Nonclassical Advantage in Metrology Established via Quantum Simulations of Hypothetical Closed Timelike Curves

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(Received 3 September 2022; revised 8 September 2023; accepted 11 September 2023; published 12 October 2023)

We construct a metrology experiment in which the metrologist can sometimes amend the input state by simulating a closed timelike curve, a worldline that travels backward in time. The existence of closed timelike curves is hypothetical. Nevertheless, they can be simulated probabilistically by quantum-teleportation circuits. We leverage such simulations to pinpoint a counterintuitive nonclassical advantage achievable with entanglement. Our experiment echoes a common information-processing task: A metrologist must prepare probes to input into an unknown quantum interaction. The goal is to infer as much information per probe as possible. If the input is optimal, the information gained per probe can exceed any value achievable classically. The problem is that, only after the interaction does the metrologist learn which input would have been optimal. The metrologist can attempt to change the input by effectively teleporting the optimal input back in time, via entanglement manipulation. The effective time travel sometimes fails but ensures that, summed over trials, the metrologist's winnings are positive. Our Gedankenexperiment demonstrates that entanglement can generate operational advantages forbidden in classical chronology-respecting theories.

DOI: 10.1103/PhysRevLett.131.150202

Introduction.—The arrow of time makes gamblers, investors, and quantum experimentalists perform actions that, in hindsight, are suboptimal. Examples arise in quantum metrology, the field of using nonclassical phenomena to estimate unknown parameters [1]. The optimal input states and final measurements are often known only once the experiment has finished [2]. Below, we present a Gedankenexperiment that circumvents this problem via entanglement manipulation.

A common metrological goal is to estimate the strength of a weak interaction between a system in a state $|\phi\rangle$ and a probe in a state $|\psi\rangle$. The interaction strength can be estimated from the data from several measured probes. Upon measuring probes at too high an intensity, detectors can saturate—cease to function until given time to reset [3–8]. Additionally, one might lack the memory needed to store all the probes [9], or lack the computational power needed to process the probes' contents after their measurement [10]. Reducing the number of probes measured is therefore often advantageous [4–7,11–14]. In such situations, one can use weak-value amplification to boost the amount of information obtained per measured probe [4,5,11,12,15–17]. In weak-value amplification, the system is initialized in a state $|\phi_{\rm i}\rangle$, the system interacts with the probe, and then the system is measured. If, and only if, the system's measurement outcome corresponds to $|\phi_{\rm f}\rangle$, the probe is measured. Successful preselection and postselection guarantees that the probe carries a large amount of information. Weak-value amplification stems from genuine nonclassicality, as reviewed below [18–20]. The nonclassicality originates in both the postinteraction measurement and the initialization, sparking discussions about chronology-violating physics [21,22].

Chronology-violating physics includes closed timelike curves (CTCs) [23–29]. A CTC is a hypothetical spacetime worldline that loops backward in time (Fig. 1). Particles that follow CTCs could travel backward in time with respect to chronology-respecting observers. Although allowed by general relativity, CTCs lead to logical paradoxes. A famous example is the grandfather paradox: A time traveler travels back in time to kill her grandfather, before he fathers any children, such that the time traveler could never have been born…. Such inconsistency can

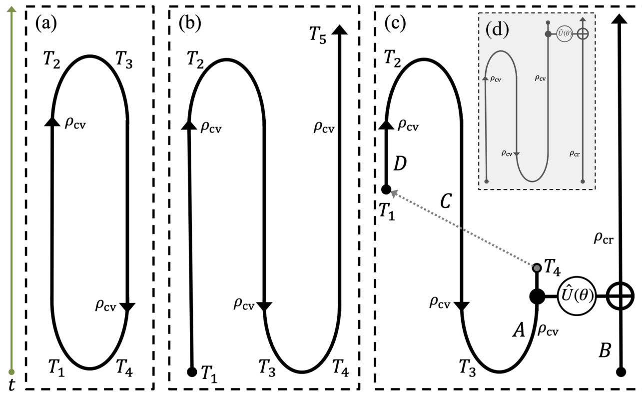

FIG. 1. Examples of chronology-violating particles traversing hypothetical CTCs. ρ_{cv} denotes such particles' states. Time t, experienced by a chronology-respecting observer, runs from bottom to top. The time-traveling particle experiences time T. (a) Closed loop. (b) ρ_{cv} returns to its past and then travels forward in time again. (c) CTC interpretation of the successful trials of our Gedankenexperiment. ρ_{cv} is created at T_1 and travels forward in time until T_2. Then, it reverses temporal direction and travels backward in time until reaching T_3. After that, it again travels forward in time. ρ_{cv} then interacts with a chronology-respecting state, ρ_{cr}, and is subsequently destroyed, prior to ρ_{cv}'s creation (T_1). For comparison, the inset (d) depicts the standard teleportation, across space, of a quantum state needed as input for an interaction.

characterize classical CTCs and has prompted scientific discussions about CTCs' likelihood of existing [25,29–33]. Two competing theories resolve such paradoxes, self-consistently reconciling general-relativistic CTCs with quantum theory [25–29,34,35]. We use the theory of postselected CTCs (PCTCs). PCTCs are equivalent to quantum circuits that involve postselection, or conditioning on certain measurement outcomes [29]. Such circuits have been realized experimentally [28]. Our results concern postselected circuits that achieve weak-value amplification.

In this work, we show that postselected quantum-teleportation circuits can effectively send useful states from the future to the past, providing access to nonclassical phenomena in quantum metrology. We propose a weak-value-amplification Gedankenexperiment for estimating the strength of an interaction between a system and a probe. Motivated by the aforementioned practical limitations, the figure of merit is the amount of information obtained per probe. As mentioned earlier, this rate can be nonclassically large if one discards the probe conditionally on an earlier measurement of the system. But this information distillation requires that the systems be initialized in a specific state. In our Gedankenexperiment, the optimal input state is unknown until after the system has been measured. We circumvent this challenge via quantum theory's ability to simulate backward time travel: One can effectively teleport the optimal state from the experiment's end to its beginning. The simulated time travel sometimes fails, but at no detriment to the figure of merit, the amount of information gleaned from the remaining probes. These probes, retained only if the simulated time travel succeeds, carry amounts of information impossible to achieve classically. Thus, in weak-value amplification, the system can be initialized *after* the system-probe interaction—paradoxically, in chronology-respecting theories. Our conceptual results pinpoint a deep connection between entanglement and effectively retrocausal correlations that enable nonclassical advantages.

Background: Closed timelike curves.—Figure 1 shows examples of CTCs—hypothetical spacetime worldlines that loop in the direction of time. Two (primary) theories entail self-consistent quantum descriptions of CTCs. The first theory is called Deutsch's CTCs (DCTCs) [25,27]. DCTCs conserve a time traveler's state but not the state's correlations (e.g., entanglement) with chronology-respecting systems.

We use a second model: PCTCs [26–29,34,35], which cast CTCs as quantum communication channels to the past [29]. The following condition defines PCTCs: Consider measuring a system that undergoes a PCTC. Whether the measurement happens before or after the PCTC does not affect the measurement statistics. Such self-consistency follows from modeling CTCs with quantum-teleportation circuits (quantum-communication channels) that involve postselection. The postselection ensures that time-traveling particles preserve their correlations with chronology-respecting systems.

Quantum circuits with entangled inputs can effectively realize PCTCs, as illustrated in Fig. 1. (The word "effectively" is used because one cannot empirically prove whether time travel *actually* happened [27].) There, the $\cup$ depicts the creation of a Bell (maximally entangled) state [36]. The $\cap$ depicts the future postselection on that Bell state. In Fig. 1(a), the two entangled particles can be viewed as the forward-traveling (left) and backward-traveling (right) parts of one chronology-violating particle's worldline.

The CTC in Fig. 1(b) can be simulated by a three-qubit quantum-teleportation circuit. With probability $1/4$, the to-be teleported qubit appears at the receiver's end, without the receiver's performing any local operation [37]. In these events, the teleported qubit was already at the receiver's end (see Ref. [39]) [26,28,38]. Postselected on these outcomes, the circuit can be viewed as mimicking one chronology-violating qubit's worldline. In the qubit's rest frame, the qubit is initialized at T_1. At T_2, it starts traveling backward according to the laboratory frame, until reaching the point of its "birth" at T_3. At T_4, the qubit reverses its temporal direction again, returning to traveling forward in time.

We do not argue for or against the physical existence of PCTCs. Rather, we identify a consequence of quantum theory's ability to simulate PCTCs: a counterintuitive metrological advantage achievable with entanglement. Below, we outline a Gedankenexperiment that achieves this advantage. First, we review weak-value amplification in quantum metrology.

Weak-value amplification for metrology.—We now describe how to estimate the strength of a weak system-probe interaction. Weak-value amplification concentrates information, boosting the amount of information obtained per probe.

Using quantum metrology, one infers the value of an unknown parameter θ by measuring N copies of a state $|\Psi_\theta\rangle$ [1]. Every such procedure implies an estimator θ_e of θ. The Cramér-Rao inequality lower-bounds the precision of every unbiased θ_e:

$$\mathrm{Var}(\theta_e) \geq \frac{1}{N \cdot \mathcal{I}_q(\theta|\Psi_\theta)}. \tag{1}$$

$\mathcal{I}_q(\theta|\Psi_\theta)$ is the quantum Fisher information, which quantifies the average amount of information learned about θ per optimal measurement [40]. The quantum Fisher information has the form

$$\mathcal{I}_q(\theta|\Psi_\theta) = 4\langle\dot{\Psi}_\theta|\dot{\Psi}_\theta\rangle - 4|\langle\Psi_\theta|\dot{\Psi}_\theta\rangle|^2, \tag{2}$$

where $\dot{x} \equiv \partial x/\partial\theta$. Common estimators saturate Eq. (1) when N is large. The larger $\mathcal{I}_q(\theta|\Psi_\theta)$ is, the more precisely one can estimate θ.

In this work, we consider estimating the strength of an interaction $\hat{U}(\theta) = e^{-i\theta\hat{\Pi}_a\otimes\hat{B}/2}$ between a system qubit in a state $|\phi\rangle_A$ and a probe qubit in a state $|\psi\rangle_B$ [41]. Here, $\theta \approx 0$ is the weak-coupling strength, and $\hat{\Pi}_a \equiv |a\rangle\langle a|$ denotes a rank-1 projector on qubit A's Hilbert space. $\hat{B} \equiv |b^+\rangle\langle b^+| - |b^-\rangle\langle b^-|$ is a Hermitian operator acting on qubit B's Hilbert space, with eigenvalues ± 1. $\hat{U}(\theta)$ evolves $|\psi\rangle_B$ with a unitary evolution generated by $\hat{B}$, conditionally on qubit A's being in the state $|a\rangle$.

To measure the coupling strength θ, we prepare the system-and-probe state $|\Psi_0\rangle_{A,B} \equiv |\phi\rangle_A|\psi\rangle_B$, evolve it under $\hat{U}(\theta)$, and then measure the qubits. An information-optimal input is $|\Psi_0^\star\rangle_{A,B} = |a\rangle_A(1/\sqrt{2})(|b^+\rangle_B + |b^-\rangle_B)$; this state acquires the greatest possible quantum Fisher information, being maximally sensitive to changes in θ. The post-interaction state is

$$|\Psi^\star(\theta)\rangle_{A,B} \equiv \hat{U}(\theta)|\Psi_0^\star\rangle_{A,B} = |a\rangle_A \frac{e^{-i\theta/2}|b^+\rangle_B + e^{i\theta/2}|b^-\rangle_B}{\sqrt{2}}. \tag{3}$$

According to Eq. (2), the average measurement yields $\mathcal{I}_q[\theta|\Psi^\star_{A,B}(\theta)] = 1$ unit of Fisher information per post-interaction state.

Usefully, one can distill much information into few probes. One measures system A and, conditionally on

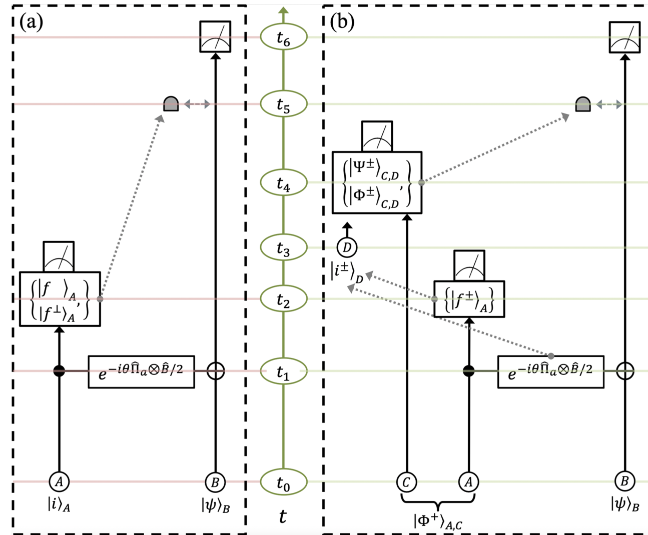

FIG. 2. Circuit diagrams for (a) standard and (b) PCTC-powered weak-value amplification. Time progresses in the laboratory's rest frame as one proceeds upward along the central, vertical axis. Black lines represent qubits. Dashed gray lines represent classical information.

the outcome, discards or keeps (postselects) the probe B. Information distillation is particularly advantageous if one's detectors saturate, if memory constraints limit one's data storage, or if computational resources limit one's postprocessing power. Then, qubit B merits measuring only if B carries much information [3–8]. We now review one such distillation scheme, weak-value amplification [4,5,11,12,15–17], depicted in Fig. 2(a).

One evolves $|\Psi_0^w\rangle_{A,B} \equiv |i\rangle_A(1/\sqrt{2})(|b^+\rangle_B + |b^-\rangle_B)$ under $\hat{U}(\theta)$, then measures A in the basis $\{|f\rangle, |f^\perp\rangle\}$. If the outcome is ${}_A\langle f|$, the blocker in Fig. 2(a) is removed, and B is measured. If not, the blocker destroys B. The postselected state is

$$|\Psi^{\mathrm{PS}}(\theta)\rangle_B = |\psi^{\mathrm{PS}}(\theta)\rangle_B/\sqrt{p_\theta^{\mathrm{PS}}}, \tag{4}$$

where $|\psi^{\mathrm{PS}}(\theta)\rangle_B \equiv ({}_A\langle f| \otimes \hat{1}_B)\hat{U}(\theta)|\Psi_0^w\rangle_{A,B}$. The probability of postselecting successfully is $p_\theta^{\mathrm{PS}} \equiv {}_B\langle\psi^{\mathrm{PS}}(\theta)|\psi^{\mathrm{PS}}(\theta)\rangle_B$. A little algebra simplifies the postselected state, if $|\theta \cdot {}_f\langle\hat{\Pi}_a\rangle_i| \ll 1$:

$$|\Psi^{\mathrm{PS}}(\theta)\rangle_B = \frac{e^{-i\theta_f\langle\hat{\Pi}_a\rangle_i/2}|b^+\rangle_B + e^{i\theta_f\langle\hat{\Pi}_a\rangle_i/2}|b^-\rangle_B}{\sqrt{2}} + \mathcal{O}(\theta^2). \tag{5}$$

The *weak value* of $\hat{\Pi}_a$ is

$${}_f\langle\hat{\Pi}_a\rangle_i \equiv \frac{{}_A\langle f|\hat{\Pi}_a|i\rangle_A}{{}_A\langle f|i\rangle_A}, \tag{6}$$

the "expectation value" of $\hat{\Pi}_a$ preselected on the state $|i\rangle_A$ and postselected on ${}_A\langle f|$. The quantum Fisher information [Eq. (2)] of $|\Psi^{\rm PS}(\theta)\rangle_B$ is

$$\mathcal{I}_{\rm q}(\theta|\Psi_B^{\rm PS}(\theta)) = \left|{}_f\langle\hat{\Pi}_a\rangle_i\right|^2 + \mathcal{O}(\theta). \tag{7}$$

Above, we found that the nonpostselected experiment's quantum Fisher information, $\mathcal{I}_{\rm q}(\theta|\Psi_{A,B}(\theta))$, has a maximum value of 1. The postselected experiment, however, can achieve a quantum Fisher information $\mathcal{I}_{\rm q}(\theta|\Psi_B^{\rm PS}(\theta)) \gg 1$. Weak-value amplification does not increase the total amount of information gained from all the probes [42,43] but distills large amounts of information into a few postselected probes.

Such anomalously large amounts of information witness nonclassical phenomena [6,7,13,44]. For small θ, Eq. (7) exceeds 1 if, and only if, the weak value $|{}_f\langle\hat{\Pi}_a\rangle_i| > 1$, i.e., the weak value's magnitude exceeds the greatest eigenvalue of $\hat{\Pi}_a$. Such a weak value is called anomalous. Anomalous weak values arise from the quantum resource contextuality: One can try to model quantum systems as being in real, but unknown, microstates like microstates in classical statistical mechanics. In such a framework, however, operationally indistinguishable quantum procedures cannot be modeled identically. This impossibility is contextuality [19,20,45,46], which is valuable. It enables weak-value amplification, which compresses many probes' metrological information into a few highly informative probes.

Metrological quantum advantage via PCTC simulation.—To perform weak-value amplification, an experimentalist must carefully choose qubit A's input state, $|i\rangle_A$, and final-measurement basis, $\{|f\rangle_A, |f^\perp\rangle_A\}$, such that $|{}_f\langle\hat{\Pi}_a\rangle_i| > 1$. Doing so requires knowledge of $\hat{\Pi}_a$. The goal is to simultaneously achieve a small weak-value denominator ${}_A\langle f|i\rangle_A$ and large numerator ${}_A\langle f|\hat{\Pi}_a|i\rangle_A$ in Eq. (6). If $\hat{\Pi}_a$ and the postselection basis are unknown, achieving the goal seems impossible.

We overcome this obstacle by implementing postselected metrology with a quantum circuit that simulates a PCTC. We assume that $\hat{\Pi}_a$ and the postselection basis $\{|f^\pm\rangle_A\}$ are unknown until just after (in the laboratory's rest frame) the interaction [47]. Can we nevertheless initialize $|\phi\rangle_A$ to leverage contextuality? We answer affirmatively, by constructing a PCTC simulation.

Given a postelection outcome ${}_A\langle f^\pm|$, we choose the input state $|i^\pm\rangle_A$ such that $|{}_{f^+}\langle\hat{\Pi}_a\rangle_{i^+}| = |{}_{f^-}\langle\hat{\Pi}_a\rangle_{i^-}| \gg 1$ [49]. As $\hat{\Pi}_a$ and $\{|f^\pm\rangle_A\}$ are known only after the interaction, we effectively create the input state $|\phi\rangle_A = |i^\pm\rangle_A$ after the interaction has taken place. Then, using a postselected quantum-teleportation circuit, we effectively transport the state backward in time, such that $|\phi\rangle_A$ serves as an input to the interaction. Figure 2(b) illustrates our experiment with a quantum circuit.

In the laboratory's rest frame, our protocol proceeds as follows:

t_0: A and C are entangled:

$$|\Phi^+\rangle_{A,C} = \frac{1}{\sqrt{2}}\big(|0\rangle_A|0\rangle_C + |1\rangle_A|1\rangle_C\big).$$

Qubit B is initialized to

$$|\Psi\rangle_B = \frac{1}{\sqrt{2}}\big(|b^+\rangle_B + |b^-\rangle_B\big).$$

t_1: A and B interact via $\hat{U}(\theta) = e^{-i\theta\hat{\Pi}_a\otimes\hat{B}/2}$. The value of θ and the form of $\hat{\Pi}_a = |a\rangle\langle a|$ are unknown.

t_2: The as-yet-unknown, optimal measurement basis $\{|f^\pm\rangle_A\}$ is revealed. A is measured in this basis.

t_3: Information about $\hat{\Pi}_a$ and about the outcome ${}_A\langle f^\pm|$ reaches D. Qubit D is created and initialized in $|i^\pm\rangle_D$.

t_4: C and D are measured in the Bell basis [36]. Outcome ${}_{C,D}\langle\Phi^+|$ effectively teleports $|i^\pm\rangle_D$ to the time-t_0 system A [50].

t_5: If, and only if, outcome ${}_{C,D}\langle\Phi^+|$ was obtained at t_4, a beam blocker is removed from B's path.

t_6: If the beam blocker was removed, B is measured in the $\{(1/\sqrt{2})(|b^+\rangle \pm |b^-\rangle)\}$ basis.

Supplemental Material [51] Note I presents the mathematical details behind our protocol's effectiveness. Supplemental Material [51] Note II (which references experimental works [52–57]) proposes an optics realization.

Repeated experiments that involve final B measurements produce an anomalously large weak value. Hence, our scheme amplifies the quantum Fisher information about θ to nonclassically large values. We have thus shown that, in weak-value amplification, the preselected system state can effectively be created *after* the interaction—even after the state has been measured and destroyed. This point is visible in Fig. 1(c), a CTC depiction of our protocol. The inset [Fig. 1(d)] shows the standard teleportation, across space, of a quantum state to be inputted into an interaction. The state's initialization is postponed, and the state's destruction is advanced, in Fig. 1(c). These changes do not affect the chronology-respecting state's final form. In each of many previous studies, classical or quantum information—but not both—traverses a PCTC. Our study differs. Quantum information (solid line) and classical information (dashed line) form the loop of our simulated CTC.

One could imagine three objections. First, some postselections—and so teleportation attempts—fail. However, these failures do not lower the figure of merit, the average amount of information per probe that passes the blocker. As further reassurance, our setup does not send classical information to the past. [The dashed line in Fig. 1(c) travels only forward in time.] The improved information-per-detection rate is available only at the end of the experiment.

Second, one might view as artificial our assumption about when the information needed to choose $|i^{\pm}\rangle$ arrives. Indeed, the assumption is artificial. Our study's purpose is foundational—to demonstrate the power of entanglement to achieve a counterintuitive metrological advantage. Nevertheless, our setup illustrates the general metrological principle that optimal input states are known only once the specifics of the interaction $\hat{U}(\theta)$ are known [40,48]. However, Supplemental Material [51] Notes III and IV contain two extensions of our protocol: one extension with a greater success probability and one extension with greater practicality.

Third, one might view our experiment as involving a preselected state $|\Phi^{+}\rangle_{C,A}|i^{\pm}\rangle_{D}$ and a postselected state ${}_{A}\langle f^{\pm}|_{D,C}\langle\Phi^{+}|$. Our experiment would entail no more effective retrocausality than earlier experiments with preselection and postselection. However, such an interpretation contradicts the definitions of preselection and postselection, as D is created after A is postselected.

The limit as $\theta \to 0$ implies more counterintuitive phenomena. First, B and the rest of the system always remain in a tensor-product bipartite state—they share no correlations, let alone entanglement. Yet B can still carry a nonclassically large amount of quantum Fisher information. Furthermore, imagine, in addition to the $\theta \to 0$ limit, measuring B between t_1 and t_2, before any other measurement and before D is initialized [58]. At time t_5, one would postprocess the data from the B measurements. One would uncover the same contextuality as in conventional weak-value amplification [Fig. 2(a)]. This conclusion paradoxically holds even though B is destroyed before A, C, and D are measured. How? If PCTCs are real (perhaps probabilistic) effects of quantum theory, the nonclassicality comes from time travel. Without real PCTCs, the paradox's resolution will depend on the power of entanglement.

Previous works have addressed the advantages offered by CTCs [28,59–67]. For example, PCTCs would boost a computer's computational power [28,59–61,63]. (Classical computers, too, could achieve such computational power if postselected.) Our metrological protocol differs, posing a paradox even in the absence of true CTCs: Probabilistically simulating PCTCs suffices for achieving the nonclassical advantage. Relatedly, Svetlichny showed that PCTC simulation can effect a Bell measurement of a state before the state is created [27]. Also, probabilistically simulating DCTCs enables nonorthogonal-state discrimination [68]. However, our result differs from these two by entailing that CTC simulation can effectively enable a truly nonclassical advantage—one sourced by contextuality—in the past.

Conclusions.—We have shown how simulating time travel with entanglement benefits the estimation of a coupling strength. A certain "key" input state is needed to unlock a quantum advantage. However, in our setup, the ideal input state is known only after the interaction takes place and the system is measured. We have shown how simulating quantum time travel allows for the key to be created at a later time and then effectively teleported back in time to serve as the experiment's input. The time travel can be simulated with postselected quantum-teleportation circuits. Our Gedankenexperiment thus draws a metrological advantage from effective retrocausation founded in entangled states. While PCTC simulations do not allow you to go back and alter your past, they do allow you to create a better tomorrow by fixing yesterday's problems today.

The authors thank Aharon Brodutch, Noah Lupu-Gladstein, and Hugo Lepage for useful discussions. This work was supported by the EPSRC, the Sweden-America Foundation, the Lars Hierta Memorial Foundation, and Girton College.

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