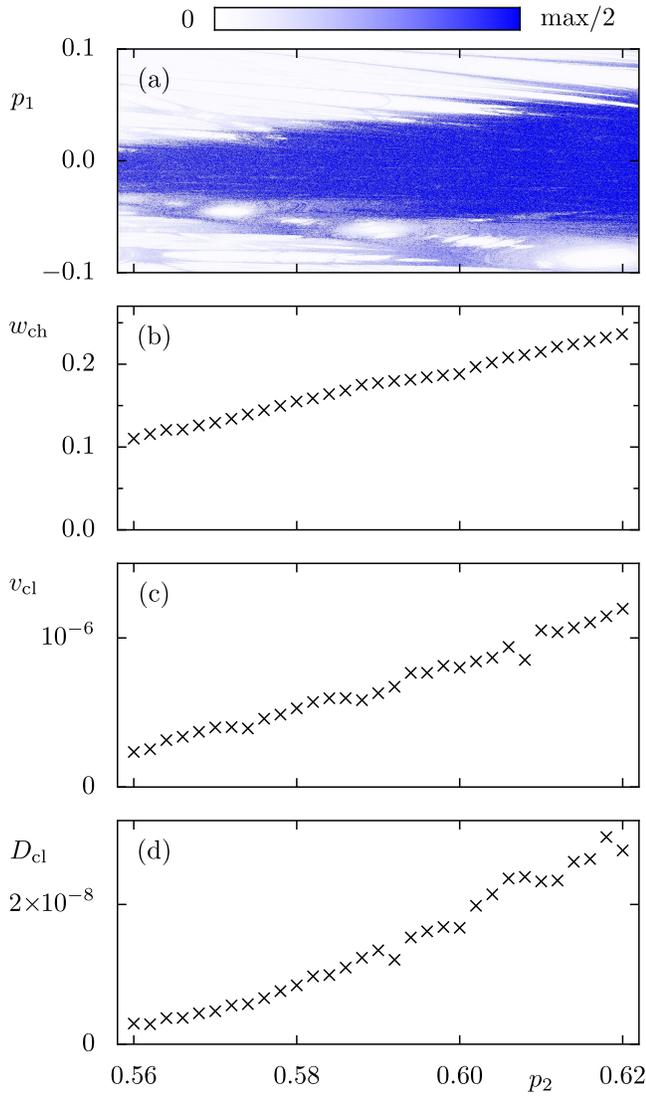


FIG. 3. Classical properties of the 4D map, Eq. (6), depending on coordinate  $x = p_2$ : (a) Fast Lyapunov indicator on the plane  $q_1 = 0$ ,  $q_2 = 0.5$  visualizing the increasing cross-sectional volume of the resonance channel, (b) chaotic 3D phase-space volume  $w_{\text{ch}}$  of the resonance channel, (c) drift velocity  $v_{\text{cl}}$ , and (d) diffusion coefficient  $D_{\text{cl}}$ .

$$\mathcal{N} = \frac{\lambda w_{\text{ch}}}{h^f}, \quad (3)$$

i.e. the chaotic phase-space volume of the resonance channel within a localization length divided by the size of a Planck cell. The quasienergies  $\varepsilon \in [0, \hbar\omega]$  with  $\omega = 2\pi/\tau$  thus have an effective mean level spacing  $\Delta\varepsilon = \hbar\omega/\mathcal{N}$ . This defines an effective Heisenberg time  $h/\Delta\varepsilon$ , which equals the break time,

$$t^* = h/\Delta\varepsilon = \mathcal{N}\tau. \quad (4)$$

Combining Eqs. (4) and (3) gives the ratio  $t^*/\lambda$ , which leads from Eq. (1) to Eq. (2), where the  $x$  dependence is

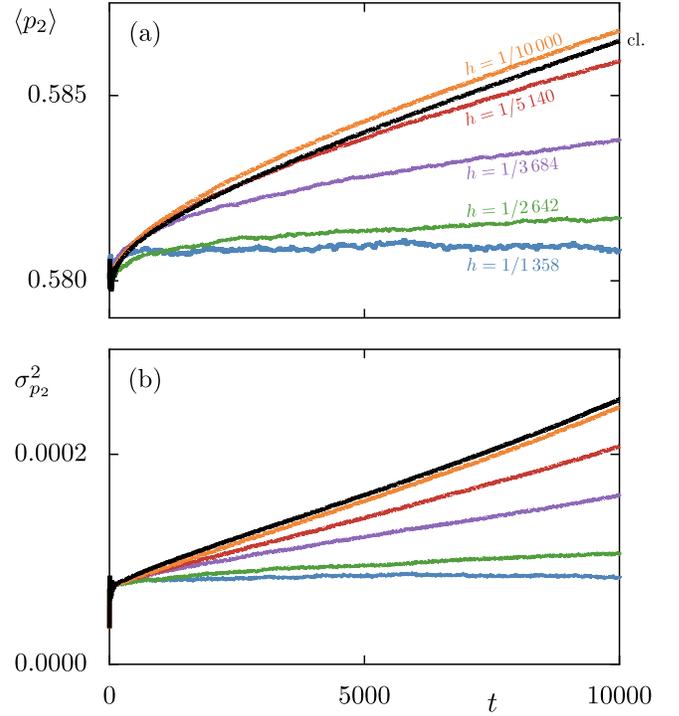


FIG. 4. Time dependence of (a) mean value  $\langle p_2 \rangle$  and (b) variance  $\sigma_{p_2}^2$  along  $p_2$  for a quantum wave packet for various Planck constants  $h$  (light colors), showing a transition from localization to mimicking a classical ensemble of initial conditions (black). Temporal fluctuations are reduced by convolution with a Gaussian of width 1 in  $t$ .

added. Note that the localization length is  $\lambda = 2D_{\text{cl}}w_{\text{ch}}/h^f$ , which follows from Eqs. (4) and (3) and by assuming that the variance of classical diffusion with diffusion coefficient  $D_{\text{cl}}$  at the break time equals the squared localization length,  $2D_{\text{cl}}t^* = \lambda^2$ .

*Hamiltonian with large resonance channel.*—The Arnold web occurs in Hamiltonian systems with at least three degrees of freedom or with at least two degrees of freedom under time-periodic driving. We concentrate on the lowest dimensional time-periodic case ( $f = 2$ ) and a single resonance channel of the Arnold web. In order to study the proposed delocalization transition we need to vary the effective Planck cell over a sufficiently large range. This range is quantum mechanically limited at its lower end due to increasing numerical effort and at its upper end by a Planck cell which is still small enough to fit into the chaotic layer of the resonance channel.

We are able to fulfill these considerations by engineering a Hamiltonian with a large resonance channel having a large chaotic layer. Additionally, the cross-sectional volume of the chaotic layer varies by construction, which induces classically a drift in addition to chaotic diffusion, as demonstrated below. We expect, that in this way the generic features of a resonance channel widening towards the chaotic region or a resonance junction are considered.

The time-periodically kicked Hamiltonian, based on the coupled standard map [43], is given (in dimensionless units) by

$$H = \frac{p_1^2}{2} + \frac{p_2^2}{2} + \frac{k(p_2)}{4\pi^2} \cos(2\pi q_1) \sum_{n \in \mathbb{Z}} \delta(t - n) + \frac{\xi}{4\pi^2} \cos(2\pi[q_1 + q_2]) \sum_{n \in \mathbb{Z}} \delta[t - (n + \epsilon)]. \quad (5)$$

The kicking strength  $k(p_2)$  in the first degree of freedom depends on  $p_2$ . The coupling strength  $\xi$  governs the coupling between the two degrees of freedom. The corresponding kicking term occurs infinitesimally after the first kicking term by choosing the limit  $\epsilon \rightarrow 0^+$  from above. A different ordering would give a qualitatively similar map. Periodic boundary conditions with period 1 are applied to all four coordinates.

*Classical phase space, drift, and diffusion.*—The kicked Hamiltonian (5) leads classically to a 4D symplectic map,

$$q'_1 = q_1 + p_1, \quad (6a)$$

$$q'_2 = q_2 + p_2 + \frac{1}{4\pi^2} \frac{dk(p_2)}{dp_2} \cos(2\pi q'_1), \quad (6b)$$

$$p'_1 = p_1 + \frac{k(p_2)}{2\pi} \sin(2\pi q'_1) + \frac{\xi}{2\pi} \sin(2\pi[q'_1 + q'_2]), \quad (6c)$$

$$p'_2 = p_2 + \frac{\xi}{2\pi} \sin(2\pi[q'_1 + q'_2]). \quad (6d)$$

If the coupling  $\xi$  is set to zero,  $\xi = 0$ , one has a Cartesian product of dynamics in  $(q_1, p_1)$  and integrable rotational dynamics in  $(q_2, p_2)$ . Such a product structure has been used, e.g., in Refs. [38,44,45]. In particular, Eq. (6) corresponds to a stack of 2D standard maps in  $p_2$  direction with kicking strength  $k(p_2)$ . We choose  $k(p_2)$  such that it gives rise to an increasing chaotic layer around the main resonance with a hyperbolic fixed point at  $(q_1, p_1) = (0, 0)$  and an elliptic fixed point at  $(0.5, 0)$ . For nonzero but small coupling,  $\xi = 0.1$ , one has a 4D phase space with the topological features of the product structure. This gives rise to a widening resonance channel along the  $p_2$  coordinate with slow chaotic diffusion due to the weak coupling  $\xi$ . For the local phase-space coordinate  $x$  along the resonance channel appearing in Eq. (2) we use  $x = p_2$ .

Specifically, we choose  $k(p_2)$  as a periodic triangular function approximated by a Fourier expansion ranging from  $k(0.5) = \bar{k} - \Delta k$  to  $k(0) = k(1) = \bar{k} + \Delta k$  with  $\bar{k} = 0.4$  and  $\Delta k = 0.3$  leading to a widening resonance channel,  $k(p_2) = \bar{k} + \Delta k C_2 \sum_{i=0}^n \cos[2\pi(2i+1)p_2]/(2i+1)^2$  with  $n = 2$  and normalization  $C_2 = (225/259)$ . The parameters  $\bar{k}$  and  $\Delta k$  are chosen such that the influence of other resonance channels crossing the main one is

minimized, while the increase of the cross-sectional volume  $w_{\text{ch}}(p_2)$  of the resonance channel governed by chaotic dynamics is as large as possible.

In Fig. 2 we present a 3D phase-space slice [46] at  $q_2 = 0.5$  of the 4D map, Eq. (6), for three ensembles with initial conditions started at different positions  $p_2$  along the resonance channel [near  $(q_1, p_1) = (0, 0)$  in the first degree of freedom and for all  $q_2$ ]. Each ensemble spreads in the first degree of freedom along the chaotic layer of the resonance while staying close to an almost invariant surface  $p_2(q_1, p_1)$ , which is flat in the limit  $\xi \rightarrow 0$ . Furthermore, it slowly diffuses in  $p_2$  direction away from this surface, which is most prominently seen for the ensemble with largest  $p_2$  values. The dynamics in  $q_2$  is purely rotational with momentum  $p_2$ .

In Fig. 3(a) the fast Lyapunov indicator [47–49] in the plane  $q_1 = 0$ ,  $q_2 = 0.5$  visualizes the increasing cross-sectional volume of the chaotic region of the resonance channel, by measuring the chaoticity of trajectories. In Fig. 3(b) this is quantified by the 3D chaotic phase-space volume  $w_{\text{ch}}(p_2)$  of the resonance channel at a given coordinate  $p_2$ . It is measured by iterating trajectories started in the chaotic sea for long times and counting the number of visited phase-space boxes. Over the considered range in  $p_2$  an increase by a factor of more than 2 can be observed. Note that we restrict the analysis to the range  $p_2 \in [0.56, 0.62]$ , for which no large resonance channels cross the main one.

Classical transport along  $p_2$  is characterized by the drift velocity  $v_{\text{cl}}(p_2)$  and the diffusion coefficient  $D_{\text{cl}}(p_2)$ . Both strongly depend on  $p_2$ , see Figs. 3(c) and 3(d). Over the considered range in  $p_2$  the drift velocity  $v_{\text{cl}}(p_2)$  increases by a factor of 5 and the diffusion coefficient  $D_{\text{cl}}(p_2)$  by a factor of 10. They are measured by fitting a linear slope to the increasing mean value and variance in  $p_2$  from ensembles of initial conditions started at each  $p_2$ ; see as an example the black lines in Fig. 4. The time interval used for the linear fit depends on the initial  $p_2$  and is chosen linearly between  $t \in [6000, 10000]$  for  $p_2 = 0.56$  and  $t \in [500, 1000]$  for  $p_2 = 0.62$ . This ensures that the trajectories (i) are spread out in the chaotic region of the almost-invariant surfaces and (ii) are still close to the initial  $p_2$ . We attribute the small fluctuations in Figs. 3(b)–3(d) to the complex geometry of the Arnold web. Note that the origin of the drift along the resonance channel is related to the increasing cross-sectional volume  $w_{\text{ch}}(p_2)$  of the channel and the increasing diffusion coefficient  $D(p_2)$ . It seems possible to quantitatively relate the  $p_2$  dependence of the drift velocity  $v_{\text{cl}}(p_2)$  to  $w_{\text{ch}}(p_2)$  and  $D(p_2)$  in the 4D map using concepts from stochastic processes [50], but this is beyond the scope of this paper.

*Quantum delocalization transition.*—In order to study the influence of the classically observed drift on dynamical localization we study the time evolution of wave packets for the kicked Hamiltonian (5) for varying effective Planck

constants  $h$ , corresponding to Hilbert space dimensions  $1/h^2$  [51–56]. Its time-evolution operator is given by

$$U = e^{-\frac{i}{h} \frac{\xi}{4\pi^2} \cos(2\pi[q_1+q_2])} e^{-\frac{i}{h} \frac{k(p_2)}{4\pi^2} \cos(2\pi q_1)} e^{-\frac{i}{h} (\frac{p_1^2}{2} + \frac{p_2^2}{2})}. \quad (7)$$

As initial wave packets we choose a product of a coherent state centered at  $(q_1, p_1) = (0, 0)$  in the first degree of freedom and of a momentum eigenstate at various  $p_2$  in the second degree of freedom, in agreement with the classical initial ensembles. In Fig. 4 the time dependence of the wave packet’s mean value  $\langle p_2 \rangle$  and its variance  $\sigma_{p_2}^2$  in  $p_2$  direction are shown. A transition from localization to drift and diffusion for decreasing  $h$  can be observed, mimicking the classical behavior for the smallest value of  $h$ .

We determine the quantum drift velocity  $v_{\text{qm}}(p_2)$  and diffusion coefficient  $D_{\text{qm}}(p_2)$  from a linear fit using the same time intervals as for the classical values. The ratios with the classical drift velocity  $v_{\text{cl}}(p_2)$  and diffusion coefficient  $D_{\text{cl}}(p_2)$ , respectively, versus the transition parameter  $\Lambda$  are shown in Fig. 1. We find a smooth transition from localization to drift-induced delocalization with increasing  $\Lambda$  and centered at  $\Lambda = 1$ . The transition is universal as it depends on  $\Lambda$ , Eq. (2), only. In particular, it is independent of the specific system parameters  $v_{\text{cl}}(p_2)$ ,  $D_{\text{cl}}(p_2)$ , and  $w_{\text{ch}}(p_2)$  for various initial  $p_2 \in \{0.56, 0.59, 0.6, 0.62\}$  and various  $h \in [1/500, 1/10000]$ . For the smallest Planck constant  $h = 10^{-4}$  the dimension of the Hilbert space is  $10^8$ , with the time evolution of the kicked Hamiltonian made possible by using 2D fast Fourier transforms [28].

*Discussion and outlook.*—We show that the classical drift along resonance channels destroys dynamical localization if it is strong enough. The proposed transition parameter  $\Lambda$ , Eqs. (1) and (2), leads to a universal description of this drift-induced delocalization transition. An important consequence is that a quantum mechanical wave packet may explore the Arnold web in larger regions than expected. Namely, the accessible region of a resonance channel, described by a one-dimensional local phase-space coordinate  $x$  along the resonance channel, is given by those points where  $\Lambda(x) \gtrsim 1$ . For strong enough drift the extent of this accessible region is larger than the localization length obtained from purely diffusive dynamics. A direct consequence is that also chaotic eigenstates extend further into the Arnold web.

A future task is to control eigenstates and wave packets in few-body and many-body systems by adjusting classical drift or relevant phase-space volume. Interesting examples include many-site Bose-Hubbard systems and quantum computers with hundreds of qubits. Extending the analysis to such high-dimensional systems poses a significant numerical challenge. Still, we expect the drift-induced delocalization transition to follow the same universal properties.

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