Coupling Fields to 3D Quantum Gravity via Chern-Simons Theory

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(Received 22 April 2023; revised 14 July 2023; accepted 22 September 2023; published 25 October 2023)

We propose a mechanism that couples matter fields to three-dimensional quantum gravity, which can be used for theories with a positive or negative cosmological constant. Our proposal is rooted in the Chern-Simons formulation of three-dimensional gravity and makes use of the Wilson spool, a collection of Wilson loops winding around closed paths of the background. We show that the Wilson spool correctly reproduces the one-loop determinant of a free massive scalar field on rotating black holes in AdS_3 and Euclidean dS_3 as $G_N \to 0$. Moreover, we describe how to incorporate quantum metric fluctuations into this formalism.

DOI: 10.1103/PhysRevLett.131.171602

Introduction.—Chern-Simons theory is a compelling approach to quantum gravity that makes manifest the topological nature of gravity in three dimensions [1,2]. For a negative cosmological constant $\Lambda < 0$, Chern-Simons theory nicely characterizes the physics of black holes in anti–de Sitter (AdS_3) space and their higher-spin generalizations [3–5], and dovetails the AdS_3/CFT_2 dictionary with the existence of edge modes [6–8]. Importantly, however, Chern-Simons gravity provides a powerful computational foothold for three-dimensional quantum gravity without recourse to holography. This is especially key in the context of $\Lambda > 0$, de Sitter (dS_3) gravity, where the corresponding dS/CFT dictionary is much less understood; see, though, [9,10] for recent developments. This context exhibits the true efficacy of Chern-Simons gravity, allowing a characterization of loop corrections that are relevant to quantum cosmology [11–18].

It has been a long-standing problem to incorporate matter into Chern-Simons gravity while retaining the topological features that make it natural as a theory of quantum gravity. In this Letter we address this problem. In short, we introduce a new object we deem the Wilson spool which provides an effective coupling of massive fields to quantum gravity directly as a gauge-invariant operator. We define it precisely below, but intuitively, the spool represents a Wilson loop winding arbitrarily many times around a closed path for which the fields have nontrivial holonomy. This represents a pivotal entry into a dictionary mapping geometric quantities to quantum operators in Chern-Simons gravity [19–24]. The Wilson spool will dictate how quantum gravity alters the physics of quantum fields in a manner that is quantitatively controlled by the gravitational coupling G_N.

We will show that the Wilson spool is a natural object regardless of the sign of cosmological constant. In the context of AdS_3 we will exactly reproduce, at tree level ($G_N \to 0$), the one-loop determinant of a scalar field on a rotating Bañados-Teitelboim-Zanelli (BTZ) black hole background. This computation is done directly "in the bulk," without reference to holography. In fact, it is in the context of dS_3, where holography is of limited utility, that we can make use of the full power of this proposal. Certain exact results in Chern-Simons theory can be meaningfully adapted to accommodate features necessary for de Sitter gravity and its massive single-particle states. This provides a principled and controlled method to computing G_N corrections to one-loop determinants of fields coupled to dynamical gravity. In this Letter, we distill key results that are presented in full detail in the companion paper [25], and also cast them in a presentation that is unified for both signs of the cosmological constant.

In the following we will present our proposal of the Wilson spool and illustrate its efficacy for AdS_3 and dS_3 gravity. We will evaluate it on AdS_3 in Lorentzian signature, where the Chern-Simons gauge group is $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$, while for dS_3 it will be treated in Euclidean signature, where the gauge group is $SU(2) \times SU(2)$. This choice facilitates making parallels between them, and hence makes evident the robustness of our proposal, while also showing contrast when appropriate.

The proposal.—To illustrate our proposal it is instructive to cast some portions in the metric formulation. Consider the path integral for a quantum scalar field ϕ, of mass m, with no self-interactions and minimally coupled to the metric field, $g_{\mu\nu}$. This would be

$$Z_{\text{scalar}}[g_{\mu\nu}] = \int [\mathcal{D}\phi] e^{iS_{\text{matter}}[\phi, g_{\mu\nu}]}. \quad (1)$$

Including the perturbative, quantum fluctuations of the metric leads to

$$\langle Z_{\rm scalar}`M]\rangle_{\rm grav} \coloneqq \int [\mathcal{D}g_{\mu\nu}]_M e^{-I_{\rm EH}[g_{\mu\nu}]} Z_{\rm scalar}[g_{\mu\nu}]. \quad (2)$$

For concreteness we wrote here a Euclidean path integral, where $I_{\rm EH}$ is the Euclidean Einstein-Hilbert action plus a cosmological constant term. The gravitational path integral is taken on a fixed topology M, and it includes all perturbative corrections around that topology.

Our proposal for quantifying the coupling of matter to gravity is captured by the following equality [26]:

$$Z_{\rm scalar}[g_{\mu\nu}] = \exp\left(\frac{1}{4}\mathbb{W}_j[A_L, A_R]\right). \quad (3)$$

On the right-hand side we have the Wilson spool

$$\mathbb{W}_j[A_L, A_R] \coloneqq i\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\alpha/2}{\sin\alpha/2} \times \mathrm{Tr}_{R_j}\left(\mathcal{P}e^{\frac{\alpha}{2\pi}\oint A_L}\right)\mathrm{Tr}_{R_j}\left(\mathcal{P}e^{-\frac{\alpha}{2\pi}\oint A_R}\right), \quad (4)$$

which captures $Z_{\rm scalar}$ by using solely objects in Chern-Simons theory.

We will give each component of (4) a natural interpretation below, but first let us explain how $\mathbb{W}_j$ is formally derived. Specifically, starting from the expression of the scalar one-loop determinant as a product over quasinormal mode spectra [27] and reinterpreting it in terms of representations of the local isometry group, one naturally arrives at (4). This was shown explicitly in [25] in the context of dS_3. In the Supplemental Material [28] we review key facets of that construction and extend it to AdS_3 as well.

We now provide an interpretation to each component of (4). Starting from the right, we have path ordered exponentials of A_L and A_R which encode $g_{\mu\nu}$ above: this is the portion that captures the information of the geometry. Next, we have traces over a representation R_j: here is where we encode the single-particle representations of the field. The mass of the field is related to the Casimir of the representation

$$c_2 = -\frac{\mathsf{m}^2}{4\Lambda}, \quad (5)$$

with Λ the cosmological constant. Finally, in (4) we have an integral over α. The measure and contour $\mathcal{C}$ of this integral serve two purposes [25]: First, they make $\mathbb{W}$ free of UV divergences. Second, as will become clear in detail, evaluating this contour integral as a sum over poles implements a sum over Wilson loops with arbitrary winding, which is what makes it a "spool." The sum over windings evokes earlier conceptions of one-loop determinants [32]; however, the contour integral generalizes it in a way that applies to gravity. The specific details of $\mathcal{C}$, and allowed deformations, depend on the holonomies of $A_{L,R}$; these will be specified for the backgrounds considered in the following sections.

In this Letter we want to uphold (4) on two fronts. First, we believe (4) applies to any smooth three-dimensional background, including but also extending beyond dS_3 quantum gravity. Second, it is a useful expression to quantify quantum gravity effects: the Chern-Simons formulation allows us to integrate out the scalar field and obtain an explicit functional in terms of the connections $A_{L,R}$. In this context, the main appeal of our proposal is the ability to quantify

$$\langle \log Z_{\rm scalar}\rangle_{\rm grav} = \frac{1}{4}\langle \mathbb{W}_j\rangle_{\rm grav} = \frac{1}{4}\int \mathcal{D}A_{L/R}\, e^{ik_L S_{\rm CS}[A_L] + ik_R S_{\rm CS}[A_R]}\,\mathbb{W}_j[A_L, A_R], \quad (6)$$

where $S_{\rm CS}[A]$ is the Chern-Simons action, $k_{L/R}$ are the levels. For brevity, here $\mathcal{D}A_{L/R}$ accounts for the measure for the two copies of the gauge group. The brackets mean that one is accounting for gravitational fluctuations around a fixed topology. In the Chern-Simons language, this means that we fix a background holonomy for $A_{L,R}$ in addition to the topology. It is important to stress that (6) is a nontrivial function of G_N and the mass of scalar field.

Wilson spools in AdS_3 *gravity.*—As a first example of the utility of our proposal we will focus on Chern-Simons gravity with negative cosmological constant $\Lambda = -\ell_{\rm AdS}^{-2}$. This is quantum gravity on spaces that are locally asymptotically anti–de Sitter (AdS). In Lorentzian signature, the relevant isometry group is $SL(2,\mathbb{R})\times SL(2,\mathbb{R})$ and massive particles in this space can be organized into $\mathfrak{sl}(2,\mathbb{R})$ representation theory. The Lorentzian Einstein-Hilbert action is given by the difference in $SL(2,\mathbb{R})$ Chern-Simons theories

$$S_{\rm EH} = k(S_{\rm CS}[A_L] - S_{\rm CS}[A_R]), \quad (7)$$

where $k = (\ell_{\rm AdS}/4G_N)$. The connections are related to the co-frame and the spin connection via

$$A_L = (\omega^a + e^a/\ell_{\rm AdS})L_a, \qquad A_R = (\omega^a - e^a/\ell_{\rm AdS})\bar{L}_a. \quad (8)$$

Above L_a and $\bar{L}_a$ generate the independent $\mathfrak{sl}(2,\mathbb{R})$ algebras. Our conventions are reviewed in the Supplemental Material [28]. At the classical level, the background geometries of interest are generated by flat background connections

$$a_L = L_0 d\rho + \left(e^{\rho} L_+ - e^{-\rho}\frac{2\pi\mathcal{L}}{k}L_-\right)dx^+,$$
$$a_R = -\bar{L}_0 d\rho - \left(e^{\rho} \bar{L}_- - e^{-\rho}\frac{2\pi\bar{\mathcal{L}}}{k}\bar{L}_+\right)dx^-, \qquad (9)$$

where $x^{\pm} = t \pm \varphi$ and $\varphi \sim \varphi + 2\pi$. Upon using (8), these are rotating BTZ black hole geometries [33,34] with mass M and angular momentum J given by

$$\mathcal{L} = \frac{M\ell_{\mathrm{AdS}} + J}{4\pi}, \qquad \bar{\mathcal{L}} = \frac{M\ell_{\mathrm{AdS}} - J}{4\pi}. \qquad (10)$$

In Euclidean signature, we can rotate (x^+, x^-) to complex coordinates $(z, -\bar{z})$. Periodicity in φ plus smoothness of the horizon implies these parametrize a complex torus $(z,\bar{z}) \sim (z + 2\pi m + 2\pi n\tau, \bar{z} + 2\pi m + 2\pi n\bar{\tau})$, $m, n \in \mathbb{Z}$ with modular parameter

$$\tau = \frac{i}{2}\sqrt{\frac{k}{2\pi\mathcal{L}}}, \qquad \bar{\tau} = -\frac{i}{2}\sqrt{\frac{k}{2\pi\bar{\mathcal{L}}}}. \qquad (11)$$

What separates this complex torus from the torus defining the boundary of thermal AdS_3 is how we "fill it in" in the bulk: in particular, the black hole geometry is filled in so that the thermal cycle is bulk contractible while the spatial cycle is not. The holonomies of the background connections $a_{L/R}$ around this cycle γ_φ are easily computed to be

$$\mathcal{P}\exp\left(\oint_{\gamma_\varphi} a_{L/R}\right) = u_{L/R}^{-1} e^{i2\pi \mathsf{h}_{L/R} L_0} u_{L/R}, \qquad (12)$$

where $u_{L/R}$ are periodic group elements and

$$\mathsf{h}_L = -\frac{1}{\tau}, \qquad \mathsf{h}_R = -\frac{1}{\bar{\tau}}. \qquad (13)$$

Next we will show that the Wilson spool of these background connections $a_{L/R}$ reproduces the one-loop determinant of a massive scalar field on the BTZ background. That is, we will test the relation

$$\log Z_{\mathrm{scalar}}[\mathrm{BTZ}] = \log\det(-\nabla^2_{\mathrm{BTZ}} + \mathsf{m}^2\ell^2_{\mathrm{AdS}})^{-1/2}$$
$$= \frac{1}{4}\mathbb{W}_j[a_L, a_R]. \qquad (14)$$

Here j labels a lowest-weight representation of $\mathfrak{sl}(2,\mathbb{R})$ related to the mass of the bulk field via

$$j = \frac{1}{2}\left(1 + \sqrt{\mathsf{m}^2\ell^2_{\mathrm{AdS}} + 1}\right) \equiv \frac{1}{2}\Delta. \qquad (15)$$

The expression for the Wilson spool, (4), can be derived for this background. Given (12), we have

$$\mathbb{W}_j[a_L, a_R] = i\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\alpha/2}{\sin\alpha/2}\chi_j\left(\frac{\alpha}{2\pi}\mathsf{h}_L\right)\chi_j\left(-\frac{\alpha}{2\pi}\mathsf{h}_R\right), \qquad (16)$$

where $\mathsf{h}_{L/R}$ are given by (13) and

$$\chi_j(z) = \mathrm{Tr}_{R_j}(e^{i2\pi z L_0}) = \frac{e^{i\pi z(2j-1)}}{2\sinh(-i\pi z)} \qquad (17)$$

is the character of the lowest-weight representation R_j. The contour $\mathcal{C}$ is given by twice the contour running up the imaginary α axis to the right of zero: $\mathcal{C} = 2\mathcal{C}_+$. This follows from the procedure in [25] but assigning an $i\epsilon$ prescription appropriate for representations and holonomies relevant to AdS_3. Further details of this are also described in the Supplemental Material [28].

Because $\tau, \bar{\tau} \in i\mathbb{R}$ (equivalently, $\mathcal{L}$ and $\bar{\mathcal{L}}$ are real and positive), all poles in the α integrand in (16) to the right of zero are simple poles at $2\pi\mathbb{Z}_{>0}$ and arise from the measure $[(\cos\alpha/2)/(\sin\alpha/2)]$. We then deform the contour $\mathcal{C}$ to the right where the α integrand is damped, picking up the residues of the simple poles. We can then write

$$\mathbb{W}_j[a_L, a_R] = \sum_{n=1}^{\infty}\frac{e^{-i\pi n(\tau^{-1} - \bar{\tau}^{-1})(2j-1)}}{n\sin(\frac{\pi n}{\tau})\sin(\frac{\pi n}{\bar{\tau}})}. \qquad (18)$$

This can be easily rewritten into

$$\frac{1}{4}\mathbb{W}_j[a_L, a_R] = \log\prod_{l,\bar{l}=0}^{\infty}\left(1 - q^{\frac{\Delta}{2}+l}\bar{q}^{\frac{\Delta}{2}+\bar{l}}\right)^{-1}, \qquad (19)$$

where $q = e^{-i(2\pi/\tau)}$ and $\bar{q} = e^{i(2\pi/\bar{\tau})}$. This matches exactly $\log Z_{\mathrm{scalar}}[\mathrm{BTZ}]$ for a real massive field propagating on the rotating BTZ background [35].

This is the tree-level $(G_N \to 0)$ contribution to $\langle \log Z_{\mathrm{scalar}}\rangle_{\mathrm{grav}}$. Promoting the background connections to dynamical fields, $a_{L/R} \to A_{L/R}$, the Chern-Simons path integral provides a way forward, in principle, for calculating perturbative G_N corrections to (19). The noncompact gauge group and the noncompact background topology make this program still difficult. However, one may still make progress using large-k Chern-Simons perturbation theory similar to [22,36,37]. Below we will see that in the context of positive cosmological constant, our ability to calculate G_N corrections is under even better control.

Wilson spools in dS_3 *gravity.*—Our next example will be Chern-Simons gravity with positive cosmological constant $\Lambda = \ell_{\mathrm{dS}}^{-2}$ which describes quantum gravity on de Sitter space. We will be working with a Euclidean action given by two copies of the $SU(2)$ Chern-Simons action; its relation to dS_3 gravity is via

$$I_{\mathrm{EH}} - i\delta I_{\mathrm{GCS}} = -ik_L S_{\mathrm{CS}}[A_L] - ik_R S_{\mathrm{CS}}[A_R], \qquad (20)$$

where $I_{\rm EH}$ is the Euclidean Einstein-Hilbert action and $I_{\rm GCS}$ is the gravitational Chern-Simons action. The Chern-Simons couplings $k_{L,R}$ are now in general complex,

$$k_L = \delta + is, \qquad k_R = \delta - is, \tag{21}$$

with an imaginary part that is related to Newton's constant $s = (\ell_{\rm dS}/4G_N)$ and a real part δ giving the coefficient of a gravitational Chern-Simons action. Quantum effects lead to a renormalization in the coupling constants [38]

$$k_L \to r_L = k_L + 2, \qquad k_R \to r_R = k_R + 2, \tag{22}$$

which amounts to a renormalization of $\delta \to \hat{\delta} = \delta + 2$.

The above matching is facilitated by relating the gauge fields to the vielbein and spin connection via

$$A_L = i\left(\omega^a + \frac{e^a}{\ell_{\rm dS}}\right)L_a, \qquad A_R = i\left(\omega^a - \frac{e^a}{\ell_{\rm dS}}\right)\bar{L}_a, \tag{23}$$

where now $\{L_a\}$ and $\{\bar{L}_a\}$ generate the two independent $\mathfrak{su}(2)$'s. We will focus presently on the round S^3 saddle whose classical background geometry is generated by background connections

$$\begin{aligned} a_L &= iL_1 d\rho + i(\sin\rho L_2 - \cos\rho L_3)(d\varphi - d\tau), \\ a_R &= -i\bar{L}_1 d\rho - i(\sin\rho \bar{L}_2 + \cos\rho \bar{L}_3)(d\varphi + d\tau). \end{aligned} \tag{24}$$

These background connections have a singularity at the causal horizon ($\rho = \pi/2$) and the holonomy around that singularity is given by [39]

$$\mathcal{P}\exp\oint_\gamma a_{L/R} = u_{L/R}^{-1} e^{i2\pi L_3 \mathsf{h}_{L/R}} u_{L/R}, \tag{25}$$

with $u_{L/R}$ periodic group elements and

$$\mathsf{h}_L = 1, \qquad \mathsf{h}_R = -1. \tag{26}$$

Now let us add matter and test our proposal. We will begin with the tree-level check which evaluates the spool on the background connections:

$$\log Z_{\rm scalar}[S^3] = \frac{1}{4}\mathbb{W}_j[a_L, a_R]. \tag{27}$$

The representation R_j that appears in $\mathbb{W}_j$ is a highest-weight representation of $\mathfrak{su}(2)$ related to the mass of the scalar field as

$$j = -\frac{1}{2}\left(1 + \sqrt{1 - \mathsf{m}^2\ell_{\rm dS}^2}\right). \tag{28}$$

Note that j is a continuous parameter and can even become complex for large enough masses. Thus these representations do not correspond to the standard finite dimensional representations of $SU(2)$. Instead they correspond to infinite dimensional "nonstandard" representations whose weight spaces line up with the dS_3 quasinormal mode spectrum [24,25]. Despite not lying in the standard $SU(2)$ representation theory, the representations obeying (28) can be equipped with an inner product such that all states have positive norm [24,25]. Additionally they admit well-defined characters:

$$\chi_j(z) = {\rm Tr}_{R_j}(e^{2\pi i z L_3}) = \frac{e^{i\pi z(2j+1)}}{2i\sin(\pi z)}. \tag{29}$$

We can then express (30) in terms of the holonomies (26) as

$$\begin{aligned} \mathbb{W}_j[a_L, a_R] &= i\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\alpha/2}{\sin\alpha/2}\chi_j\left(\frac{\alpha}{2\pi}\mathsf{h}_L\right)\chi_j\left(-\frac{\alpha}{2\pi}\mathsf{h}_R\right) \\ &= -\frac{i}{4}\int_{\mathcal{C}}\frac{d\alpha}{\alpha}\frac{\cos\alpha/2}{\sin^3\alpha/2}e^{i(2j+1)\alpha}. \end{aligned} \tag{30}$$

The $i\epsilon$ prescription appropriate for de Sitter results in a contour $\mathcal{C} = \mathcal{C}_- \cup \mathcal{C}_+$, that is, the union of contours running up the imaginary α axis both to the left ($\mathcal{C}_-$) and right ($\mathcal{C}_+$) of zero [25]. We pull both $\mathcal{C}_\pm$ toward the positive real axis to pick up residue of the quadruple pole at $\alpha = 0$ as well as twice the residues of the poles at $\alpha \in 2\pi\mathbb{Z}_{>0}$ along the positive real line; these latter poles are now third order. Doing so we find

$$\begin{aligned} \frac{1}{4}\mathbb{W}_j[a_L, a_R] &= i\frac{\pi(2j+1)^3}{6} - \frac{1}{4\pi^2}{\rm Li}_3\left(e^{i2\pi(2j+1)}\right) \\ &\quad + i\frac{(2j+1)}{2\pi}{\rm Li}_2\left(e^{i2\pi(2j+1)}\right) \\ &\quad - \frac{(2j+1)^2}{2}{\rm Li}_1\left(e^{i2\pi(2j+1)}\right), \end{aligned} \tag{31}$$

where ${\rm Li}_q(x) = \sum_{n=1}^\infty (x^n/n^q)$ are polylogarithm functions. Upon using (28), this answer matches precisely the finite contribution to the scalar one-loop determinant on S^3; see for instance [17]. This provides a second nontrivial check of the physical relevance of the Wilson spool.

In the context of de Sitter gravity we can actually make substantial progress in discussing the Wilson spool beyond its classical expectation value. This is because there exists a library of "exact techniques" for $SU(2)$ Chern-Simons theories on compact topologies. These techniques reduce the Chern-Simons path integral, as well as the expectation values of certain operators, to ordinary integrals. In the present context, care is needed to alter these methods to accommodate the features necessary for Chern-Simons gravity (i.e., complex levels, nonzero background connections, and nonstandard representations). However several exact methods remain amenable to these alterations [25]. For instance, a suitable alteration of Abelianization [40–42] reduces the expectation value of $\mathbb{W}_j[A_L, A_R]$ with dynamical connections to a pair of simple integrals [25]:

$$\begin{aligned}\frac{1}{4}\langle \mathbb{W}_j\rangle_{\text{grav}} &= \frac{i}{4} e^{ir_L S_{\text{CS}}[a_L]+ir_R S_{\text{CS}}[a_R]} \\ &\times \int d\sigma_L d\sigma_R \Bigg\{ e^{i\frac{\pi}{2}r_L\sigma_L^2+i\frac{\pi}{2}r_R\sigma_R^2}\sin^2(\pi\sigma_L)\sin^2(\pi\sigma_R) \\ &\times \int_C \frac{d\alpha}{\alpha}\frac{\cos\alpha/2}{\sin\alpha/2}\chi_j\left(\frac{\alpha(\sigma_L+\mathsf{h}_L)}{2\pi}\right)\chi_j\left(-\frac{\alpha(\sigma_R+\mathsf{h}_R)}{2\pi}\right)\Bigg\},\end{aligned} \tag{32}$$

which we can equate to the quantum gravity corrected one-loop determinant about the S^3 saddle $\langle \log Z_{\text{scalar}}[S^3]\rangle_{\text{grav}}$. The integral appearing in (32) is difficult to evaluate analytically; however, it can systematically be evaluated in a $s^{-1} = 4G_N/\ell_{\text{dS}}$ Taylor expansion. This provides a controlled procedure to computing G_N corrections to $\log Z_{\text{scalar}}[S^3]$. These corrections are naturally identified with a mass renormalization. To extract them one simply computes $\langle \log Z_{\text{scalar}}[S^3]\rangle_{\text{grav}}$, normalized by the gravitational path integral, using (32); the resulting renormalized mass to $O(G_N^2)$ is given by [43]

$$\mathsf{m}_R^2\ell_{\text{dS}}^2 = \mathsf{m}^2\ell_{\text{dS}}^2 + \frac{96}{5}\mathsf{m}^4\ell_{\text{dS}}^4 e^{-2\pi|\mathsf{m}\ell_{\text{dS}}|}\left(\frac{G_N}{\ell_{\text{dS}}}\right)^2 + \dots. \tag{33}$$

Above we have kept the leading term in a large mass expansion $\mathsf{m}^2\ell_{\text{dS}}^2 \gg 1$; however, (32) provides an expression for the G_N^2 renormalization of the mass that can be calculated analytically [25]. We emphasize that this is a concrete predictive statement about how dynamical quantum gravity renormalizes quantum field theory.

Discussion.—We have introduced a new object, the Wilson spool, which allows one to have matter fields in the Chern-Simons formulation of three-dimensional quantum gravity while keeping manifest the key topological aspects of gravity. To test this object, we have shown that, for $G_N \to 0$, it reproduces correctly one-loop determinants of massive scalar fields on a curved background. The proposal works for spacetimes that are widely different, such as the spinning BTZ black hole and Euclidean de Sitter spacetime.

If our proposal merely provided a way to match onto known one-loop determinants, it would have limited utility. However, it is also possible to use this technique to make a *prediction* for quantum corrections to $\log Z_{\text{grav}}$, having allowed for quantum fluctuations of the metric around a classical background. For certain AdS geometries (see, e.g., [22]), the $1/c$ corrections to Wilson lines have been computed in a holographic setting. Our techniques give us a way to extend this beyond the holographic setting to quantum gravity in spacetimes like de Sitter, which are more realistic approximations of our observed universe but where techniques of holography are largely out of reach.

There are two future directions in this area that we would like to highlight. A more in depth discussion is presented in [25].

Massive spinning fields: We have focused on massive scalar fields in this Letter. But both AdS_3 and dS_3 admit massive fields of arbitrary integer spin. We can ask if the Wilson spool is of utility in coupling these excitations to dynamical gravity. In the context of AdS_3, the definition of the Wilson spool (4) can be intuitively extended to reproduce the one-loop determinant for a massive spinning field on the rotating BTZ background. The details of this can be found in the Supplemental Material [28]. This is a strong indication that $\mathbb{W}$ is useful for the physics of spinning fields. A proper treatment of spinning fields in dS_3 requires more care: one must first construct the nonstandard $\mathfrak{su}(2)$ representations accommodating spin, which we leave for future work. We include a discussion on the subtleties that arise in continuing the AdS_3 result to dS_3 in the Supplemental Material [28].

Sum over topologies: Here we have kept the topology of the background, and holonomies of the connections, fixed. It is of great interest to allow for these nonperturbative contributions in (2), albeit it comes with difficulties: if only metric degrees of freedom are incorporated, both the AdS_3 thermal partition function [44,45] and the Euclidean dS_3 path integral [16] suffer from pathologies. But it is also expected that adding matter can fix some of these problems [46]. It would be interesting to investigate the fate of the Wilson spool under these pathologies, i.e., loop in matter in the sum over manifolds and quantify its imprint. This is specially of interest in dS_3, where a holographic dictionary is still nascent, but the Wilson spool gives a concise path to quantify the effect of fields at all orders in G_N.

We would like to thank Kurt Hinterbichler and Sergei Cherkis for useful comments and discussions. J. R. F. thanks the University of Amsterdam for hospitality. The work of A. C. and J. R. F. has been partially supported by STFC consolidated Grant No. ST/T000694/1. The work of J. R. F. has been also partially supported by Simons Foundation Award No. 620869. I. C. has been partially supported by the ERC starting grant H2020 ERC StG No. 640159. C. Z. has been supported by a UM Duluth Higholt Professorship, and acknowledges a Heising-Simons Fellowship as part of the "Observational Signatures of Quantum Gravity" collaboration Grant No. 2021-2818.

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