Quantum and Classical Spin-Network Algorithms for q-Deformed Kogut-Susskind Gauge Theories

Torsten V. Zache,* Daniel González-Cuadra, and Peter Zoller
Institute for Theoretical Physics, University of Innsbruck, 6020 Innsbruck, Austria
and Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, 6020 Innsbruck, Austria

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Treating the infinite-dimensional Hilbert space of non-Abelian gauge theories is an outstanding challenge for classical and quantum simulations. Here, we employ *q-deformed Kogut-Susskind lattice gauge theories*, obtained by deforming the defining symmetry algebra to a quantum group. In contrast to other formulations, this approach simultaneously provides a controlled regularization of the infinite-dimensional local Hilbert space while preserving essential symmetry-related properties. This enables the development of both quantum as well as quantum-inspired classical spin-network algorithms for q-deformed gauge theories. To be explicit, we focus on $\mathrm{SU}(2)_k$ gauge theories with $k \in \mathbb{N}$ that are controlled by the deformation parameter $q = e^{2\pi i/(k+2)}$, a root of unity, and converge to the standard SU(2) Kogut-Susskind model as $k \to \infty$. In particular, we demonstrate that this formulation is well suited for efficient tensor network representations by variational ground-state simulations in 2D, providing first evidence that the continuum limit can be reached with $k = \mathcal{O}(10)$. Finally, we develop a scalable quantum algorithm for Trotterized real-time evolution by analytically diagonalizing the $\mathrm{SU}(2)_k$ plaquette interactions. Our work gives a new perspective for the application of tensor network methods to high-energy physics and paves the way for quantum simulations of non-Abelian gauge theories far from equilibrium where no other methods are currently available.

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Introduction.—Lattice gauge theories (LGTs) constitute the foundation of our fundamental understanding of nature, as formulated in the standard model of particle physics [1] and the spin foam approach to quantum gravity [2]. LGTs also find applications for topologically ordered phases in condensed matter physics [3] and quantum information processing [4]. The lattice formulation [5–7], discretizing space-time while preserving the relevant symmetries of the theory, allowed to put gauge theories on computers, eventually leading to remarkable predictions in QCD [8]. These well-established methods are, however, hindered by numerical sign problems [9] that arise, e.g., for real-time dynamics or in the presence of fermionic matter.

In recent years, quantum-inspired classical methods, such as tensor networks that target physically relevant low-entangled states [10], have emerged as promising alternatives to simulate LGTs without sign problems [11–13]. On the other hand, quantum computers and simulators can more efficiently tackle highly entangled regimes [14–18]; see Refs. [19–31] for experimental realizations of LGTs. While the simulation of non-Abelian LGTs is arguably one of the most promising targets for a potential quantum advantage [32], treating the infinite-dimensional Hilbert space of non-Abelian theories remains an outstanding theoretical challenge [17,33–44] and previous approaches have suffered from fundamental drawbacks: in particular, (i) finite subgroup truncations [35,36,45–50] ultimately lead to uncontrolled errors because any non-Abelian Lie group has a largest finite subgroup; (ii) quantum link models [17,41,51–53] give up unitarity of the plaquette operator, rendering known efficient decompositions inapplicable; and (iii) hard cutoffs in the "representation" basis [38,54–60] typically require more sophisticated quantum algorithms as subroutines leading to hardware requirements beyond the realm of current "noisy intermediate-scale quantum" devices [61]. For a recent comparison of different Hamiltonian formulations of LGTs, we refer to the literature [62–65].

In this Letter, we propose to overcome these problems by employing another LGT formulation [3,43,66–68], which is tailored for quantum algorithms but also serves as a natural starting point for quantum-inspired classical methods. In addition to the spatial lattice regularization underlying the Kogut-Susskind (KS) formulation [69], we regularize the infinite-dimensional Hilbert space resulting from non-Abelian Lie groups by replacing the corresponding Lie algebra with a quantum group [70–73] with deformation parameter q, a root of unity [74]. In a basis of gauge-invariant spin network (SN) states, we thus define a truncated model, which we call *q-deformed Kogut-Susskind (qKS) LGT*, and argue that it preserves essential symmetry-related properties, while the KS theory is

recovered by tuning a single control parameter $k \in \mathbb{N}$. While closely related q-deformed gauge theories have been studied in the past (see, e.g., [3,43,66–68] and references therein), here we point out their relevance for both classical and quantum simulations.

In particular, we study the case of $SU(2)_k$ LGT in two spatial dimensions in detail and first show the convergence of the $k \to \infty$ limit with exact results for a single plaquette. We then illustrate the advantages of this formulation by developing both classical and quantum spin-network algorithms for q-deformed gauge theories (SNAQs). In the classical case, we perform tensor network simulations based on an infinite projected entangled pair states (iPEPS) [10] ansatz, indicating quantitative agreement with continuum results for $k = \mathcal{O}(10)$. Concerning quantum simulations, we design a scalable digital quantum algorithm for real-time evolution using an analytical Trotter decomposition, enabled by an exact diagonalization of the plaquette operator using local basis transformations on a SN register. Our resource estimates for this algorithm demonstrate the potential of *qudit* quantum computers [75–79] for high-energy physics [49,80–85].

Model and truncation.—To be specific, we consider SU(2) LGT in two spatial dimensions, but the approach applies to SU(N) LGTs in arbitrary dimensions. In preparation for the q-deformed theory, we start with the KS Hamiltonian [69,86]

$$H_{\mathrm{KS}} = \frac{g^2}{2a}\sum_{\ell} E_{\ell}^2 - \frac{1}{2ag^2}\sum_{\square}(\mathcal{U}_{\square} + \mathcal{U}_{\square}^{\dagger}), \qquad (1)$$

where g^2 is the dimensionless bare coupling constant and a denotes the spatial lattice spacing. Here, E_{ℓ}^2 is the electric energy operator acting on every link ℓ of a 2D square lattice, while $\mathcal{U}_{\square}$ acts on four links forming an elementary plaquette [see Fig. 1(a)]. In the Hamiltonian formulation, gauge invariance is expressed by Gauss' law operators G_+, associated to every vertex $+$ of the lattice, such that $[H_{\mathrm{KS}}, G_+] = 0 \ \forall \ +$, and the gauge-invariant Hilbert space is spanned by all states $|\psi\rangle$ that fulfill Gauss' law $G_+|\psi\rangle = 0$ (in the absence of static charges).

Before defining the qKS theory in a gauge-invariant basis formed by spin-network (SN) states, we recall this construction for the standard KS model [86]. These states are obtained by solving Gauss' law in terms of spin singlets at every four-vertex. To keep track of inequivalent singlets, it is convenient to work on a tri-valent lattice obtained by "point splitting" every four-vertex into two three-vertices as indicated in Fig. 1(b), a construction that is also heavily used in the loop-string-hadron (LSH) formulation [40,87]. The fundamental nonuniqueness of this procedure implies the existence of local basis changes [see Fig. 1(d)], which will become essential for SNAQs. A general SU(2) SN state has the form $|\boldsymbol{j}\rangle = \otimes'_{\ell} |j_{\ell}\rangle$ with one SU(2) representation label $j_{\ell} \in \{0, \frac{1}{2}, 1, \ldots\}$ assigned to every link of the resulting lattice. The rules of angular momentum addition lead to an additional "triangle" constraint $|j_1 - j_2| \le j_3 \le j_1 + j_2$, together with $j_1 + j_2 + j_3 \in \mathbb{N}$, which has to be satisfied by all triples of spins (j_1, j_2, j_3) that meet at a vertex, which we indicate by the primed product. One can show that the collection of all such SN states forms an orthonormal basis of the gauge-invariant Hilbert space (see Ref. [86] and Supplemental Material (SM) [88]).

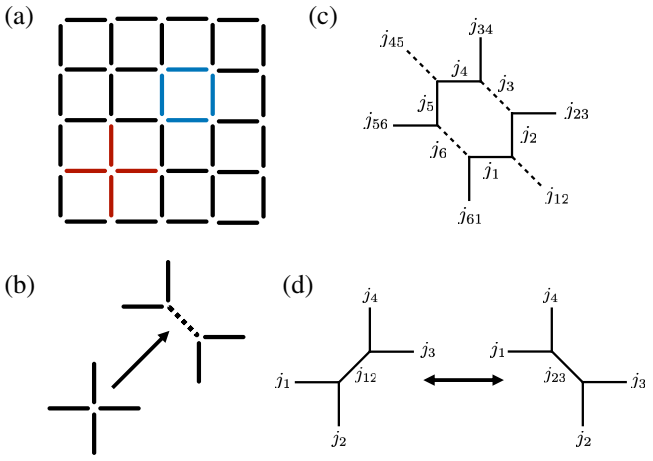

FIG. 1. (a) In $2+1$D Kogut-Susskind LGT [Eq. (1)] gauge fields live on links of a spatial 2D square lattice, which contains elementary plaquettes (blue) and four-vertices (red). (b) For the gauge-invariant SN basis (see main text) every four-vertex is split into two three-vertices, resulting in an additional link (dashed). (c) The elementary plaquette operator on the point-split lattice acts on elementary hexagons according to Eq. (2). (d) A key feature preserved by the proposed q-deformed regularization are local unitary transformations ("F moves") that effect a basis transformation between inequivalent ways of point splitting [see Eq. (6)].

We regularize the KS model by deforming the corresponding Lie *algebra*. In the present example, we proceed by replacing the data arising from the representation theory of SU(2) with analogous expressions for the quantum group $SU(2)_k$ (see, e.g., [70] and the SM). Here, it is crucial to work with deformation parameter $q = e^{2\pi i/(k+2)}$, a root of unity that leads to a closed fusion ring with irreducible representations of finite dimension [95,96]. This allows us to define generalized SN states with $j_{\ell} \in \{0, \frac{1}{2}, 1, \ldots, (k/2)\}$, truncating the local Hilbert space dimension that physically corresponds to a maximum electric flux $j_{\max} = (k/2)$ with $k \in \mathbb{N}$. Additionally, the triangle constraint for triples (j_1, j_2, j_3) is replaced by the $SU(2)_k$ fusion rule: $j_1 + j_2 \ge j_3$ and $j_1 + j_2 + j_3 \le k$. To remain close to the original KS model, we define the electric energy operator E_{ℓ}^2 analogously and only truncate it to admissible states. That is, E_{ℓ}^2 is diagonal and acts only on the links ℓ that are also present in the original square lattice (additional links introduced by point splitting do not carry electric energy), where $E_{\ell}^2|j_{\ell}\rangle = \mathcal{E}(j_{\ell})|j_{\ell}\rangle$ with $\mathcal{E}(j) = j(j+1)$.

To complete the construction, recall that in the SN basis, the plaquette operator acts nontrivially on the six inner links of a plaquette, depending on the six outer links [see Fig. 1(c)] [97]. The nonvanishing matrix elements are conveniently expressed using F matrices (see SM for an explicit formula via Wigner's $6j$ symbols) as

$$\langle \boldsymbol{j}'|\mathcal{U}_\square|\boldsymbol{j}\rangle = F^{j_{12}j_1j_2}_{\frac{1}{2}j_2'j_1'} F^{j_{23}j_2j_3}_{\frac{1}{2}j_3'j_2'} F^{j_{34}j_3j_4}_{\frac{1}{2}j_4'j_3'} \times F^{j_{45}j_4j_5}_{\frac{1}{2}j_5'j_4'} F^{j_{56}j_5j_6}_{\frac{1}{2}j_6'j_5'} F^{j_{61}j_6j_1}_{\frac{1}{2}j_1'j_6'}, \tag{2}$$

where a trivial action for links ℓ not touching the plaquette $\square$ is implicit and the index "1/2" comes from the fact that $\mathcal{U}_\square$ changes the flux by $j = \frac{1}{2}$. For the q-deformed theory, we define the action of plaquette operators in the $\mathrm{SU}(2)_k$ SN basis by Eq. (2) with F matrices replaced by their counterparts for $\mathrm{SU}(2)_k$ (see Ref. [70] and the SM).

The resulting theory, which we call the "q-deformed" Kogut-Susskind model ($H_{q\mathrm{KS}}$), can be interpreted as a perturbation of the stringnet models introduced in [3]. A related q-deformed truncation of 3D SU(2) lattice Yang-Mills theory was studied with tensor networks in [43]. While the present discussion builds on gauge-covariant bases in the Hamiltonian formulation introduced in [86], note that similar constructions were used for the LSH formulation [40]. Gauge-invariant bases were also constructed for SU(2) quantum link models, enabling efficient quantum Monte Carlo simulations through an equivalent dual model [98] (see also [99] for a dual formulation of SU(2) lattice Yang-Mills theory).

As we demonstrate in the rest of the Letter, the qKS formulation is very promising for simulations with quantum technologies. In particular, it is constructed to recover the KS description of LGTs as $k \to \infty$ in contrast to, e.g., finite subgroup truncations. In that sense, the qKS model is closely related to a hard cutoff truncation with irreducible representation $j \le (k/2)$ (see SM for a detailed comparison) and mainly differs in the matrix elements of plaquette operators. This difference of the q-deformed theory preserves the structure of local unitary transformations of the SN basis in terms of so-called F moves [see Fig. 1(d)], which enables a relatively simple decomposition of plaquette operators in contrast to, e.g., quantum link models. This feature also enables the construction of efficient quantum algorithms.

Exact results for a single plaquette.—We next illustrate the convergence of the proposed truncation. Consider a single plaquette with open boundary conditions and fixed zero electric flux at the boundaries as indicated by the SN diagram in the inset of Fig. 2. Then the gauge-invariant Hilbert space becomes $(k+1)$-dimensional, spanned by SN states $|j\rangle$ with a single label j. The Hamiltonian (rescaling $H'_{q\mathrm{KS}} = (2a/g^2) \times H_{q\mathrm{KS}}$) reads

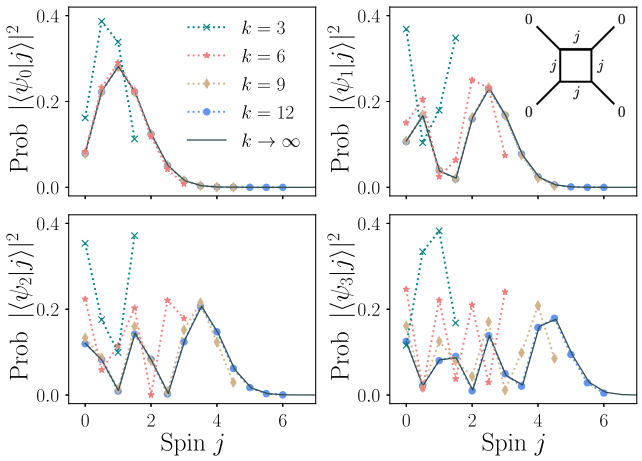

FIG. 2. The probability distributions $|\langle\psi|j\rangle|^2$ for ground state (top left), first (top right), second (bottom left), and third (bottom right) excited states for $g^2 = 0.1$ of a single plaquette converge to the $k \to \infty$ limit quickly once the cutoff $j_{\max} = (k/2)$ is large enough to support the bulk of the wave function. The inset in the top right panel illustrates the SN basis for a single plaquette with open boundary conditions and zero incoming flux.

$$H'_{q\mathrm{KS}} = \sum_{j=0}^{k/2} 4\mathcal{E}(j)|j\rangle\langle j| - \frac{2}{g^4}\sum_{j=0}^{(k-1)/2}\left(\left|j+\frac{1}{2}\right\rangle\langle j| + \mathrm{H.c.}\right), \tag{3}$$

where the effect of working with the generalized $\mathrm{SU}(2)_k$ theory is particularly transparent as it just imposes a cutoff $j_{\max} = (k/2)$ on the largest flux allowed on the plaquette. The qKS formulation thus reduces to a hard cutoff truncation in this case.

In Fig. 2, we plot the probability distributions $|\langle\psi|j\rangle|^2$ of the ground state $|\psi_0\rangle$ and the first three excited states $|\psi_{1/2/3}\rangle$ for fixed coupling. These results, obtained by exact diagonalization, are compared to analytical results in terms of Mathieu functions of the limit $k \to \infty$ (see SM). We observe that the wave functions converge rapidly for sufficiently large values of k, where the threshold is dictated by the total energy and shifts to larger values for higher excited states. Similarly, larger k will be needed to reach small g^2 required for scaling toward the continuum limit.

Classical SNAQ for ground states.—The continuum field theory limit is approached by increasing the lattice size and sending $g^2 \to 0$. In the following, we provide first estimates of how to scale k to reach the continuum limit.

We make a variational ansatz $|\boldsymbol{\psi}\rangle$ for the ground state of an infinite system

$$|\boldsymbol{\psi}\rangle = \prod_\square \left[\sum_{j=0}^{k/2} \psi_j \mathcal{U}^{(j)}_\square\right]|\mathbf{0}\rangle, \tag{4}$$

generalizing the one used in Refs. [100,101]. Here, $|\mathbf{0}\rangle$ is the SN vacuum state, $\mathcal{U}^{(j)}_\square$ the plaquette operator that creates

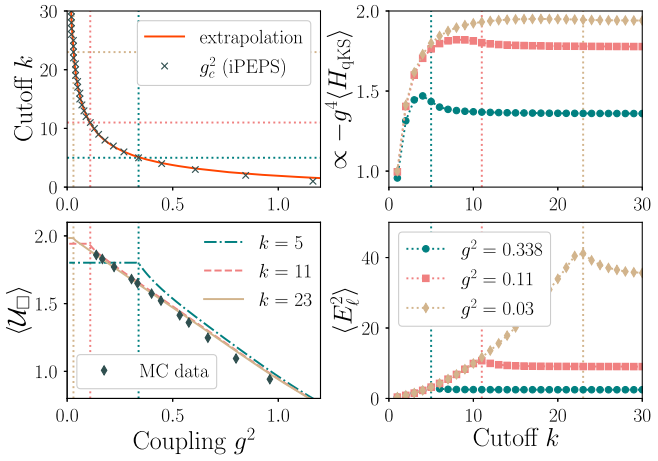

FIG. 3. The top left panel shows the critical coupling g_c^2 as a function of k, extracted from the nonanalytic behavior of $\langle \mathcal{U}_\square \rangle = \langle \boldsymbol{\psi}_{\mathrm{opt}} | \mathcal{U}_\square | \boldsymbol{\psi}_{\mathrm{opt}} \rangle$ (lower left panel) in the optimized iPEPS $|\boldsymbol{\psi}_{\mathrm{opt}}\rangle$. Both the total energy (top right) and the electric energy (bottom right) converge rapidly with increasing k, once the threshold k_c is surpassed. The dotted vertical and horizontal lines indicate the relation between the values of k_c and g_c^2. We compare our iPEPS results for $\langle \mathcal{U}_\square \rangle$ with MC data taken from Table VIII of Ref. [106].

a j flux loop on the plaquette $\square$, i.e., replacing $1/2$ by j in Eq. (2). The ψ_j are variational parameters, normalized as $\sum_{j=0}^{k/2} |\psi_j|^2 = 1$.

There are several reasons for this ansatz (see SM for details): first, it can exactly represent ground states in the limits $g^2 = 0$ and $g^2 \to \infty$. Second, we can evaluate the expectation value of $H_{q\mathrm{KS}}$ analytically and find

$$\langle \boldsymbol{\psi} | H'_{q\mathrm{KS}} | \boldsymbol{\psi} \rangle \propto \sum_{j_1 j_2 j_3} |\psi_{j_1}|^2 |\psi_{j_2}|^2 \frac{j_3(j_3+1)d_{j_3}}{d_{j_1} d_{j_2}} \delta_{j_1 j_2 j_3} - \frac{1}{g^4} \sum_{j_1 j_2} \psi_{j_1}^* \psi_{j_2} \delta_{j_1 j_2 \frac{1}{2}}. \tag{5}$$

Here, $\delta_{j_1 j_2 j_3}$ abbreviates the fusion constraint that $(j_1 j_2 j_3)$ forms an admissible vertex and d_j is the quantum dimension of j. We emphasize that even though the ansatz has a "mean-field-like" character, it in general represents a highly entangled state. Technically, it can be interpreted as an iPEPS (see also [54,101–104]). We expect that generalizations of this tensor network ansatz will be useful for future investigations with classical high-performance computing and with quantum hardware, or hybrid variational approaches.

We find an approximation of the ground state as a function of g^2 for several k by numerically minimizing the average energy [Eq. (5)]. Our results are summarized in Fig. 3. For large g^2, the system is in a confined phase as expected for a strong electric field energy, which is also the phase expected for the continuum theory [105]. For finite values of k, however, we observe indications of a phase transition for small g^2. For $k = 1$, this phase is expected to be topologically ordered, i.e., deconfined, with $\mathbb{Z}_2$ (Toric code) topological order [3] and we argue in the SM that for general k the exact $g^2 = 0$ ground state has $\mathrm{SU}(2)_k$ topological order. Note that the undesired phases (from a high-energy physics point of view) shrink toward $g^2 \to 0$ as $k \to \infty$.

As illustrated in Fig. 3, we find fast convergence of local observables with increasing k once the system is in the anticipated "correct" phase. This motivates to consider the location $g_c^2 = g_c^2(k)$ of the transition as an estimate for the value $k_c = k_c(g^2)$ when the model significantly deviates from the desired continuum behavior. For a given coupling g^2, we expect to converge to the continuum limit rapidly for $k \gtrsim k_c(g^2)$. Our findings are consistent with a simple power-law behavior $g_c^2 = \left[g_0/(k + k_0)\right]^2$ with $g_0 \approx 4.4$ and $k_0 \approx 2.5$, which agrees with the expectation that $g_c^2 \to 0$ as $k \to \infty$. This suggests that a moderately small coupling like $g^2 = 0.1$ requires $k = g_0/g - k_0 \sim \mathcal{O}(10)$, which lies within reach of trapped-ion qudit computers [78] by encoding a single link into a single qudit (see, e.g., [75] for a general review of qudit quantum computing, [77–79] for experimental realizations, and [49,80–85] for applications of qudits in the context of high-energy physics).

In practice, it is sufficient to decrease g^2 until the *scaling regime* is reached, where continuum physics can be reliably extracted. For the $2+1$D SU(2) KS model, we compare our simulations to Euclidean Monte Carlo (MC) results for the plaquette expectation value [106]. We obtain quantitative agreement with the MC data in the regime $k \gtrsim 15$ and $0.1 \lesssim g^2 \lesssim 0.5$, indicating that our tensor network ansatz—despite its simplicity—captures the essential degrees of freedom correctly.

Quantum SNAQ for real-time evolution.—To illustrate the usefulness of our proposal for quantum simulation, we now present a quantum SNAQ that provides an exact Trotter decomposition of the time-evolution operator of the q-deformed theory. The algorithm is formulated on a *SN register*, where we associate one degree of freedom $|j_\ell\rangle$ to every link ℓ of the hexagonal graph obtained from point splitting the original lattice. We refer to $|j_\ell\rangle$ as a local qudit, but decomposition into qubits is of course possible. Note that this computational basis is overcomplete because it contains states violating the fusion constraints. We keep this redundancy because it simplifies gate parallelization within SNAQ, making the approach scalable to large system sizes. Furthermore, since the constraints imposed by the fusion rules are diagonal in the computational basis, configurations that do not correspond to valid SN states can be identified easily.

The core elements of this SNAQ are local basis changes (F moves), which allow diagonalizing the plaquette

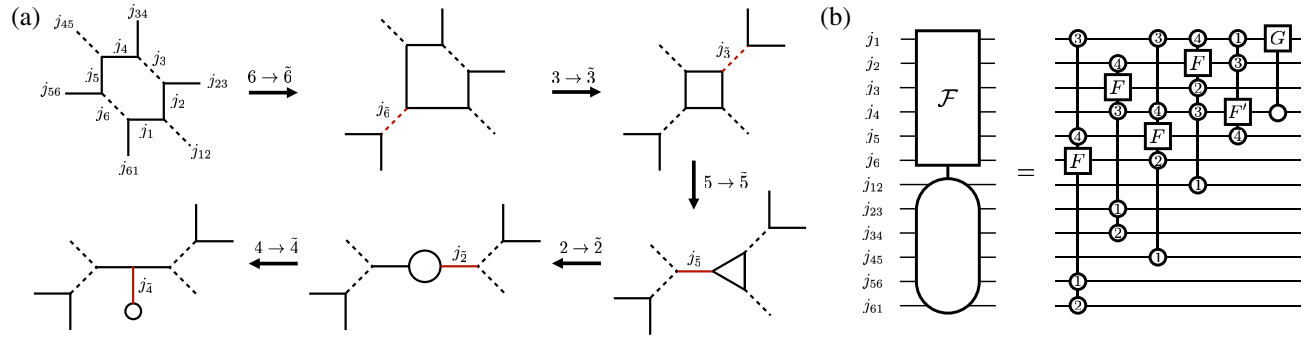

FIG. 4. (a) Sequence of five F moves that partially diagonalizes the plaquette term for a hexagon-shaped SN. Dashed lines indicate auxiliary links that arise from virtual point splitting of four-vertices on a 2D square lattice. Links affected by a single F move are highlighted in red in the resulting SN diagram. For example, the first F move denoted as $6 \to \tilde{6}$ involves the links j_{61}, j_1, j_5, j_{56} and changes j_6 to $j_{\tilde{6}}$. (b) Quantum circuit decomposition derived from the local unitary transformation illustrated in (a).

operator. On the SN register, an F move corresponds to a multiply controlled unitary operator that changes the state of one target qudit, depending on the state of four control qudits [see Fig. 1(d)] as

$$F|j_1 j_2 j_3 j_4 j\rangle = |j_1 j_2 j_3 j_4 j'\rangle, \tag{6}$$

where F is defined by the matrix elements $\left(F^{j_1 j_2}_{j_3 j_4}\right)_{j,j'} = F^{j_1 j_2 j'}_{j_3 j_4 j}$. This five-qudit operator F induces other controlled unitaries with fewer controls. Explicitly, we will need a four-qudit operator F' defined through the matrix elements $\left(F'^{j_1}_{j_3 j_4}\right)_{j,j'} = F^{j_1 j_1 j'}_{j_3 j_4 j}$, identifying $j_1 = j_2$. Finally, we introduce a controlled two-qudit operator G, which diagonalizes the matrix $(F''_J)_{j'j} = F^{Jjj}_{\frac{1}{2}j'j'}$, whose eigenvalues we denote by $\omega^{(J)}_j$.

We arrive at a key observation: there exists a sequence of F moves, shown in Fig. 4(a), that partially diagonalizes the plaquette operator on an elementary hexagon. Intuitively, the properties of F matrices allow to shrink the loop down to $\widetilde{F}^{j_4 j_1 j_1}_{\frac{1}{2}j'_1 j'_1}$ (see SM for details). To the best of our knowledge, this property was first observed in [86] for the original KS theory and later translated to quantum circuits for stringnet models [107]. Our proposal to employ a q-deformed regularization is tailored to preserve this property. As a direct consequence, we obtain the controlled unitary quantum circuit $\mathcal{F}$ shown in Fig. 4(b). The operator $\mathcal{F}$ acts on inner qudits of a hexagon $j_1, \ldots, j_6$ and takes the outer qudits $j_{12}, \ldots, j_{61}$ as controls. This decomposition enables an analytic control over the plaquette operator—made possible by the unitarity of F moves, a property that is lost in other formulations—which we expect will be beneficial in many quantum algorithms for LGTs. In the SM we provide explicit decompositions of the involved unitaries into controlled two-qudit gates, demonstrating a simple and transparent implementation on a *qudit* quantum computer [49,75–79].

An immediate application is a SNAQ using an analytical Trotter decomposition of the evolution operator $U_{q\mathrm{KS}}(\tau) = e^{-i\tau H_{q\mathrm{KS}}}$. Explicitly, we write a Trotter step of a single plaquette term as

$$e^{i\tau\frac{2}{ag^2}\mathcal{U}_\square} = \mathcal{F}\Omega(\tau)\mathcal{F}^\dagger, \qquad \Omega(\tau)|j_1 j_4\rangle = e^{i\tau\frac{2}{ag^2}\omega^{(j_4)}_{j_1}}|j_1 j_4\rangle, \tag{7}$$

where $\Omega(\tau)$ denotes a two-qudit phase gate. For a 2D square lattice this Trotter step can be applied in parallel on half of all plaquettes, yielding an exact realization of the magnetic part $U_B(\tau) = e^{+i\tau(2/ag^2)\sum_\square \mathcal{U}_\square} = \prod_\square e^{+i\tau(2/ag^2)\mathcal{U}_\square}$. The electric part $U_E(\tau) = e^{-i\tau(g^2/2a)\sum_\ell E^2_\ell} = \prod_\ell e^{-i\tau(g^2/2a)E^2_\ell}$ can be parallelized in terms of single-qudit phase gates $e^{-i\tau(g^2/2a)E^2_\ell}|j_\ell\rangle = e^{-i\tau(g^2/2a)j_\ell(j_\ell+1)}|j_\ell\rangle$ on physical links. $U_{q\mathrm{KS}}$ can then be approximated using U_B and U_E as usual.

For example, we analyze the resources of a second-order algorithm $U_{q\mathrm{KS}}(\tau) = U_E(\tau/2) \times U_B(\tau) \times U_E(\tau/2) + \mathcal{O}(\tau^2)$, assuming that every link is encoded into a single qudit of size $k+1$. Including parallelizations, a single Trotter step has a circuit depth determined by 2 electric phase gates (E^2_ℓ), 2 magnetic phase gates (Ω), 4 applications of G and F', and 12 F gates. Quantifying the circuit complexity C by the number of controlled two-qudit unitaries using the decompositions shown in SM yields a polynomial scaling of $C \leq 4 + 28(k+1)^3 + 108(k+1)^4 \sim \mathcal{O}(k^4)$. We note that the choice of qudit gate set can affect the scaling, leading to, e.g., $\mathcal{O}(k^5)$ controlled-increment (CINC) gate entangling gates (see SM). Using the properties of the F matrices, we expect that the gate count can be drastically improved, and we leave further optimizations for future work.

Outlook.—Our work sets the stage for several follow-up investigations. First, an extension to general SU(N), in particular $N = 3$, LGTs is desirable. In this case, a technical obstacle are multiplicities in the generalized

Clebsch-Gordon series, which could be tackled using a graphical calculus [108,109] adapted to the q-deformed case. Second, as briefly outlined in the SM, it appears straightforward to incorporate matter, fermionic or Higgs fields, into our approach, which will add matter-specific gates to SNAQ [82,110]. Third, the close similarities to the spin-foam approach to quantum gravity [2,111] suggest to explore related classical and quantum simulations of gravity [112,113]. From a condensed matter perspective, q-deformed KS LGTs deserve further study in their own right as interesting topologically ordered phases [3] and critical phenomena [114] can be expected, and we refer to [107,115,116] for related methods to simulate anyons on a quantum computer.

Classically, we expect that gauge-invariant tensor networks [12,54,101,111,117,118] will play a crucial role in simulations of LGTs. For the theories studied in this work, extensions of Eq. (4) to inhomogeneous or time-dependent scenarios could be useful to study the dynamics of (de)confined flux strings and string breaking. On the quantum side, near-term hardware, especially based on qudits [49,75–79,82,110], provides the means for implementing our algorithm or variants, such as hybrid variational SNAQs, that are expected to be more robust against experimental noise [119].

Note added.—After the completion of our work a closely related article [120] appeared on arXiv where many-body scars were studied numerically in non-Abelian stringnet models using exact diagonalization, including the qKS formulation discussed here. In a subsequent work by the same authors [121], they also applied the ansatz of Eq. (4) to a $\mathrm{SU}(3)_k$ qKS model.

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*Corresponding author: torsten.zache@uibk.ac.at

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