Editors' Suggestion

Are Gluon Showers inside a Quark-Gluon Plasma Strongly Coupled? A Theorist's Test

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We study whether in-medium showers of high-energy gluons can be treated as a sequence of individual splitting processes $g \to gg$, or whether there is significant quantum overlap between where one splitting ends and the next begins. Accounting for the Landau-Pomeranchuk-Migdal (LPM) effect, we calculate such overlap effects to leading order in high-energy $\alpha_s(\mu)$ for the simplest theoretical situation. We investigate a measure of overlap effects that is independent of physics that can be absorbed into an effective value $\hat{q}_{\text{eff}}$ of the jet-quenching parameter $\hat{q}$.

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When passing through matter, very high energy particles lose energy by showering, via the splitting processes of hard bremsstrahlung and pair production induced by small-angle scatterings from the medium. Figure 1 shows a cartoon of shower development, where the energy E_0 of the initial high-energy particle is split among more and more particles as time goes by, until eventually the remaining particles have such low energy that they thermalize with the medium (if the medium is thick enough to stop them before they leave). We will focus on showers of very high energy ($E \gg T$) partons traversing a quark-gluon plasma of temperature T. The quantum mechanical duration of a high-energy splitting in the rest frame of the plasma is known as the formation time. We have drawn ovals in Fig. 1 to represent the formation time (or, equivalently, formation length) of each splitting, depicting the formation times as small compared to the time between splittings. In that case, if one has results for individual medium-induced splitting rates, one may statistically model shower development by treating high-energy particles classically between splittings, and rolling dice based on the splitting rates to decide when and how each particle splits. We call this a "weakly coupled" picture of in-medium shower development.

Alternatively, if formation times are large compared to times between splittings, one may not treat different splittings as quantum mechanically independent, and any classical picture of shower development breaks down. We will call that a "strongly coupled" shower, which has been studied theoretically for certain QCD-like theories (such as $\mathcal{N} = 4$ supersymmetric QCD) that can be studied with gauge-gravity duality [1–4].

As we will review, the distinction between weakly and strongly coupled pictures of shower development is controlled by the size of the running coupling $\alpha_s(\mu)$ at the transverse momentum scale μ associated with high-energy splittings. We will devise and calculate a theoretical measure of how large $\alpha_s(\mu)$ can be before the weakly coupled picture of shower development breaks down. Roughly, our approach will be to treat $\alpha_s(\mu)$ as small but calculate the correction to the qualitative picture of Fig. 1 by computing the correction from *overlapping* formation times of two consecutive splittings. We will have to carefully sharpen the question we ask in order to factorize out effects of soft bremsstrahlung. This Letter aims to give a broad overview of our method and conclusion, with many details and derivations left to a companion paper [5].

The formalism for making such calculations is challenging, and so we take the simplest possible theoretical situation. (i) Imagine a quark-gluon plasma that is static, homogeneous, and large enough to completely stop the shower. (ii) Imagine that we start with a single high-energy

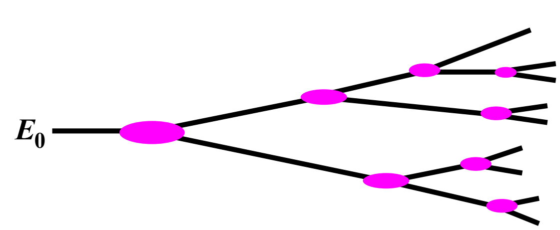

FIG. 1. Schematic depiction of a high-energy shower in a medium. The splittings are nearly collinear, but tiny splitting angles have been exaggerated to make the drawing readable.

parton that is very close to on shell. (This ignores, for example, the initial shower of decreasing virtuality that takes place when a high-energy parton is scattered out of a nucleon in a relativistic collision.) (iii) Treat the elastic scattering of high-energy partons from the medium in multiple-scattering ($\hat{q}$) approximation, which is that the typical total transverse momentum change $p_\perp$ after traveling through a length L of medium behaves like a random walk, $\langle p_\perp^2 \rangle = \hat{q}L$, where the proportionality constant $\hat{q}$ is determined by the medium. (iv) Take the large-N_c limit, where N_c is the number of quark colors. (v) Focus on gluon-initiated showers, and so the only relevant splittings in Fig. 1 are $g \to gg$ in the large-N_c limit. All of these assumptions could, in principle, be relaxed in the formalism that we use, but that would make the calculations much more difficult.

Before proceeding, we review some parametric scales associated with single splittings (such as $g \to gg$), shower development, and the weakly-coupled picture of Fig. 1. Formation times grow with energy. At sufficiently high energy ($E \gg T$ in our case), the formation time $t_{\rm form}$ of high-energy splittings becomes large compared to the mean free time $\tau_{\rm scatt}$ for elastic scattering from the medium; many scatterings take place during a single splitting, which causes a very significant reduction of the splitting rate, known as the Landau-Pomeranchuk-Migdal (LPM) effect [6–9]. The treatment of the LPM effect in QCD was originally worked out by Baier, Dokshitzer, Mueller, Peigne, and Schiff [10–12] and by Zakharov [13,14] (BDMPS-Z). In that limit, the formation time scales parametrically as $t_{\rm form} \sim \sqrt{\omega/\hat{q}}$ in QCD, where $\omega \gg T$ is the energy of the least-energetic daughter of the splitting. The typical scale μ of transverse momentum transferred from the medium during the formation time is of order

$$\mu \sim \sqrt{\hat{q} t_{\rm form}} \sim (\hat{q}\omega)^{1/4}. \tag{1}$$

This is also the typical scale of the relative transverse momenta of the two daughters of the splitting.

For simplicity, focus for now on *democratic* splitting of a particle with energy E, meaning that the two daughters have roughly comparable energies. In the high-energy limit, the probability of a democratic splitting is parametrically of order $\alpha(\mu)$ per formation time, where $\alpha(\mu)$ is the running QCD coupling. Note that μ grows with energy $\omega \sim E/2$ in (1). Now consider two, consecutive, democratic splittings. Then the energies and so formation lengths characteristic of the two consecutive splittings are the same order of magnitude. Since the probability of a splitting is parametrically α per formation time, the typical distance between splittings will be of order $t_{\rm form}/\alpha$, and the probability of the two consecutive splittings overlapping will be order α. So, naively, the weak-coupling picture of showers corresponds parametrically to $\alpha(\mu)$ small, and that picture fails when $\alpha(\mu)$ is large.

That is a naive statement because the preceding argument was for democratic splittings. References [15–17] have shown that the probability of a hard splitting overlapping with *soft* bremsstrahlung is enhanced by a large double logarithm in QCD, similar to double logarithms in small-x physics but with some kinematic limits different. They found that, even if $\alpha_s(\mu)$ is small, such overlaps have large effects on energy loss when the double logarithm compensates. In our case of splitting of a high-energy particle of energy E in a thick quark-gluon plasma of temperature T, "soft" gluon energy ω means $T \ll \omega \ll E$, which is the range that contributes to the double logarithm. So overlap effects become significant when $\alpha_s(\mu)\ln^2(E/T)$ is large, which can happen even if $\alpha_s(\mu)$ is somewhat small. But they also found that these double log effects can be absorbed into a redefinition of the medium parameter $\hat{q}$. In our situation here, that means that the potentially large effects of a soft gluon bremsstrahlung overlapping a hard splitting process can be absorbed into the original LPM/BDMPS-Z calculation of the hard $g \to gg$ splitting rate by taking $\hat{q} \to \hat{q}_{\rm eff}(E) = \hat{q} + \delta\hat{q}$ in that calculation, where $\delta\hat{q}(E) \sim \alpha_s \hat{q} \ln^2(E/T)$. They also showed (following [18]) how to resum leading logs to all orders in $\alpha_s(\mu)$.

Refining the question.—The goal of this Letter is to construct a measure of the size of overlap effects that *cannot* be factorized away and absorbed into an effective value for the medium parameter $\hat{q}$. We start with an idea proposed in Ref. [19]. For simplicity, imagine for a moment a shower composed of democratic splittings. The distance between consecutive splittings is of order $t_{\rm form}/\alpha \sim \alpha^{-1}\sqrt{E/\hat{q}}$, where the typical energy E of the individual shower particles decreases rapidly as the shower develops. A shower initiated by a single particle of energy E_0, moving in the z direction, will therefore stop and deposit all its energy into the medium in a distance of order $\ell_{\rm stop} \sim \alpha^{-1}\sqrt{E_0/\hat{q}}$, which depends on $\hat{q}$. As a thought experiment, imagine measuring the distribution $\epsilon(z)$ in z of where that energy is deposited into the medium, statistically averaged over many such showers. (We do not track the parametrically small spread of the shower in the transverse directions.) A qualitative picture is shown in Fig. 2. We define $\ell_{\rm stop}$ as the first moment $\langle z \rangle \equiv E_0^{-1} \int dz\, z\epsilon(z)$ of this distribution. Other features of the distribution, such as its

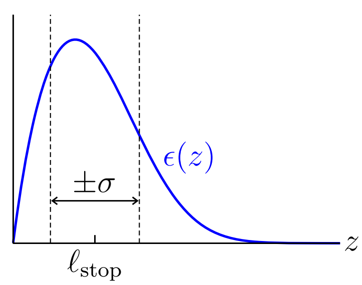

FIG. 2. Energy deposition distribution $\epsilon(z)$.

width $\sigma = \sqrt{\langle z^2\rangle - \langle z\rangle^2}$, are parametrically the same order as $\ell_{\rm stop} \sim \alpha^{-1}\sqrt{E_0/\hat{q}}$. Naively, the dependence on $\hat{q}$ would then *cancel* in a ratio such as $\sigma/\ell_{\rm stop}$. More generally, one may study any aspect of what we will call the "shape" $S(Z)$ of the energy deposition distribution $\epsilon(z)$. By shape, we mean Fig. 2 rescaled to units where $\ell_{\rm stop} = 1$ and normalized to have unit area under the curve:

$$S(Z) \equiv \frac{\langle z\rangle}{E_0}\epsilon(\langle z\rangle Z), \tag{2}$$

where $Z \equiv z/\langle z\rangle$. Naively, this shape function is insensitive to any physics (such as soft bremsstrahlung) that can be absorbed into the value of $\hat{q}$.

The shape $S(Z)$ and its moments are insensitive to *constant* shifts $\delta\hat{q}$ to $\hat{q}$. However, the potentially large double log correction, arising from a soft bremsstrahlung overlapping a hard splitting, is not constant: it depends logarithmically on the energy scale E of the underlying hard splitting. So $\delta\hat{q}$ is different for different splittings in the shower, and those differences do not exactly cancel in $S(Z)$. As discussed in Ref. [19] in the specific context of $\sigma/\ell_{\rm stop}$ (which is the second reduced moment of S), the energy dependence of the double-log corrections from overlapping soft bremsstrahlung will lead to potentially large single-log corrections to the shape—that is, corrections that are $O[\alpha_{\rm s}\ln(E_0/T)]$ instead of $O(\alpha_{\rm s})$. The naive calculation of overlap corrections to $S(Z)$ will not be completely independent of soft bremsstrahlung physics.

To proceed, consider a loose analogy with parton distribution functions (PDFs) in the context of deep inelastic scattering (DIS) and other inclusive processes. The cross sections factorize into (i) constituent cross sections of the partons and (ii) PDFs. Beyond leading order (LO), the constituent cross sections have initial-state collinear divergences in perturbation theory that must be absorbed into the PDFs. This requires introducing a factorization scale $M_{\rm fac}$ to specify exactly how much to absorb (analogous to introducing the renormalization scale μ when absorbing ultraviolet divergences) [20]. In next-to-leading order (NLO) perturbative calculations, the answer depends on the choice of $M_{\rm fac}$, just as it depends on the choice of renormalization scale μ. Theorists typically set $M_{\rm fac}$ and μ to be the same and of order of the appropriate physics scale of the problem (e.g., $\sqrt{|Q^2|}$ in DIS) in order to avoid large logarithms in the perturbative expansion. Typically, the exact choice of scale is varied over a reasonable range to give a theory guess of uncertainty. The higher the order in perturbation theory, the less sensitive the result to that variation.

We adopt a similar strategy. We define $\hat{q}_{\rm eff}(\Lambda_{\rm fac})$ to exactly absorb all double and subleading single log behavior from overlapping soft bremsstrahlung that has $\omega_{\rm soft} \le \Lambda_{\rm fac}$. We will choose $\Lambda_{\rm fac}$ to be of the order of the relevant energy scale of the problem (in the rest frame of the plasma). Similar to (1), the corresponding transverse momentum scale is $M_{\rm fac} \sim (\hat{q}\Lambda_{\rm fac})^{1/4}$. We will calculate all effects of overlapping formation times on $S(Z)$ that have not already been absorbed into $q_{\rm eff}(\Lambda_{\rm fac})$. Later, we will see that the question of whether overlap effects that *cannot* be absorbed into $\hat{q}$ are large or small is very insensitive to the exact choice of $\Lambda_{\rm fac}$.

Shower evolution.—The rates that contribute to LO + NLO, large-$N_{\rm c}$, gluon shower evolution are called $[d\Gamma/dx]^{\rm LO}$, $[\Delta d\Gamma/dx]^{\rm NLO}_{g\to gg}$, and $[\Delta d\Gamma/dxdy]_{g\to ggg}$. Formulas for these rates are given in Refs. [22,23], culminating the development of Refs. [24–28]. Here, leading order refers to the LPM/BDMPS-Z rate for a single, *nonoverlapping* splitting $g \to gg$, such as each individual splitting shown in Fig. 1. Our "LO" rate encompasses an arbitrary number of scatterings from the medium and does not assume that the coupling $\alpha_{\rm s}(T)$ of the quark-gluon plasma is perturbatively small. Here, LO vs NLO refers only to how many powers of direct interactions $\alpha_{\rm s}(\mu)$ between *high-energy* ($E \gg T$) partons are involved in the splitting. $[d\Gamma/dx]^{\rm LO}$ is the differential LO rate for the energy to split as $E \to xE + (1-x)E$. $[\Delta d\Gamma/dxdy]_{g\to ggg}$ is a rate representing the *overlap correction* to any two consecutive splittings, $g \to gg \to ggg$, with final energy split as $E \to xE + yE + (1-x-y)E$. (We also include $g \to ggg$ from direct 4-gluon vertices in $[\Delta d\Gamma/dxdy]_{g\to ggg}$ [23].) $[\Delta d\Gamma/dx]^{\rm NLO}_{g\to gg}$ gives related one-loop corrections to single splitting, such as from $g \to gg \to ggg \to gg$. These rates are designed so that one may evolve the shower using classical statistics for an evolution that contains both $1 \to 2$ splittings, with differential rate $[d\Gamma/dx]_{1\to 2} = [d\Gamma/dx]^{\rm LO} + [\Delta d\Gamma/dx]^{\rm NLO}_{g\to gg}$, and $1 \to 3$ splittings, with rate $[\Delta d\Gamma/dxdy]_{g\to ggg}$. The latter rate can sometimes be negative because it contains the overlap *correction*, which can have either sign [25]. Negative $[\Delta d\Gamma/dxdy]_{g\to ggg}$ will not cause any problem for the NLO analysis in this Letter.

In our convention, final-state identical particle symmetry factors are not included in the differential rates above. So, since all our high-energy particles are gluons, the total splitting rate would be formally (ignoring the fact that it is infrared divergent)

$$\Gamma = \frac{1}{2!}\int_0^1 dx \left[\frac{d\Gamma}{dx}\right]_{1\to 2} + \frac{1}{3!}\int_0^1 dx \int_0^{1-x} dy \left[\frac{d\Gamma}{dxdy}\right]_{1\to 3}. \tag{3}$$

When a shower involves more than just $1 \to 2$ splitting processes, the shower evolution equation can be neatly packaged in terms of what we call the "net" rate $[d\Gamma/dx]_{\rm net}$ [22] for a splitting to produce one daughter of energy xE (plus any other daughters) from a parent of energy E. In the case of generic $1 \to 2$ and $1 \to 3$ splittings,

$$\left[\frac{d\Gamma}{dx}\right]_{\rm net} = \left[\frac{d\Gamma}{dx}\right]_{1\to 2} + \frac{1}{2!}\int_0^{1-x} dy \left[\frac{d\Gamma}{dxdy}\right]_{1\to 3}. \quad (4)$$

Note that the integral of $[d\Gamma/dx]_{\rm net}$ over x is *not* the total rate Γ. Instead, there is a very useful alternative relation [5]: $\Gamma = \int dx\, x[d\Gamma/dx]_{\rm net}$. In terms of $[d\Gamma/dx]_{\rm net}$, the shower evolution equation is [5,22]

$$\frac{\partial}{\partial t} n(\zeta, E_0, t) = \int_0^1 dx \left\{ -x \left[\frac{d\Gamma}{dx}(x, \zeta E_0)\right]_{\rm net} n(\zeta, E_0, t) + \frac{\theta(x-\zeta)}{x}\left[\frac{d\Gamma}{dx}\left(x, \frac{\zeta}{x}E_0\right)\right]_{\rm net} n\left(\frac{\zeta}{x}, E_0, t\right)\right\}, \quad (5)$$

where $n(\zeta, E_0, t)$ is the number density in ζ of gluons with energy ζE_0 at time t. $[d\Gamma(x,E)/dx]_{\rm net}$ is the net splitting rate (4), and θ is the unit step function.

We have implicitly integrated over final (postoverlap) transverse momenta both in our rate calculations and in $n(\zeta, E_0, t)$, and chosen a $p_\perp$ insensitive test of overlap effects, because implicit $p_\perp$ integration drastically simplifies the calculation of rates [29].

We want to factor out (and absorb into $\hat{q}$) the double and single logs arising from soft bremsstrahlung with energy $\omega' \le \Lambda_{\rm fac}$, and so, at NLO, we use a factorized version of the net rate in evolution equations like (5). In the multiple-scattering ($\hat{q}$) approximation we have used, the net rate (4) is double-log infrared divergent, but the factorized net rate will not be. The computations [22,23] of splitting rates used a small infrared (IR) cutoff $\omega_{\rm min}$ on soft gluon energy. With that IR regulator, the *factorized* rate is then

$$\left[\frac{d\Gamma}{dx}\right]^{\rm fac}_{\rm net} = \left[\frac{d\Gamma}{dx}\right]_{\rm net} - \frac{C_{\rm A}\alpha_{\rm s}}{4\pi}\left[\frac{d\Gamma}{dx}\right]^{\rm LO} \times \int_{\omega_{\rm min}}^{\Lambda_{\rm fac}} \frac{d\omega'}{\omega'}\left\{\ln\left(\frac{E}{\omega'}\right) - \bar{s}(x)\right\}, \quad (6)$$

where $C_{\rm A} = N_{\rm c}$ is the adjoint Casimir, the integral of the first term in braces produces a double logarithm, and the single log coefficient $\bar{s}(x)$ is given explicitly in Refs. [30,31]. The combination (6) is finite as $\omega_{\rm min} \to 0$ and should be independent of the details of the *actual* physics [18,32] that cuts off the double logarithm in the infrared.

The evolution equation (5) can be simplified if the (factorized) net rate scales with energy as exactly $E^{-1/2}$ for fixed x. This depends on the details of how one chooses $\Lambda_{\rm fac}$. One choice might be (i) $\Lambda_{\rm fac} \propto E_0$, the energy of the entire shower. Absorbing double logs into $\hat{q}$, the "leading-order" description would then use $\hat{q}_{\rm eff}(E_0)$ for all splittings in the shower. A more refined choice would be (ii) $\Lambda_{\rm fac} \propto E$, and so use $\hat{q}_{\rm eff}(E)$ for each splitting, adjusted for the parent's energy E of that particular splitting. An even more refined choice would be to recognize that the formation time and transverse momentum kicks associated with a $g \to gg$ splitting are determined (regarding the LPM effect) by the energy of the softest daughter, and so take (iii) $\Lambda_{\rm fac} \sim \min[xE, (1-x)E]$. In case (i), due to the mismatch of $\Lambda_{\rm fac}$ and the energy of individual splittings, a part of (6) will scale like $E^{-1/2}\ln^2(\Lambda_{\rm fac}/E)$, which does not allow simplification of the evolution equation. Both cases (ii) and (iii) avoid logarithmic dependence on E. Because (iii) is the most natural choice, we stick to that here. Specifically, we choose $\Lambda_{\rm fac} \propto x(1-x)E$, which is a smooth function of x with the desired parametric behavior.

To simplify the shower evolution equation, scale $E^{-1/2}$ out of the rate by rewriting $d\Gamma(x,E) = E^{-1/2} d\tilde{\Gamma}(x)$, $t = E_0^{1/2}\tilde{t}$, and $n(\zeta, E_0, t) = \tilde{n}(\zeta, \tilde{t})$. Then

$$\frac{\partial}{\partial \tilde{t}}\tilde{n}(\zeta, \tilde{t}) = \zeta^{-1/2}\int_0^1 dx \left[\frac{d\tilde{\Gamma}}{dx}(x)\right]^{\rm fac}_{\rm net} \times \left\{ -x\tilde{n}(\zeta, \tilde{t}) + \frac{\theta(x-\zeta)}{x^{1/2}}\tilde{n}\left(\frac{\zeta}{x}, \tilde{t}\right)\right\}. \quad (7)$$

An even simpler equation can be found for the (rescaled) energy deposition distribution [5,19],

$$\frac{\partial \tilde{\epsilon}(\tilde{z})}{\partial \tilde{z}} = \int_0^1 dx\, x \left[\frac{d\tilde{\Gamma}}{dx}(x)\right]^{\rm fac}_{\rm net} \{x^{-1/2}\tilde{\epsilon}(x^{-1/2}\tilde{z}) - \tilde{\epsilon}(\tilde{z})\}, \quad (8)$$

where $\tilde{\epsilon}(\tilde{z}) \equiv E_0^{-1/2}\epsilon(E_0^{1/2}\tilde{z})$ is normalized so that $\int_0^\infty d\tilde{z}\,\tilde{\epsilon}(\tilde{z}) = 1$. Simpler yet, the *moments* of this distribution are given recursively in terms of integrals of the net rate [5,19]:

$$\langle \tilde{z}^n \rangle = \frac{n\langle \tilde{z}^{n-1}\rangle}{\int_0^1 dx\, x(1-x^{n/2})[\frac{d\tilde{\Gamma}}{dx}]^{\rm fac}_{\rm net}}. \quad (9)$$

Results and conclusions.—We find that the width $\sigma_S = \sigma/\ell_{\rm stop}$ of the shape distribution $S(Z)$ is

$$\frac{\sigma}{\ell_{\rm stop}} = \left[\frac{\sigma}{\ell_{\rm stop}}\right]^{\rm LO}_{\rm eff}(1 + \chi\alpha_{\rm s} + \text{higher order}), \quad (10)$$

where the relative size of overlapping formation-time corrections not absorbed into $\hat{q}_{\rm eff}$ is

$$\chi\alpha_{\rm s} = (-0.019 \pm 0.001 \ln\kappa) C_{\rm A}\alpha_{\rm s}(\mu) \quad (11)$$

for $\Lambda_{\rm fac} = \kappa x(1-x)E$ and $\mu = (\hat{q}_{\rm A}\Lambda_{\rm fac})^{1/4}$, where our canonical choice is $\kappa = 1$. Even for $N_{\rm c}\alpha_{\rm s}(\mu) = 1$, (11) is a tiny, few-percent effect (for any reasonable choice of κ).

Reference [5] gives some results for higher moments of $S(Z)$ for gluon showers, but it is more interesting to just look at how the function $S(Z)$ itself changes. Let $\delta S(Z)$ be the change in the shape function to first order in overlap effects, i.e., to first order in $\alpha_{\rm s}(\mu)$. Figure 3 depicts $S^{\rm LO}(Z)$

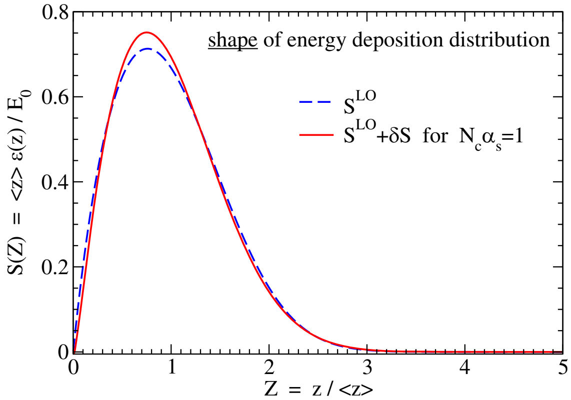

FIG. 3. Energy deposition shape with and without first-order overlapping formation time effects δS, for $N_c\alpha_s = 1$.

vs $S^{\rm LO}(Z) + \delta S(Z)$ for $N_c\alpha_s = 1$. The difference is very small and will be proportionally smaller for smaller $N_c\alpha_s(\mu)$.

Though the corrections to the shape $S(Z)$ are very small for large-N_c gluon showers (Fig. 3), the corrections to quantities that *do* depend directly on $\hat{q}$ are substantial, even when factorized. The relative difference between $[d\Gamma/dx]^{\rm fac}_{\rm net}$ and $[d\Gamma/dx]^{\rm LO}$ can be of order $N_c\alpha_s \times 100\%$ for democratic splittings and is fairly sensitive to the choice of $\Lambda_{\rm fac}$ [5].

We should clarify that, when we use measurements of the shape function to "ignore all effects that can be absorbed into $\hat{q}$," we are not claiming that those exact same effects also affect transverse momentum broadening (the basis for our original definition of $\hat{q}$). For our purpose here, think of $\hat{q}_{\rm eff}$ as an effective "jet quenching" parameter rather than a precisely defined effective "transverse momentum broadening" parameter. It is known that the coefficient of the IR double logs are universal in the sense that they affect both the same way [15–17]. At least in the large-N_c limit, there is a (more subtle) universality for subleading, IR single logs as well [30]. But we are unaware of any reason for such universality to hold beyond logarithms.

In dramatic contrast to (11), Ref. [19] analyzed $\sigma/\ell_{\rm stop}$ for *charge* (rather than energy) deposition of an *electron-initiated* shower in large-N_f QED, and the analog of (11) was found to be $\chi\alpha_{\rm EM} = -0.87 N_f \alpha_{\rm EM}(\mu)$ (and no factorization scale need be introduced). This is a large effect for $N_f\alpha_{\rm EM} = 1$. Reference [5] offers some crude, incomplete, after-the-fact insight about the qualitative difference with (11) and motivates future study of (i) whether adding quarks to our analysis would qualitatively change our conclusion and (ii) whether overlap effects for energy vs charge stopping are qualitatively different.

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