Anyon Statistics through Conductance Measurements of Time-Domain Interferometry

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We propose a method to extract the mutual exchange statistics of the anyonic excitations of a general Abelian fractional quantum Hall state, by comparing the tunneling characteristics of a quantum point contact in two different experimental conditions. In the first, the tunneling current between two edges at different chemical potentials is measured. In the second, one of these edges is strongly diluted by an earlier point contact. We describe the case of the dilute beam in terms of a time-domain interferometer between the anyons flowing along the edge and quasiparticle-quasihole excitations created at the tunneling quantum point contact. In both cases, temperature is kept large, such that the measured current is given to linear response. Remarkably, our proposal does not require the measurement of current correlations, and allows us to carefully separate effects of the fractional charge and statistics from effects of intra- and interedge interactions.

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Introduction.—It has been almost four decades since the initial proposal that the elementary quasiparticles of fractional quantum Hall (FQH) systems obey anyonic statistics [1]. Despite the apparent maturity of the field, the pursuit to definitively observe the physical quantities and quantum numbers characterizing anyons [2,3] is constantly being reinvigorated [4–21]. In particular, early 2020 saw two major experimental steps forward: the observation of anyonic braiding in a Fabry-Perot interferometer [22], and demonstration of a so-called "anyon collider" [23,24] using cross-correlation measurements.

Here we show that anyonic statistics can be inferred directly from conductance measurements, without requiring current correlation measurements or explicitly building an interferometer. The configuration we propose to obtain this result consists of a quantum point contact (QPC) between two edges of a general Abelian FQH state at filling factor ν. The edges may be driven off equilibrium by one of three methods: injecting a single quasiparticle into one of the edges; injecting a Poissonian, dilute beam of quasiparticles into one of the edges; and placing a finite bias voltage between the edges.

Our proposed setup, shown in Fig. 1(a), allows a smooth transition between the dilute Poissonian beam and a full beam at finite bias voltage. This is obtained by tuning a second, injection QPC from fully open (a differential conductance, $G_{\mathrm{inj}} \equiv dI_{\mathrm{inj}}/dV$, satisfying $G_{\mathrm{inj}}/\sigma_{xy} \to 0$) to fully closed ($G_{\mathrm{inj}}/\sigma_{xy} \to 1$). We henceforth refer to these as the dilute and full limits, respectively.

We propose sweeping G_{inj} through this range, and measuring the ratio I/I_{inj}, where I is the measured current after the tunneling QPC, and I_{inj} is the injected incident current, as defined in Fig. 1(a). Comparing the values at the dilute and full limits cancels out nonuniversal constants, yielding the relation,

$$\left[\frac{I(T)}{I_{\mathrm{inj}}(T)}\right]_{\mathrm{dilute}} = \frac{\nu e^2}{2\pi e_1^* e_2^*} \sin 2\theta_{12} \left[\frac{I(T)}{I_{\mathrm{inj}}(T)}\right]_{\mathrm{full}} + \frac{G_{\mathrm{direct}}}{G_{\mathrm{inj}}}. \quad (1)$$

Here, $e_{1/2}^*$ is the tunneling or injected quasiparticle charge, θ_{12} is the mutual statistics phase between the injected and tunneling quasiparticles, T is temperature, and G_{direct} is a residual conductance corresponding to direct tunneling [25–27] through both QPCs. A comparison between these two limits is shown schematically in Fig. 1(b).

The mechanism leading to this result is a time-domain interferometer at the tunneling QPC, created by the dilute incident beam. The interference is between two processes, in which a quasiparticle-quasihole excitation occurs at the tunneling QPC either before or after the arrival of an injected quasiparticle (see Fig. 2). A similar physical picture has been shown in Refs. [26,28,29]. We further find that this interference is sensitive to the *mutual statistics* phase between the injected and the tunneling quasiparticles, θ_{12}. We emphasize that these quasiparticles are not necessarily of the same type, although they must be supported by the same FQH liquid.

The key point of our analysis is the identification of the phase differences in the two interfering of two amplitudes, which differ from one another by the orderings of events. These are determined by the quasiparticle charge e^*, which is a fraction of the electron charge for noninteger values of ν [4–6]; the scaling dimension δ, which defines the zero-temperature time correlations of the quasiparticle via the relation $\langle \psi^\dagger(\tau)\psi(0)\rangle \sim \tau^{-2\delta}$ [30–33]; and the exchange

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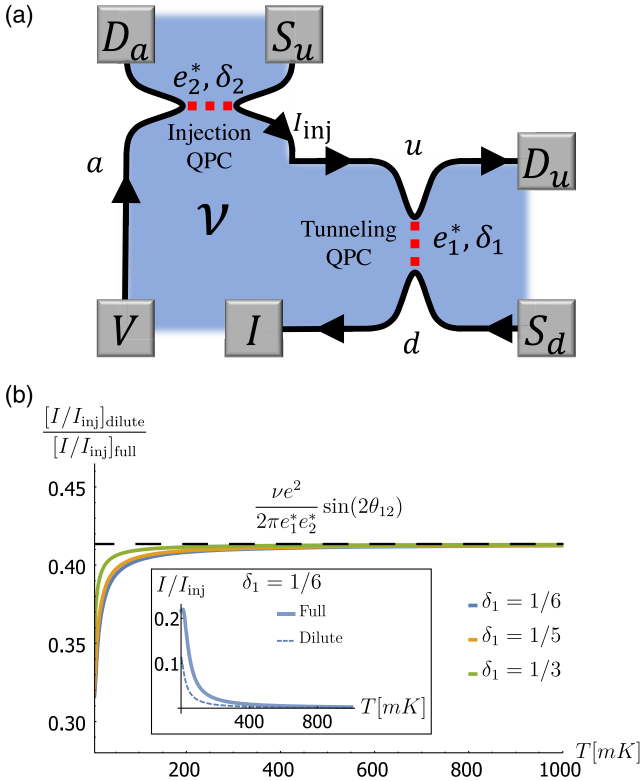

FIG. 1. (a) Two counterpropagating edge modes (u/d) of a fractional quantum Hall droplet at filling factor ν are connected by a quantum point contact, through which quasiparticles of charge e_1^* and scaling dimension δ_1 can tunnel. Current is measured at the lower edge's drain, denoted by I. A current of $I_{\rm inj}$ is injected into the upper edge via a second, injection QPC, e.g., from a third auxiliary edge mode (a). The injection QPC is placed at a bias voltage of V, and allows tunneling of quasiparticles of charge e_2^* and scaling dimension δ_2. All other sources and drains are grounded. (b) The ratio between $I/I_{\rm inj}$ in the dilute case and $I/I_{\rm inj}$ in the full case, as a function of temperature, for $\nu = e_1^*/e = e_2^*/e = 1/3$, and for different scaling dimensions δ_1. For the dilute case, we use $I_{\rm inj} = 10$ pA, and assume $k_B T \ll eV$ for all relevant temperatures, such that the contribution from $G_{\rm direct}$ to Eq. (1) is negligible. In the full case, we use $V = 10$ μV. Both cases use $\xi = 72$ mK, $\tau_c = 10^{-13}$ s. When the dilute case satisfies $\hbar I_{\rm inj}/e \ll k_B T \ll eV \ll \hbar/\tau_c$, and the full case satisfies $\hbar I_{\rm inj}/e = \nu eV/2\pi \ll k_B T \ll \hbar/\tau_c$, the ratio approaches an asymptote that does not depend on scaling dimension, allowing extraction of the mutual statistics θ_{12}. Inset: $I/I_{\rm inj}$ for the dilute and full cases as a function of temperature for $\delta_1 = 1/6$, the canonical value for a Laughlin 1/3 state.

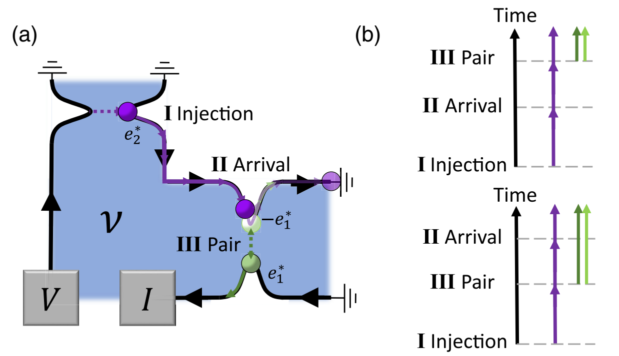

FIG. 2. Time-domain interferometry. (a) I: A quasiparticle is injected from the sourced, left edge, through the injection QPC, and into the upper edge. II: The injected quasiparticle, by virtue of its chiral motion along the edge, arrives at the tunneling QPC. III: A quasiparticle-quasihole pair is created at the tunneling QPC. (b) The two processes by which charge carriers may ultimately arrive at the drain. The injected quasiparticle arrives at the tunneling QPC either before (upper panel) or after (lower panel) the creation quasiparticle-quasihole pair. These two processes interfere, with a relative phase dictated by the mutual statistics phase, $e^{i2\theta_{12}}$.

statistics phase θ, which for anyons take special values beyond the fermionic π and the bosonic 2π [1–3].

We are interested here in isolating the effect of θ. In particular, we would like to separate it from the effect of δ. For noninteracting, fully chiral edges, $2\pi\delta = \theta$; however, in general δ is affected by nonuniversal factors, such as intraedge and interedge interactions, $1/f$ noise or neutral modes [34–39]. This in stark contrast to the charge, exchange statistics phase, or filling factor, which are universal.

We separate the effect of θ from that of δ by tuning the system to a regime where δ only affects observables through a nonuniversal prefactor, which then cancels out in the ratio of currents given in Eq. (1). This is done via a careful ordering of the energy scales in the system, such that $\hbar I_{\rm inj}/e \ll k_B T$ at both the dilute and full limits. This ensures that the current I is given to linear response in $I_{\rm inj}$. As the dominant energy scale in the system is now the temperature, any renormalization that the edges and the scaling dimension undergo will be cut off by the same energy scale. Thus, even if the scaling dimension deviates from predictions [40,41], it will do so in the same manner at both limits, and the cancellation described holds. We present an analytic expression generalizing Eq. (1) outside of this regime [42].

While in the full limit the edge that enters the tunneling QPC is in equilibrium at chemical potential e^*V, at the dilute limit we need the injection current to be Poissonian and rare. Said differently, this limit must satisfy $I_{\rm inj} \ll \sigma_{xy} V$. The beam must remain dilute when arriving at the tunneling QPC. Hence, the distance between the QPCs must be sufficiently small that no equilibration or dephasing occurs along the way. Finally, we assume tuning the injection QPC does not affect the transparency of the tunneling QPC.

Easy extraction of θ_{12} requires $G_{\rm direct}$ to be sub-dominant [see Eq. (1)]. Quantitatively, this is the case if both $k_B T \ll eV$ and $4\delta_1 < 2$ are satisfied. These constraints result from the direct tunneling process being dominated by short timescales. Naive theories describing quasiparticles may satisfy this condition even if the aforementioned nonuniversal effects change the scaling dimension quite significantly. For example, theory gives $\delta = 1/2m$ for Laughlin quasiparticles.

Edge theory.—We now define the system's Hamiltonian and derive the current. As shown by Wen, the edge theory of a general Abelian FQH state can be described by n-boson fields, $\boldsymbol{\phi}(x,t) \equiv (\phi_1, \phi_2, \cdots \phi_n)^T$ [2]. These define the theory in conjunction with a charge vector, $\boldsymbol{q}$, which determines the electric charge carried by each boson field, and the so-called K matrix, which determines the commutation relations between the boson fields,

$$[\phi_i(x), \partial_{x'}\phi_j(x')] = i2\pi(K^{-1})_{ij}\delta(x-x'). \tag{2}$$

The filling factor is given by $\nu = \boldsymbol{q}^T K^{-1}\boldsymbol{q}$, and the charge density by $\rho = -(1/2\pi)\boldsymbol{q}\cdot\partial_x\boldsymbol{\phi}$. In terms of these fields, the Hamiltonian of a single FQH edge mode is given by

$$\mathcal{H}_{\mathrm{edge}} = \frac{1}{4\pi}\sum_{i,j=1}^{n}\int dx \partial_x\phi_i V_{ij}\partial_x\phi_j, \tag{3}$$

where $\hat{V}$ is a positive definite matrix describing the velocities of the modes and intraedge interactions. These edges support quasiparticles of the form $\psi_{\boldsymbol{l}} \sim e^{i\boldsymbol{l}\cdot\boldsymbol{\phi}}$, where $\boldsymbol{l}$ is a vector of integers. The charge of these quasiparticles is then given by $e^*_{\boldsymbol{l}} = \boldsymbol{q}^T K^{-1}\boldsymbol{l}$.

The configuration of Fig. 1(a) involves two edges, u and d, tunnel coupled by a QPC. This is described by two copies of the Hamiltonian $\mathcal{H}_{\mathrm{edge}}$, and a tunneling term, $\mathcal{H}_T$, treated perturbatively. Assuming only one type of quasiparticle, denoted by the vector $\boldsymbol{l}_1$ and carrying charge e^*_1, tunnels between the edges, this is given by

$$\mathcal{H}_T = \xi\left[\hat{A} + \hat{A}^\dagger\right]; \qquad \hat{A}(t) \equiv e^{i\left(\boldsymbol{l}_1\cdot\boldsymbol{\phi}^{(u)}(0,t) - \boldsymbol{l}_1\cdot\boldsymbol{\phi}^{(d)}(0,t)\right)}. \tag{4}$$

Here, ξ is a small tunneling amplitude, assumed to be real, and $\boldsymbol{\phi}^{(u/d)}$ are the bosonic field operators on the upper or lower edge. We project the auxiliary edge a out of the Hamiltonian, as it is only used to "initialize" the state of the edge u.

The current that tunnels from the upper edge to the lower edge is then given by the operator, $\hat{I}_T(t) = i\xi e^*_1\left[\hat{A}^\dagger(t) - \hat{A}(t)\right]$. Since the lower edge is grounded, we henceforth identify $I = \langle\hat{I}_T\rangle$. Expanding to leading order in ξ, the current is given by

$$I(t) = e^*_1\xi^2\int_{-\infty}^{t} dt'\left\langle\left[\hat{A}^\dagger(t), \hat{A}(t')\right] + \left[\hat{A}^\dagger(t'), \hat{A}(t)\right]\right\rangle. \tag{5}$$

Here, $[\cdot,\cdot]$ denotes commutation, and expectation values are calculated with respect to the Hamiltonian in the absence of tunneling.

Deviation from equilibrium.—It is clear from Eq. (5) that one needs to derive correlation functions such as $\langle\hat{A}^\dagger(t)\hat{A}(t')\rangle$. In equilibrium, at temperature T, the system is particle-hole symmetric, and the correlation functions are given by [2,43]

$$\langle\hat{A}^\dagger(t)\hat{A}(t')\rangle_0 = \langle\hat{A}(t)\hat{A}^\dagger(t')\rangle_0 = \left[\frac{\pi T\tau_c}{\sinh(\pi T|t-t'|)}\right]^{4\delta_1} e^{-i2\pi\delta_1\mathrm{sgn}(t-t')}, \tag{6}$$

where δ_1 is the scaling dimension of the quasiparticle $\boldsymbol{l}_1$, and $\tau_c > 0$ is a short time cutoff.

We now consider two nonequibrium cases. In the first we introduce a constant bias voltage $V \equiv V_u - V_d$ between the edges. In the setup of Fig. 1(a), this corresponds to a fully closed injection QPC, i.e., $I_{\mathrm{inj}} = \sigma_{xy}V$. This can be formally absorbed into the boson fields by a simple gauge transformation, which maps $\boldsymbol{\phi}^{(u/d)}(x,t) \mapsto \boldsymbol{\phi}^{(u/d)}(x,t) + K^{-1}\boldsymbol{q}V_{u/d}(t \mp x/v)/\hbar$. The correlation functions accordingly gain a phase factor

$$\langle\hat{A}^\dagger(t)\hat{A}(t')\rangle_{\mathrm{full}} = \langle\hat{A}^\dagger(t)\hat{A}(t')\rangle_0 e^{i\frac{e^*_1 V}{\hbar}(t-t')},$$
$$\langle\hat{A}(t)\hat{A}^\dagger(t')\rangle_{\mathrm{full}} = \langle\hat{A}(t)\hat{A}^\dagger(t')\rangle_0 e^{-i\frac{e^*_1 V}{\hbar}(t-t')}. \tag{7}$$

In the second nonequilibrium driving, we consider injecting a single quasiparticle, denoted by the vector $\boldsymbol{l}_2$, into the upper edge, at the location $x_{\mathrm{inj}} < 0$ and time t_{inj}. This is shown schematically in Fig. 2(a). From the commutation relations (2), applying the quasiparticle creation operator $e^{-i\boldsymbol{l}_2\cdot\boldsymbol{\phi}^{(u)}(x_{\mathrm{inj}},t_{\mathrm{inj}})}$ on the edge creates a soliton in each of the boson fields,

$$\boldsymbol{\phi}^{(u)}(x,t_{\mathrm{inj}}) \mapsto \boldsymbol{\phi}^{(u)}(x,t_{\mathrm{inj}}) - 2\pi K^{-1}\boldsymbol{l}_2\Theta(x - x_{\mathrm{inj}}). \tag{8}$$

We assume here the injection happens instantaneously. This assumption will be relaxed to find the subleading term of Eq. (1).

The fields at general times are obtained using the equations of motion dictated by the Hamiltonian, Eq. (3). If all modes are chiral with the same velocity v, this amounts to replacing $x - x_{\mathrm{inj}} \to x - x_{\mathrm{inj}} - v(t - t_{\mathrm{inj}})$. The soliton thus arrives at the QPC, $x = 0$, at time $t_0 \equiv t_{\mathrm{inj}} - x_{\mathrm{inj}}/v$.

The c-number shift in the bosonic field of Eq. (8) leads to a phase shift in the correlator Eq. (6). We see directly from the definition of the operator $\hat{A}$ in Eq. (4) that

$$\langle\hat{A}^\dagger(t)\hat{A}(t')\rangle_{\mathrm{qp}} = \langle\hat{A}^\dagger(t)\hat{A}(t')\rangle_0 e^{2\pi i\boldsymbol{l}_1 K^{-1}\boldsymbol{l}_2[\Theta(t-t_0)-\Theta(t'-t_0)]},$$
$$\langle\hat{A}(t)\hat{A}^\dagger(t')\rangle_{\mathrm{qp}} = \langle\hat{A}(t)\hat{A}^\dagger(t')\rangle_0 e^{-2\pi i\boldsymbol{l}_1 K^{-1}\boldsymbol{l}_2[\Theta(t-t_0)-\Theta(t'-t_0)]}. \tag{9}$$

The phase we obtain is the standard definition of mutual braiding statistics between two quasiparticles, $\theta_{12} \equiv \pi\boldsymbol{l}_1 K^{-1}\boldsymbol{l}_2$ [2]. Equation (9) shows that the product gains a phase of $e^{2i\theta_{12}\mathrm{sgn}(t-t')}$ if the arrival time t_0 is between the times t' and t, and a trivial phase of 1 otherwise. We emphasize how naturally this result came from the underlying theory: the only assumptions necessary to obtain this are the commutation relations, (2), and the existence of quasiparticles in the edge's excitation spectrum.

This result holds for different boson modes with different velocities if all solitons arrive at the tunneling QPC more or less concurrently, avoiding dephasing. This is the case if $|x_{\mathrm{inj}}|\Delta(v^{-1}) \ll \hbar/T$, where $\Delta(v^{-1})$ is the inverse velocity difference between the fastest and the slowest modes.

Time-domain interferometry.—The appearance of the phase, θ_{12}, can be understood as time-domain interferometry of the two distinct $\pm e_1^*$ quasiparticle-quasihole excitations, before and after the injected e_2^* quasiparticle arrives at the QPC. A similar physical picture has been shown in Refs. [26,28,29].

To show this we consider the configuration of a single injected particle, as described in Fig. 2(a). In this case the nonequilibrium correlation function takes the form,

$$\langle \hat{A}^\dagger(t)\hat{A}(t')\rangle_{\mathrm{qp}} = \langle \psi_{I_2}(t_0)\hat{A}^\dagger(t)\hat{A}(t')\psi_{I_2}^\dagger(t_0)\rangle_0, \quad (10)$$

i.e., the expectation value is calculated with respect to the state resulting from exciting the ground state $|0\rangle$ with a single quasiparticle. Here we omit the position variable from the quasiparticle injection operator $\psi_{I_2}^\dagger(t_0)$, and assume it arrives at the tunneling QPC $x = 0$ at time t_0.

The current in Eq. (5) is then given by integration over multiple terms of the form in Eq. (10). Defining $|t, t_0\rangle_- \equiv \hat{A}(t)\psi_{I_2}^\dagger(t_0)|0\rangle$ and $|t, t_0\rangle_+ \equiv \hat{A}^\dagger(t)\psi_{I_2}^\dagger(t_0)|0\rangle$, Eq. (5) can be rewritten as

$$I \propto -\int_{-\infty}^{t} dt' \sum_{b=\pm} b||t, t_0\rangle_b + |t', t_0\rangle_b|^2. \quad (11)$$

This expression involves two interference terms. The $b = -$ term involves interference between creation of $-e_1^*$ quasiholes on the upper edge at the QPC at times t and t'. The two interfering processes are shown schematically in Fig. 2(b). From Eq. (9), these two processes are distinguished by a non-trivial phase of $e^{i2\theta_{12}}$ if the arrival time t_0 is in between the quasiholes' creation times, $t' < t_0 < t$. Combined with the equilibrium correlation function Eq. (6), this gives an interference term proportional to $\cos(2\theta_{12} - 2\pi\delta)$. Using similar arguments, the $b = +$ term gives an interference term proportional to $\cos(2\theta_{12} + 2\pi\delta)$. The total contribution from the two terms in Eq. (11) is thus proportional to $\sin(2\theta_{12})\sin(2\pi\delta)$. A full derivation of the contributions from all processes, as well as processes in which a $-e_2^*$ quasihole is injected into the upper edge, is given in Appendix A of the Supplemental Material [42].

This interference happens entirely in the time domain, and along only one edge. It is, however, crucial that this edge be part of a two-dimensional bulk. This is important both because the second edge is required to absorb the leftover quasiparticle or quasihole resulting from the pair creation at the QPC, and because the injected quasiparticle must be created within a bulk FQH droplet. Furthermore, the bulk is intimately related to the edge through bulk-edge correspondence [2]. This dictates that the statistical phase contributing to time-domain interference along a single edge, which our setup measures, is the same as the phase obtained from spatial exchange. Indeed, gauge invariance dictates that the same K matrix defines both the quasiparticle statistics on the edge and the topological quantum field theory in the bulk. The latter determines the statistics in the bulk.

It is easy to generalize this to injection of multiple quasiparticles: as long as all injected quasiparticles are mutually independent, each injected quasiparticle contributes a phase of $e^{2i\theta_{12}}$ if and only if the arrival time at the point contact was between t' and t. If this process is Poissonian, with a quasiparticle injection rate of I_{inj}/e_2^*, we obtain for $t > 0$

$$\frac{\langle \hat{A}^\dagger(t)\hat{A}(0)\rangle_{\mathrm{dilute}}}{\langle \hat{A}^\dagger(t)\hat{A}(0)\rangle_0} = \sum_{n=0}^{\infty} \frac{(tI_{\mathrm{inj}}/e_2^*)^n e^{-tI_{\mathrm{inj}}/e_2^*}}{n!} e^{2in\theta_{12}} = e^{-tI_{\mathrm{inj}}/e_2^*(1-e^{2i\theta_{12}})}, \quad (12)$$

cf. Refs. [24,26]. Adding injected quasiparticles to the lower edge and generalizing for $t < 0$ are straightforward using the same arguments.

Currents.—The effect of driving the system out of equilibrium is completely encapsulated in the correlation functions obtained above. These can then be used to derive any observable of interest. We present the results of such a calculation for the charge current at the lower drain, denoted as I in Fig. 1, and how they can be used to obtain the mutual statistics θ_{12}.

We focus on the regime where the temperature is large compared to the injected current, $\hbar I_{\mathrm{inj}}/e \ll k_B T$. For the full limit, this assumption guarantees linear response in the voltage and in the injected current, which in this limit is $I_{\mathrm{inj}} = \sigma_{xy}V$. For the dilute limit, the exponential suppression of the equilibrium correlation function at times larger than $\hbar/T$, guarantees that the exponent in Eq. (12) may be expanded to first order in I_{inj}. Consequently,

$$\frac{\langle \hat{A}^\dagger(t)\hat{A}(t')\rangle_{\mathrm{full/dilute}}}{\langle \hat{A}^\dagger(t)\hat{A}(t')\rangle_0} \approx 1 + i\omega_{\mathrm{f/d}}(t - t'), \quad (13)$$

where the frequencies $\omega_{\mathrm{f/d}}$ are given by

$$\omega_{\mathrm{f}} = \frac{e_1^* V}{\hbar} = \frac{e_1^*}{\hbar}\frac{I_{\mathrm{inj}}}{\sigma_{xy}}; \qquad \omega_{\mathrm{d}} = i\frac{I_{\mathrm{inj}}}{e_2^*}(1 - e^{2i\theta_{12}}). \quad (14)$$

The zeroth order term corresponds to the equilibrium state and does not contribute to the current. The ratio of the two first order contributions is Eq. (1).

Explicit calculation of the resulting current in Eq. (5), given in Appendix A [42], finds that

$$I_{\text{full/dilute}} = 2\pi e_1^*(\xi\tau_c)^2(2\pi T\tau_c)^{4\delta_1-2}\mathcal{B}(2\delta_1, 2\delta_1)\text{Re}[\omega_{\text{f/d}}], \tag{15}$$

where $\mathcal{B}(x, y)$ is the Euler beta function. Thus, by focusing on the ratio between the full and dilute beams, all dependence on δ_1, T and ξ drops out. Examining the ratio I/I_{inj}, we thus obtain Eq. (1).

For general temperatures, we are no longer in the linear response regime, and we obtain the typical power laws characterizing tunneling in Luttinger liquids [2,35,44,45]. Comparing measurements of the full and dilute limits can still give a quantity related to the mutual statistics θ_{12}, but will explicitly depend on the value of δ_1. We present general expressions for the current in this case in Appendix A [42].

For a fermionic $\theta_{12} = \pi$, Eq. (15) gives no current at all for a dilute electron beam. However, Landauer-Buttiker-Imry scattering theory [46] tells us the current is given by the product of the transparencies of the two QPCs along the electron's path, regardless of whether they are close to full transmission or full reflection. This requires accounting for the direct tunneling term in Eq. (1), which now becomes the leading contribution.

We do this by accounting for the finite width of the soliton. This leads to the expected result of $I_{\text{dilute}} = 4\pi^2\tau_c^2\xi^2 I_{\text{inj}}$. The intuition behind this solution is that tunneling without time-domain interferometry, dubbed the direct tunneling process in [25,26], is dominated by short times. Performing these calculations explicitly in Appendix B [42], we show that the ratio between the first term in Eq. (1) and G_{direct} is $\propto (T\tau_s)^{4\delta_1-2}$, where τ_s is the soliton width. It has been shown [25,26] that $\tau_s^{-1} \propto \max\{eV, k_B T\}$; as such, to ensure G_{direct} is subdominant, the dilute limit must be measured when $k_B T \ll eV$ and $4\delta_1 < 2$.

Several contemporary experimental setups use the equivalent of noninteracting fermionic formulas to reasonable success [47], corresponding to the limiting value of $2\delta_1 = 1$. In this case, the second term of Eq. (1) is a numerical coefficient of order one, which may depend solely on e^*, δ_1, and θ_{12}. For noninteracting fermions, this coefficient is easily found by comparing to known Landauer-Buttiker-Imry scattering theory [46], but it is straightforward to generalize. We discuss this coefficient further in Appendix B [42].

Discussion.—Both the exchange statistics θ_{11} of the tunneling quasiparticle, and θ_{22} of the injection quasiparticle, do not appear in our derivation. Rather, it is the two particles' *mutual statistics*, θ_{12} that affect the modified correlation functions, and hence, the physical observables. Likewise, only δ_1 and e_1^* directly effect observables, although properties of the injected quasiparticles may implicitly enter through the injection rate.

Exchange statistics for a single quasiparticle type are only obtained if the injected and tunneling quasiparticles are identical, $\boldsymbol{l}_1 = \boldsymbol{l}_2$. This is indeed the case in the experiment of Ref. [23], where all quasiparticles are Laughlin $e^* = e/3$ anyons, and subsequent recreations for the $\nu = 1/3$ and $\nu = 2/5$ cases [27,48,49]. Another recent experiment employing a similar setup, where the injected quasiparticle was a $e/3$ anyon and the tunneling quasiparticle was an electron, observed Andreev-like reflection [50]. This is consistent with a mutual statistics phase of $\theta_{12} = \pi$, for which Eq. (1) gives no time-domain interferometry signal.

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