Strong Backreaction Regime in Axion Inflation

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We study the nonlinear dynamics of axion inflation, capturing for the first time the inhomogeneity and full dynamical range during strong backreaction, till the end of inflation. Accounting for inhomogeneous effects leads to a number of new relevant results, compared to spatially homogeneous studies: (i) the number of extra efoldings beyond slow-roll inflation increases very rapidly with the coupling, (ii) oscillations of the inflaton velocity are attenuated, (iii) the tachyonic gauge field helicity spectrum is smoothed out (i.e., the spectral oscillatory features disappear), broadened, and shifted to smaller scales, and (iv) the nontachyonic helicity is excited, reducing the chiral asymmetry, now scale dependent. Our results are expected to impact strongly on the phenomenology and observability of axion inflation, including gravitational wave generation and primordial black hole production.

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Introduction.—As inflationary constructions are very sensitive to unknown ultraviolet (UV) physics, a promising candidate for an inflaton is an *axionlike particle* that enjoys a shift symmetry. Possible interactions of such inflaton with other species are then very restricted, protecting the inflationary dynamics from unknown UV physics. While several implementations of axion-driven inflation scenarios have been proposed [1–7], we will focus on scenarios where the lowest dimensional shift-symmetric interaction between an inflaton ϕ and a hidden Abelian gauge sector, $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$, is present, with $F_{\mu\nu}$ the field strength of a *dark photon* A_μ, and $\tilde{F}_{\mu\nu}$ its dual. These scenarios are typically referred to as *axion inflation*.

In axion inflation, an exponential production of one of the gauge field helicities is expected during the inflationary period [8–14]. The excited helicity can lead to rich phenomenology such as the production of large density perturbations [12,15–20] and chiral tensor modes [13,15,21–24]. Such perturbations can be probed by the cosmic microwave background (CMB) [12,21,25], searches for primordial black holes (PBHs) [14,18,26–32], and gravitational wave (GW) detection experiments [17,33–35]. In addition, fermion production [36–38], thermal effects [39,40], magnetogenesis [9,10,41,42], baryon asymmetry [43–48], and (p)reheating [49–53] mechanisms, can also be efficiently realized.

Axion inflation dynamics and methodology.—We consider a total action $S_{\rm tot} = S_{\rm g} + S_{\rm m}$, with standard Hilbert-Einstein *gravity* $S_{\rm g} \equiv \int dx^4 \sqrt{-g}\frac{1}{2}m_p^2 R$, and *matter* action $\mathcal{S}_{\rm m} = -\int dx^4\sqrt{-g}\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha_\Lambda}{4}\frac{\phi}{m_p}F_{\mu\nu}\tilde{F}^{\mu\nu}\}$, where m_p is the reduced Planck mass and $1/\Lambda$ the axion-gauge coupling ($\alpha_\Lambda \equiv m_p/\Lambda$). Although our methodology can be applied to arbitrary potentials, in order to compare with results in the literature, we will consider a quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$, with $m/m_p \simeq 6.16 \times 10^{-6}$. The variation of $S_{\rm tot}$, specializing the metric to an isotropic and homogeneous spatially flat expanding background, leads to

$$\ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot\vec{B}, \quad (1)$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{\alpha_\Lambda}{a m_p}(\dot{\phi}\vec{B} - \vec{\nabla}\phi\times\vec{E}), \quad (2)$$

$$\ddot{a} = -\frac{a}{3m_p^2}(2\rho_{\rm K} - \rho_{\rm V} + \rho_{\rm EM}), \quad (3)$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{\alpha_\Lambda}{a m_p}\vec{\nabla}\phi\cdot\vec{B}, \quad (4)$$

$$H^2 = \frac{1}{3m_p^2}(\rho_{\rm K} + \rho_{\rm G} + \rho_{\rm V} + \rho_{\rm EM}), \quad (5)$$

with $\dot{} \equiv \partial/\partial t$, t cosmic time, $a(t)$ the scale factor, $H(t) = \dot{a}/a$, and where we have defined the magnetic field as $\vec{B} \equiv \vec{\nabla}\times\vec{A}$, the electric field (in the temporal gauge $A_0 = 0$) as $\vec{E} \equiv \partial_t\vec{A}$, as well as the electromagnetic $\rho_{\rm EM} \equiv (1/2a^4)\langle a^2\vec{E}^2 + \vec{B}^2\rangle$ and inflaton's kinetic $\rho_{\rm K} \equiv \frac{1}{2}\langle\dot{\phi}^2\rangle$, potential $\rho_{\rm V} \equiv \langle V\rangle$, and gradient $\rho_{\rm G} \equiv (1/2a^2)\langle(\vec{\nabla}\phi)^2\rangle$ homogeneous energy densities, with

$\langle\ldots\rangle$ denoting volume averaging. While (4)–(5) are constraint equations, Eqs. (1)–(3) describe the system dynamics, which can be studied under the following successive levels of approximation.

Linear regime: Deep inside inflation, the impact of the gauge field on the inflationary dynamics is negligible, which allows one to consistently neglect the spatial inhomogeneity of the inflaton. However, as the inflaton slowly rolls its potential (we take $\dot{\phi} < 0$ without loss of generality), the interaction $\dot{\phi}\vec{B}$ in Eq. (2) induces an exponential growth in the photon helicity $A_i^{(+)}$, while $A_i^{(-)}$ remains in vacuum. Such chiral instability is controlled by

$$\xi = -\frac{\langle\dot{\phi}\rangle}{2H\Lambda}, \tag{6}$$

so that the gauge field spectrum develops a bump with exponentially growing amplitude, tracking the Hubble scale around $(k/aH) \sim (1/\xi)$, for $\xi \gtrsim 1$ [11]. The linear regime eventually breaks down when the gauge field backreacts on the system, turning the overall dynamics nonlinear. The larger the value of α_Λ, the earlier the gauge field backreacts on the dynamics.

Homogeneous backreaction: In this approximation, the backreaction of the gauge field is considered while enforcing the inflaton to remain homogeneous. This is achieved by neglecting the terms $\propto \vec{\nabla}^2\phi$, $\vec{\nabla}\phi \times \vec{E}$, and $\langle(\vec{\nabla}\phi)^2\rangle$ in Eqs. (1), (2), and (5), respectively, while promoting, for consistency, $\vec{E}\cdot\vec{B} \to \langle\vec{E}\cdot\vec{B}\rangle$, in Eq. (1). Although this regime was originally tackled only approximately, assuming $\dot{\phi}$ as constant, such limitation was later on surpassed by two methods: (i) solving self-consistently the resulting integrodifferential iterative equations [14,19,54–56], and (ii) solving the time evolution of the relevant bilinear electromagnetic functions in a *gradient expansion* formalism [57,58]. The two improved methods reached similar conclusions: once backreaction becomes relevant, a resonant enhancement of the helical gauge field production is observed, resulting in oscillatory features in the inflaton velocity, as well as in the gauge field spectrum [14,19,54–56]. This was later understood as due to the time delay between the maximum excitation rate of $A_i^{(+)}$ at slightly sub-Hubble scales, and its backreaction onto the inflaton, dominated by slightly super-Hubble modes [19,56].

We remark that in the homogeneous backreaction picture, the gauge field remains *maximally helical* (i.e., only $A_i^{(+)}$ is exponentially excited), and inflation is sustained for a number of extra efoldings $\Delta\mathcal{N}_{\rm br}$ beyond the would be end of (inflaton driven) slow-roll inflation.

Inhomogeneous backreaction: In order to address correctly the nonlinear dynamics, we need to solve Eqs. (1)–(3) fully maintaining spatial inhomogeneity, restoring all inflaton gradient terms, and using the local expression of $\vec{E}\cdot\vec{B}$ for the backreaction. For this, we have implemented in CosmoLattice (CL) [59,60] a lattice version of Eqs. (1)–(5), following the lattice gauge-invariant and shift-symmetric formalism of Refs. [51,61] (see also Refs. [62–64]). We use a second order Runge-Kutta time integrator to evolve Eqs. (1)–(3), monitoring that the constraints [Eqs. (4) and (5)] are always verified to better than $\mathcal{O}(10^{-4})$. Details on our lattice formulation can be found in the Supplemental Material [65] and in Ref. [66]. For an alternative nonshift-symmetric lattice formulation, see Refs. [20,67].

We start our simulations in the linear regime, with all comoving modes captured between the infrared (IR) and UV lattice cutoff scales, $k_{\rm IR} \le k \le k_{\rm UV}$, well inside the initial comoving Hubble radius $1/aH$. By setting initially $k_{\rm IR}/(aH) \simeq 10$, all gauge field modes of both helicities are initialized in a *Bunch-Davies* (BD) quantum vacuum state $A^{(\pm)} \simeq e^{ik/aH}/\sqrt{2k}$. The initial fluctuations serve as a seed for the tachyonic instability of $A_i^{(+)}$: as the modes approach the Hubble scale, their amplitude starts growing exponentially. In order to capture the dynamics correctly, we first solve, in the lattice, the linear regime of the gauge field, up to a given cutoff $k < k_{\rm BD}$, with $k_{\rm IR} \ll k_{\rm BD} \ll k_{\rm UV}$. We let the most IR modes grow until they dominate over the BD tail within the range $k_{\rm IR} \le k < k_{\rm BD}$. Then, we *switch* to evolve the nonlinear Eqs. (1)–(3), allowing all fields to be excited in the full lattice range $k \in [k_{\rm IR}, k_{\rm UV}]$. After the switch, the system still remains in the linear regime for a while, until the backreaction of the gauge field becomes noticeable on both the inflaton and the expansion dynamics. From that moment the system dynamics becomes fully nonlinear, entering, for sufficiently large couplings, into the strong backreaction regime.

Results.—We present our study on the strong backreaction regime, which requires $\alpha_\Lambda \gtrsim 15$, capturing the inhomogeneity and full dynamical range of the system, until the end of inflation. A detailed description of our procedure and results will be presented in Ref. [66].

We list our run parameters in Table I, where N is the number of lattice sites per dimension, $\tilde{L} = mL$ the comoving lattice length, $\kappa_{\rm UV} = k_{\rm UV}/m$ the lattice UV scale, $\kappa_{\rm BD}$ the BD cutoff scale (set by trial and error), $\mathcal{N}_{\rm start}$ the number of efolds before the end of slow-roll inflation (marked as $\mathcal{N} = 0$) when we start our simulation, and $\mathcal{N}_{\rm switch}$ the moment when all inhomogeneous terms are activated. For convenience we set $a = 1$ at $\mathcal{N} = 0$.

Our results are summarized by a series of figures, where we compare the outcome of our simulations for the linear,

TABLE I. Parameters used in the simulations.

	N	$\tilde{L}$	$\kappa_{\rm UV}$	$\kappa_{\rm BD}$	$\mathcal{N}_{\rm start}$	$\mathcal{N}_{\rm switch}$
$\alpha_\Lambda = 15$	640	32.524	106.981	46	−4.5	−1.1
$\alpha_\Lambda = 18$	1600	32.524	267.594	10	−4.5	−1.8
$\alpha_\Lambda = 20$	2340	50.971	170.746	9	−5	−2.4

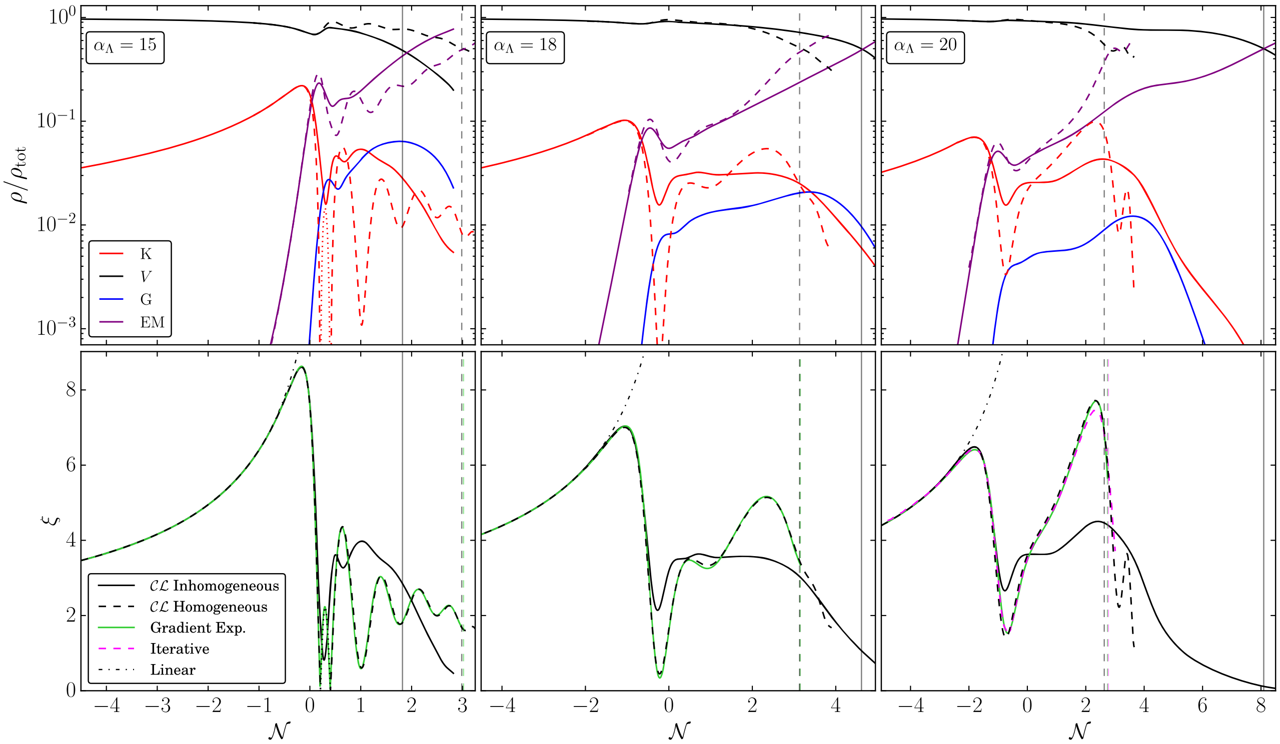

FIG. 1. Top: evolution of the electromagnetic (purple) and inflaton potential (black), kinetic (red) and gradient (blue) energy densities, all normalized to the total energy density of the system, for $\alpha_\Lambda = 15$, 18, 20. Solid (dashed) lines correspond to lattice simulations with inhomogeneous (homogeneous) backreaction. Bottom: evolution of ξ for the same coupling constants, corresponding to simulations with inhomogeneous (black solid) and homogeneous (black dashed) backreaction, and to gradient expansion [58,68] (green solid) and iterative method [19] (magenta dashed, only for $\alpha_\Lambda = 20$). Solid and dashed vertical lines signal the end of inflation in each case. Evolution in the linear regime (black dash dotted) is also shown for completeness.

homogeneous backreaction, and inhomogeneous backreaction regimes. In the top panel of Fig. 1, we plot the evolution of the electromagnetic and inflaton's kinetic, gradient, and potential homogeneous energy densities (normalized by the total energy density), whereas in the bottom panel, we show the evolution of ξ. In both panels we show, for each coupling considered, the system evolution as a function of the number of efoldings $\mathcal{N}$, from the initial moment of the simulation in the linear regime, till the end of inflation in the strong backreaction regime. While $\mathcal{N} = 0$ signals the end of slow-roll inflation, the dashed and solid vertical lines indicate the end of inflation, identified as $\epsilon_H \equiv -\dot{H}/H^2 = 1$, according to the homogeneous and inhomogeneous backreaction regimes, respectively. Whenever possible, we compare with the outcome from the gradient expansion formalism [58,68] and from the iterative method [19]. Incidentally, our code reproduces accurately the linear and homogeneous backreaction regimes in their corresponding limits, confirming the validity of the code.

We define the power spectrum of the gauge field as $\Delta_A^{(\lambda)}(k,t) \equiv (k^3/2\pi^2)\mathcal{P}_A^{(\lambda)}(k,t)$, where $\langle \vec{A}^{(\lambda)}(\vec{k},t)\vec{A}^{(\lambda')*}(\vec{k}',t)\rangle \equiv (2\pi)^3\mathcal{P}_A^{(\lambda)}(k,t)\delta_{\lambda\lambda'}\delta_{\rm D}(\vec{k}-\vec{k}')$ represents an ensemble average. In Fig. 2 we plot various power spectra for a fiducial value $\alpha_\Lambda = 18$, and compare the outcome of our inhomogeneous treatment against the solutions of the homogeneous backreaction and linear regimes. In Fig. 3 we also show the helicity imbalance measured through a *normalized spectral helicity* observable defined as

$$\mathcal{H}(k,t) \equiv \frac{\Delta_A^{(+)} - \Delta_A^{(-)}}{\Delta_A^{(+)} + \Delta_A^{(-)}}. \quad (7)$$

The inhomogeneous terms bring considerable novelties into the dynamics: (1) The gauge energy $\rho_{\rm EM}$ grows exponentially fast during the linear regime, until it reaches a *few* percentages of $\rho_{\rm K}$. The latter, that had been previously slowly growing on a slow-roll trajectory, starts then decreasing, signaling the onset of backreaction. In the homogeneous case, $\rho_{\rm EM}$ and $\rho_{\rm K}$ may perform some large oscillations [19,56], almost in opposite phase. Such oscillations are however damped in the inhomogeneous dynamics, where the gradient energy $\rho_{\rm G}$ is also significantly excited, with its contribution potentially comparable or even higher than $\rho_{\rm K}$. This could never be captured in the homogeneous regime, where by construction $\rho_{\rm G} = 0$. In the homogeneous case, for some couplings (e.g., $\alpha_\Lambda = 15$) the first and largest oscillation leads $\langle\dot{\phi}\rangle$ to even flip its sign, with ξ crossing zero back and forth (depicted in the figure

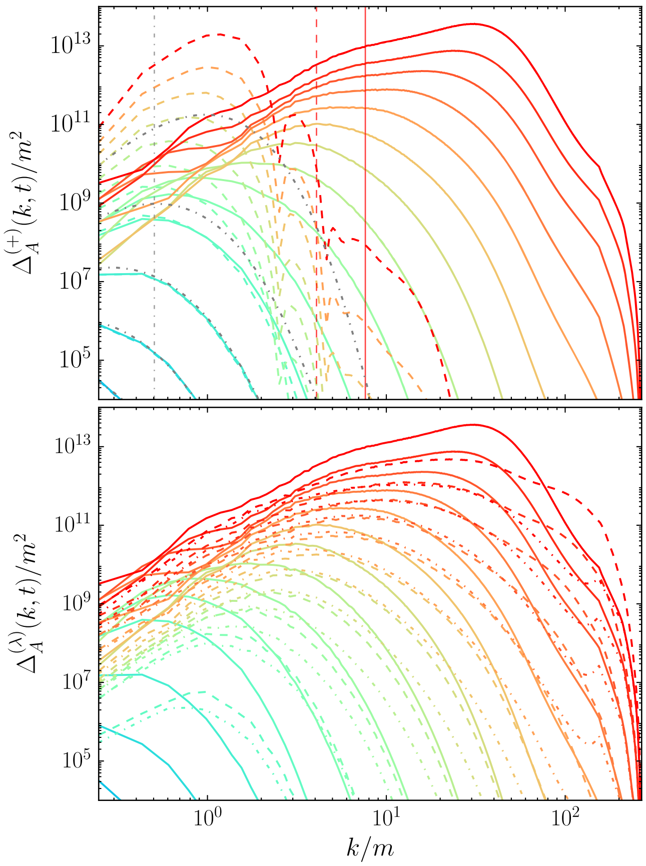

FIG. 2. Evolution of the gauge field power spectra for $\alpha_\Lambda = 18$. Top: $\Delta_A^{(+)}(k,t)$ spectra from simulations in the linear regime (gray dash-dotted lines), and with homogeneous (dashed lines) and inhomogeneous (solid lines) backreaction. Vertical lines represent the comoving Hubble scale at the end of inflation in each case. Bottom: different gauge polarization power spectra from a simulation with inhomogeneous backreaction: $\Delta_A^{(+)}(k,t)$ (solid lines), $\Delta_A^{(-)}(k,t)$ (dash-dotted lines), and $\Delta_A^{(\mathrm{L})}(k,t)$ (dashed lines). In all panels, lines are separated by $\Delta\mathcal{N} = 0.5$ from earlier times to later ones, from colder to hotter, except in the linear regime. The reddest color corresponds to the end of inflation for each case.

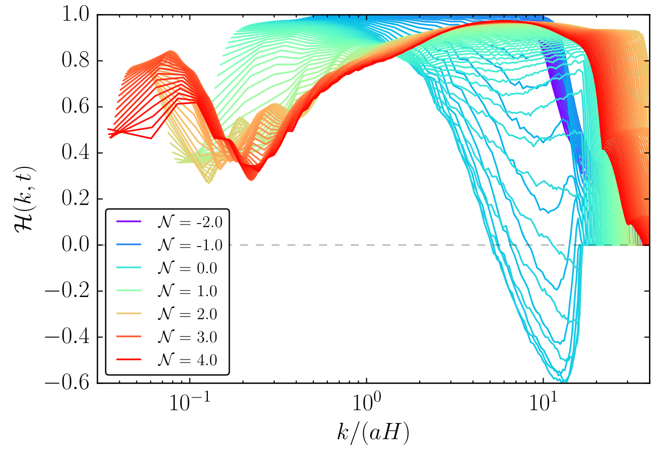

FIG. 3. Nonlinear evolution of the *normalized spectral helicity* as defined in Eq. (7) vs $k/(aH)$ for $\alpha_\Lambda = 18$. Color coding goes from earliest (colder) to latest (hotter) times in the simulation. We start plotting from $\mathcal{N}_{\mathrm{switch}}$ onward, and the separation between different lines is $\Delta\mathcal{N} = 0.05$ efoldings.

by dotted lines), signaling that the inflaton climbs its own potential. This, however, never happens in the inhomogeneous case, where the growth of ρ_{G} damps the oscillation amplitude, and prevents ξ from becoming negative. (2) For all couplings considered, inflation ends when ρ_{EM} becomes comparable to ρ_{V}, resulting in a reheated Universe at that moment, which is actually consistent with previous preheating studies for $\alpha_\Lambda \lesssim 15$ [49–53]. In the homogeneous case, the number of extra efoldings is $\Delta\mathcal{N}_{\mathrm{br}} \approx 3$ for all couplings considered. In contrast, in the inhomogeneous dynamics, the number of extra efoldings grows strongly and monotonically with α_Λ, from $\Delta\mathcal{N}_{\mathrm{br}} \approx 2$ for $\alpha_\Lambda = 15$ to $\Delta\mathcal{N}_{\mathrm{br}} \approx 8$ for $\alpha_\Lambda = 20$. The larger the coupling, the earlier backreaction happens (i.e., ρ_{EM} surpassing ρ_{K}), roughly at the same time in both approaches. In the inhomogeneous case, the earlier the crossover happens, the longer inflation is prolonged in a quasi-de Sitter regime dominated by ρ_V and ρ_{EM}. (3) In the linear regime, the power spectrum of the unstable helicity $\Delta_A^{(+)}(k,t)$ develops an exponentially growing peak, tracking the Hubble scale at $k/a \sim H/\xi$. However, as the top panel of Fig. 2 shows, the shape of the power spectra changes considerably when backreaction is considered. In the homogeneous case (dashed), the spectrum peak grows resonantly in amplitude once backreaction starts, but shifts mildly its (slightly) superhorizon position reached at the onset of backreaction. The spectrum also develops an oscillatory pattern at scales around the Hubble radius in its UV tail. In the inhomogeneous case (solid), on the contrary, oscillatory features are never imprinted in the spectrum, which now spreads power into UV scales, shifting gradually its peak to smaller (slightly subhorizon) scales, as inflation carries on. As a result, the spectrum becomes smoother and wider. The homogeneous and inhomogeneous spectra demonstrate that the two approaches capture very different physics. (4) The bottom panel of Fig. 2 features another new result. As the inflaton gradients are developed, the terms $\propto \vec{\nabla}\phi \times \vec{E}$ in Eq. (2) drive the excitation of the *longitudinal* mode $A_i^{(\mathrm{L})}$, as well as of the other circular polarization $A_i^{(-)}$, which had previously remained in vacuum. Furthermore, the term $\propto \dot{\phi}\vec{B}$ also contributes to stimulate $A_i^{(-)}$, thanks to the inhomogeneity of $\dot{\phi}$. When we switch our simulations to an evolution with Eqs. (1)–(3), $A_i^{(\mathrm{L})}$ and $A_i^{(-)}$ start with a nonvanishing amplitude much smaller than $A_i^{(+)}$. However, toward the end of inflation, once strong backreaction is at play, $A_i^{(-)}$ and $A_i^{(\mathrm{L})}$ become comparable to (when not larger than) $A_i^{(+)}$, depending on the scale. To quantify this result, we plot in Fig. 3 the spectral helicity [cf. Eq. (7)] for a fiducial $\alpha_\Lambda = 18$. Whereas in the homogeneous case the gauge field excitation is maximally chiral [$\mathcal{H}(k,t) = 1$], this is no longer the case when inhomogeneities are allowed. For instance, Fig. 3 shows that at the end of inflation, $\Delta_A^{(+)} \approx 3\Delta_A^{(-)}$ [i.e., $\mathcal{H}(k,t) \approx 1/2$] at slightly super-Hubble scales. Remarkably, the evolution around $\mathcal{N} \sim 0$ shows that $A_i^{(-)}$ dominates over $A_i^{(+)}$ at $k/a \sim 10H$, with $\mathcal{H}(k,t) \gtrsim -1/2$. We shall discuss further the excitation mechanism of $A_i^{(\mathrm{L})}$ and $A_i^{(-)}$ in Ref. [66]. We note that analogous helicity restoration effects at subhorizon scales have also been reported in preheating studies [41,49], for the milder coupling regime $9 \lesssim \alpha_\Lambda \lesssim 14$.

Discussion.—Observable CMB scales leave the Hubble radius during inflation, when the gauge dynamics is well described by the linear regime, and backreation is negligible. Backreaction becomes typically important toward the end of inflation, when large tensor [13,15,21–24,33–35] and scalar [12,15–18,20,26–32] perturbations can be generated. These can lead to potentially observable quantities, such as a population of PBHs and a stochastic background of GWs, both crucial predictions to probe axion inflation scenarios. Therefore, it is of the utmost importance to describe correctly the system dynamics when backreaction cannot be neglected.

In this Letter we report the results of using a gauge-invariant and shift-symmetric lattice formalism, capturing for the first time the inhomogeneity and full dynamical range during strong backreaction, till the end of inflation. We explore the parameter space $\alpha_\Lambda \gtrsim 15$, which has never been studied during the whole inflationary period while incorporating inhomogeneous effects. Such large coupling regime is crucial to understanding the generation of scalar perturbations during inflation, which later on lead to PBH formation. While GW production during preheating constrains the coupling down to $\alpha_\Lambda \lesssim 15$ [52,53], this depends on the details of the last stages of inflation and of a potential early PBH dominated phase ensued after inflation [19]. As the strong backreaction inflationary phenomenology uncovered in our work is (likely) expected to affect this limit, the exploration of couplings beyond current preheating bounds becomes well justified and crucial to understanding observational constraints of axion inflation.

One of the most relevant aspects of our results is the observed "*exponential UV sensitivity*" of the dynamics to small coupling increments. As longer inflationary periods emerge for larger couplings, successively smaller scales need to be resolved. Our simulation data show that when UV scales are not properly resolved, neither the width nor the peak location of the gauge spectra are well obtained (a detailed IR-UV lattice study to highlight this aspect will be presented in Ref. [66]). A simultaneous capture of IR and UV scales is required: this is why we limited our current study to $\alpha_\Lambda \leq 20$, as $\alpha_\Lambda = 20$ already required $N > 2300$ sites/dimension to capture correctly all IR-UV scales. Our results show that a correct description of the dynamics can only be provided if inhomogeneities are completely resolved at all scales of interest. In this respect, we notice that the study of the strong backreaction regime for $\alpha_\Lambda = 25$ by Ref. [20], given the lattice sizes reported, cannot capture the full dynamical range required to characterize the nonlinear dynamics until the end of inflation.

To summarize, we stress that the effect of the inhomogeneity is highly nontrivial and requires a dedicated study for each coupling. In general, the excitation and backreaction of the gauge field is no longer controlled by a homogeneous ξ parameter, and resonant oscillatory backreaction features reported by previous homogeneous analyses [19,56,58], are quite attenuated. The resulting gauge field spectra during inhomogeneous backreaction become smoother than in the homogeneous case, as no spectral oscillatory features are developed. Furthermore, gauge spectra become wider, spreading power into shorter scales, as the peak spectrum trails the Hubble scale during the $\Delta\mathcal{N}_{\mathrm{br}}$ extra efoldings, which grows very strongly with the coupling.

We conclude that the novelties of consistently taking into account the inhomogeneity of the system during strong backreaction will inevitably have an impact on the properties of the scalar and tensor perturbations derived considering homogeneous backreaction, e.g., Refs. [19,23,24]. Furthermore, the completely new feature of scale-dependent gauge chirality makes the possibility of probing these scenarios through their observational windows even more interesting. The observability and phenomenology of axion inflation scenarios will require a complete revision of the state-of-the-art predictions, which we plan to address in future work.

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