Verifying the Magnitude Dependence in Earthquake Occurrence

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The existence of magnitude dependence in earthquake triggering has been reported. Such a correlation is linked to the issue of seismic predictability and it remains under intense debate whether it is physical or is caused by incomplete data due to the missing short-term aftershocks. Working firstly with a synthetic catalog generated by a numerical model that captures most statistical features of earthquakes and then with a high-resolution earthquake catalog for the Amatrice-Norcia (2016) sequence in Italy, where for the latter case we employ the stochastic declustering method to reconstruct the family tree among seismic events and limit our analysis to events above the magnitude of completeness, we found that the hypothesis of magnitude correlation can be rejected.

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Introduction.—The question of whether earthquakes can be predicted is one of the most important in both social and scientific contexts [1,2]. The study of earthquake occurrence phenomena is of great interest and involves multiple fields of research and technology, including engineering, geophysics, seismology, statistical mechanics, and more. It has been well known that seismicity is not completely random and the biggest predictable component in seismicity is clustering. The epidemic-type aftershock sequence (ETAS) model is considered as the standard baseline for modeling earthquake clusters [3–9] and short-term aftershock forecasting. In the traditional ETAS model, all the event magnitudes are assumed to be independent from the occurrence times and identically from the same random distribution—the Gutenberg-Richter law, which in fact implies the complete randomness of earthquake magnitude in predictability.

Recently, some researchers reported the presence of correlations between seismic magnitudes within an earthquake sequence [10–14], i.e., subsequent events tend to have more similar magnitudes than expected based on the Gutenberg-Richter law. This implies that there is some predictability from complete randomness in forecast earthquake magnitude. That is, it is possible to predict, to some extent, the magnitude of an earthquake before its rupture process completes by using past seismicity.

However, some have argued that such an apparent correlation is an artifact of the short-term aftershock incompleteness (STAI) [10,15], which refers to the lack of recorded earthquakes following a major event due to overlapping coda waves, particularly in the immediate aftermath of a large earthquake [16–21]. Previous studies have demonstrated that STAI not only introduces bias in the estimation of model parameters [22] and forecasting but also could give rise to apparent magnitude correlations [15].

In an alternate hypothesis, the existence of magnitude dependence offers an alternative explanation for STAI. In other words, the lack of recorded earthquakes following a major event may not be a issue of recording bias but rather a real preference to trigger earthquakes of a certain magnitude. It is important to note that the incompleteness of the instrumental seismic catalog due to the overlapping of coda waves is a well-established effect, and the supporters of the existence of correlations between magnitudes do not deny the existence of STAI. Rather, they attribute the absence of minor events to both instrumental issues and magnitude clustering due to physical phenomena not captured in the ETAS model.

The traditional ETAS model does not account for either STAI or magnitude dependence. To improve the ETAS model's ability to describe seismicity, we need to make a choice between these two different approaches. The first is to tackle the influence of incompleteness, by "obscuring" events produced by a simulated ideal ETAS catalog to reproduce the correct sequence of events present in the real catalog, which has short-term incompleteness (as suggested in [18,19,23]) or by "reconstructing" the complete catalog by reintroducing missing events (as suggested in [24,25]). The second approach introduces a "constrained" magnitude frequency distribution $P(m|m^*)$ for the aftershocks that are triggered directly by a parent event magnitude m^* in the ETAS model to account for the existence of correlations between seismic magnitudes. Both approaches appear to improve the ETAS model's ability to describe seismicity, but it is still unclear which one corresponds to reality and should be used for the next generation of seismic forecasting statistical models.

In this Letter, we study the magnitude correlations for a synthetic seismic catalog produced with a two-layer Olami-Feder-Christensen (OFC) model ([26]) which is able to

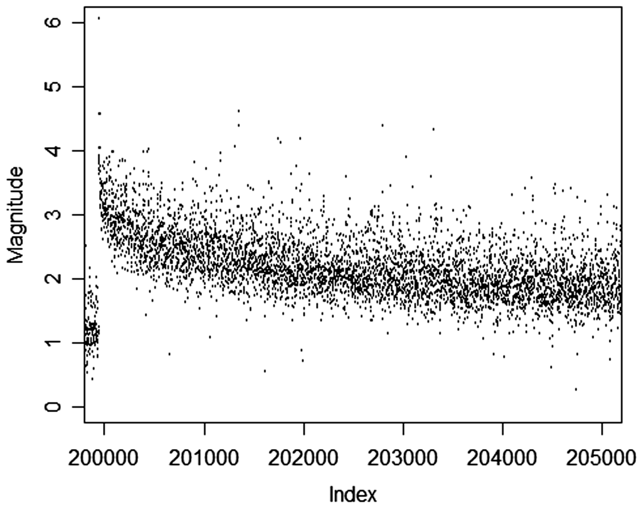

FIG. 1. A slice of the Italian seismic catalog plotting the magnitude versus the index of the event. The effect of short-term incompleteness is clearly recognizable.

produce realistic earthquake statistics. Therefore, we propose a direct correlation analysis of a machine learning high-resolution catalog for the Amatrice-Norcia (2016) sequence in Italy while avoiding biases due to uncertainty about descendants in the triggering phase. In fact, using the stochastic declustering technique [27,28], we assign to each event j a probability of being the offspring of a previous event i, or a background event. By declustering the instrumental catalog, we calculate the effect of correlations on weighting the results based on the probability of the two events being correlated. After establishing the completeness magnitude of the catalog and performing statistical analysis on correlated pairs, we can test whether the magnitude correlation hypothesis can be rejected with a high level of confidence.

The physical model and magnitude correlation.—We implement the two-layer model defined in [26,29,30] and tested in [31] composed by two elastic layers. The upper layer represents the brittle fault and the lower layer represents a ductile substrate in the deeper lithosphere. The aftershocks on the fault are indirectly triggered by the interaction with the second layer. We consider a rectangular fault modeled as a lattice of blocks of size $L_x = 1000$ and $L_y = 400$. The stress acting on the ith block is the sum of two contributions which take into account for the intralayer and interlayer interaction. The friction in the two layers is different, being velocity weakening (modeled as the Coulomb failure criterion) in the brittle layer and velocity strengthening in the ductile layer. It is important to note that the presence of the ductile layer is crucial for triggering aftershocks (and foreshocks). In fact, the limit of zero interaction between the two layers (or simply considering only the brittle layer), yields no occurrence of aftershocks (and foreshocks), moving the exponents of the statistical laws of earthquakes away from universality. For more details on the model, see Ref. [26]. The catalog produced via the two-block model has no issue of completeness and it is easy to distinguish between triggered and background events without resorting to delcustering techniques. These characteristics make it an ideal candidate as a null hypothesis for comparison with the real catalog for testing the correlation between magnitude (Fig. 4). The output seismic catalog we use in this study contains ~5 000 000 events.

Completeness of the Amatrice-Norcia seismic catalog.—The Machine-Learning-Based High-Resolution Earthquake Catalog consists of 885616 events spanning a one year period, based on arrival times derived using a deep neural network-based picker [32]. It is well known that immediately after a large earthquake, many aftershocks cannot be recorded (Fig. 1). The seismic waveforms generated by the aftershocks, many of which occur shortly after the mainshock, overlap with each other and cannot be accurately distinguished. Therefore, catalog completeness is quantified in terms of a minimum threshold m_c defined as the magnitude above which all events are identified and included in the seismic catalog. The value of m_c depends on the level of noise present in the seismic data and on the distance between the earthquake epicenter and the recording seismic stations [33]. Several methods have been proposed to estimate m_c [34–40] but many of these have limitations. To address the problem of calculating m_c, we estimate the completeness magnitude of the catalog by plotting the normalized quantity

$$F_M(t, m|m_{th}) = \sum_{i=1}^{N} \frac{\mathbf{1}(t < t_i, m < m_{th})}{\mathbf{1}(m < m_{th})} \tag{1}$$

where $\mathbf{1}$ is the indicator function and m_{th} is the threshold magnitude chosen for the calculation of the quantity. In Fig. 2 it is easy to see how that for small values of m_c, the curves are distinctly separate, whereas they blur for larger values ($m_c \geq 2$). Neglecting the small noise, a complete collapse of all curves means that the catalog is complete and all occurred events have been recorded. Here, we consider complete the catalog considering only earthquakes with $m > 3$. A comparison with other methods of calculating the magnitude of completeness is presented in the Supplemental Material [41].

Stochastic declustering.—The main weakness of a direct statistical approach in calculating the correlations between magnitudes is that the calculation is performed by ordering the earthquakes chronologically and for a fixed spatial region. Thus there is a nonzero probability that related events occurring close in time are spatially distant. Conversely related events occurring close in space can be separated by a very large time interval. For this reason a simple space-time window selection is not suitable for this kind of study. To overcome this problem we employ the stochastic declustering methodology introduced by [27], with which it is possible to estimate the probability that an

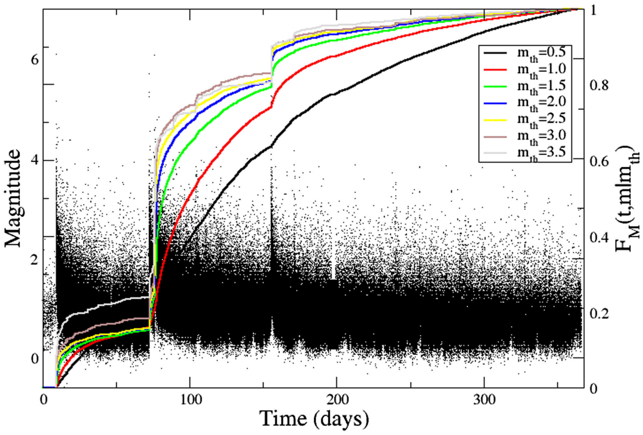

FIG. 2. The Amatrice-Norcia catalog (black dots) and the related $F_M(t, m|m_{th})$ function versus the time (colored lines). Different colors represent a different choice of m_{th} (see the legend).

event is a spontaneous event or is instead triggered by others. We define as ρ_{ij} the probability that an event i is an offspring of an event j. Since we are only interested in understanding whether there is magnitude clustering between the triggering events, we remove all background events from the computation, i.e., all events having ρ_{ij} with $i = j$. After the procedure, we obtain a probability tree among the events. In particular, we built a matrix (i, j, ρ_{ij}), where j is the possible mother of i, ranging from 1 up to the total number of mothers, while i is the index of the possible offspring related to j ranging from 0 up to the total number offsprings. We obtain $\mathrm{Nc} = 706\,266$ combinations of events with magnitude $m \geq 3$.

Correlations of the empirical magnitudes.—Instead of looking at the pairs (m_i, m_j) directly, we estimate the counts $(\mathrm{EM}_i, \mathrm{EM}_j) = [e\mathrm{CDF}_{m_{1:n}}(m_i), e\mathrm{CDF}_{m_{1:n}}(m_j)]$ in the unit square $[0, 1] \times [0, 1]$ on a regular grid weighted considering the probability ρ_{ij} (see Supplemental Material). If there is no magnitude dependence, these points are distributed completely homogeneously in the square unit without any regular patterns [see right panel of Fig. 3] conversely, with correlation between the events an accumulation of points is clear along the diagonal and the bottom-left and right-up corners (see left panel of Fig. 3). To statistically test whether a correlation exists, we can compute a ρ_{ij}-weighted histogram of the differences between EM values, $\Delta_{ij} = \mathrm{EM}_i - \mathrm{EM}_j$ [Figs. 4(a) and 4(b)]. For the null hypothesis of no correlation, Δ_i has a probability density function (PDF) with a triangular shape: $\Delta_i + 1$ if $-1 < \Delta_i < 0$, and $1 - \Delta_i$ if $0 < \Delta_i < 1$ (see Supplemental Material). Conversely, if m_i and m_j are positively correlated, then the PDF of Δ_{ij} will be more concentrated around 0. In Figs. 4(c) and 4(d) the cumulative density function (CDF) of Δ_i is compared with the theoretical one for the null hypothesis. We find that the hypothesis of magnitude dependence is rejected for $m \geq 3$, conversely, for $m < 3$ a concentration of points around 0 is more evident and the hypothesis of magnitude correlation cannot be rejected (see Supplemental Material for the study at minimum magnitudes less than 3). We then justify the observed correlations observed if we consider all the events in the catalog as "spurious" and caused by the lack of events with minor magnitudes not present in the catalog. We conclude that the apparent magnitude dependence that we found in the machine learning Amatrice-Norcia catalog might be due to the missing short-term aftershock and cannot be attributed to a real dependence between magnitudes.

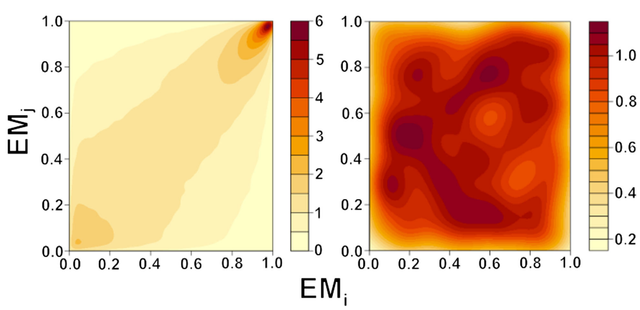

FIG. 3. A biscale empirical transformation (BEPIT) of the quantities $e\mathrm{CDF}(m_i)$ and $e\mathrm{CDF}(m_j)$. Left: all the events with magnitude m greater than 0. Right: all the events with magnitude m greater than 3; here the points are distributed randomly in the square $[0, 1] \times [0, 1]$ and no pattern is recognizable.

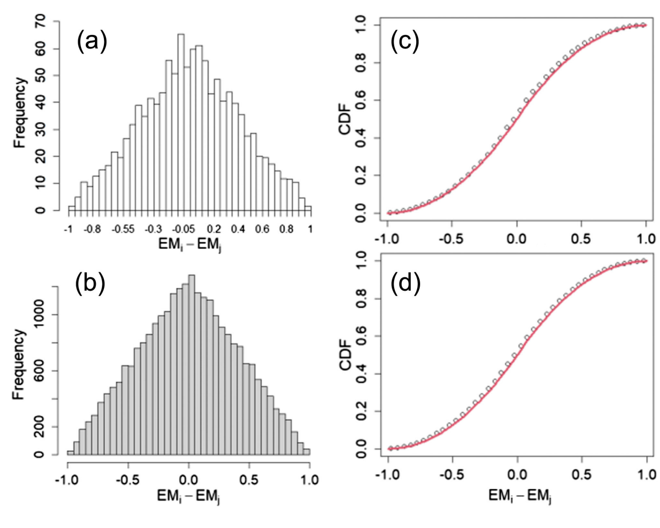

FIG. 4. The probability density function (PDF) of the quantity $\Delta_{ij} = \mathrm{EM}_i - \mathrm{EM}_j$ for the instrumental catalog (a) and for the numerical catalog (b). The triangular shape of the distribution suggests independence between the magnitudes. The cumulative density function (CDF) of $\Delta_{ij} = \mathrm{EM}_i - \mathrm{EM}_j$ (black empty circles) and the theoretical independent CDF (solid red line) for the instrumental catalog (c) and for the numerical catalog (d).

Conclusions.—Resolving the magnitude correlation debate is crucial so that seismologists can focus on developing a next-generation epidemic model for operational earthquake forecasting during such sequences [43]. Moreover, the presence of correlations is also intrinsically linked to greater predictability of a seismic event. In this Letter, we have shown how the correlation between magnitudes is an artifact of the catalog due to the incompleteness of the instrumental catalog caused by the overlapping of seismic waveforms. In addition to what has been performed in the literature, we propose three improvements: (1) we study the correlations on a synthetic catalog produced by a physical model that captures the real statistical features of earthquakes; (2) use a state of the art high-resolution experimental machine learning catalog as input data; and (3) stochastic declustering to be sure of calculating the correlation between the right pairs of events (father and descendants). We want to underline that the proposed ETAS models with magnitude correlation may still perform well, however, it is likely that they do not capture the real process behind it.

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