Disordered Quantum Critical Fixed Points from Holography

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Using holographic duality, we present an analytically controlled theory of quantum critical points without quasiparticles, at finite disorder and finite charge density. These fixed points are obtained by perturbing a disorder-free quantum critical point with relevant disorder whose operator dimension is perturbatively close to Harris marginal. We analyze these fixed points both using field theoretic arguments, and by solving the bulk equations of motion in holography. We calculate the critical exponents of the IR theory, together with thermoelectric transport coefficients. Our predictions for the critical exponents of the disordered fixed point are consistent with previous work, both in holographic and nonholograpic models.

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Introduction.—Quantum field theory has proven to be a powerful tool to study and classify quantum phases of matter [1]. In real experiments, of course, there is always disorder; the Harris criterion [2] determines whether such disorder qualitatively changes the IR fixed point (whether it is relevant or irrelevant). When disorder is Harris relevant, it is challenging to understand the *intrinsically disordered* IR fixed points that arise. Existing constructions in higher dimensions are often analyzed close to fixed points with quasiparticles, such as free theories or large-N vector models [3–12]. The problem is especially difficult in theories at finite charge density, and/or with a Fermi surface, where controlled field theories of strongly interacting non-Fermi liquids are difficult to construct [1].

This Letter presents a controlled calculation, wherein we perturb a UV quantum critical point by Harris-relevant disorder, and analytically deduce the properties (critical exponents and transport coefficients) of the resulting compressible IR fixed point. Our construction relies on holographic duality [13,14], which maps certain models of "matrix large-N" strongly interacting quantum field theories to classical gravity in one higher dimension. These models holographically describe maximally chaotic [14,15] field theories, which do not have any (known) quasiparticles. Through a careful nonperturbative analysis of the nonlinear gravitational equations, we determine the scaling exponents and transport coefficients of the emergent IR fixed point, at finite disorder and finite density.

Main result.—Let us summarize the main physical conclusions of the calculations. We consider theories perturbed by disorder which couples to scalar operator $\mathcal{O}$:

$$S = S_0 + \int \mathrm{d}t\mathrm{d}^d x h(\boldsymbol{x})\mathcal{O}(\boldsymbol{x}, t). \tag{1}$$

with S_0 a disorder-free action describing a quantum critical point with dynamical critical exponent z and hyperscaling violation θ. $h(\boldsymbol{x})$ is zero-mean Gaussian disorder:

$$\overline{h(\boldsymbol{x})h(\boldsymbol{y})} \approx D\delta(\boldsymbol{x}-\boldsymbol{y}). \tag{2}$$

The Harris criterion [2] tells us that disorder is relevant when the operator dimension $[D] > 0$. If the operator dimension of $\mathcal{O}$ is Δ, defined by $\langle \mathcal{O}(x,0)\mathcal{O}(0,0)\rangle \sim |x|^{-2\Delta}$, then [16]

$$[D] = -2\Delta + d - \theta + 2z. \tag{3}$$

It is useful to write

$$\Delta = \frac{d-\theta}{2} + z - \nu, \tag{4}$$

so that $\nu = 0$ corresponds to Harris-marginal disorder, while $\nu > 0$ implies Harris-relevant disorder. For convenience, we also require $\mathcal{O}$ not to be described by alternate quantization in holography, so $\Delta > (d+z)/2$ [16].

We first discuss a minimal theory: a charge-neutral conformal field theory (CFT) in $d = 1$ spatial dimension, perturbed by disorder as in (1), with $\nu = 0$. After a series of works [9,17–21], it was shown that disorder is marginally *irrelevant*: the scale-dependent disorder strength is captured by a beta function,

$$\beta_D = \frac{\mathrm{d}D}{\mathrm{d}\log E} = \frac{|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}} D^2; \tag{5}$$

$C_{\mathcal{O}\mathcal{O}T}$, C_{TT} are operator product expansion coefficients within the CFT.

This Letter concludes this search for a disordered fixed point without quasiparticles as follows. Just as the Wilson-Fisher fixed point can be perturbatively accessed in

$d = 3 - \epsilon$ spatial dimensions [22], with ϵ perturbatively small, if we turn on a perturbatively small ν in (4),

$$\beta_D = \frac{|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}} D^2 - 2\nu D. \tag{6}$$

This flow equation has a stable fixed point as $E \to 0$ if $\nu > 0$: the value of disorder at the critical point is finite and nonzero, and takes the universal value

$$D^* = \frac{2\nu C_{TT}}{|C_{\mathcal{O}\mathcal{O}T}|}. \tag{7}$$

Invoking a universal relation [9] between D^* and z^*, valid for perturbations away from a conformal field theory, we obtain dynamical critical exponent

$$z^* = 1 + \frac{|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}} D^* = 1 + 2\nu. \tag{8}$$

The argument above can be justified both using our holographic models, and using conformal perturbation theory to derive the exact prefactor of (5): see the Supplemental Material [23] for the latter. However, we do not know any field theoretic tools to generalize (8) to perturbations of scaling theories where $z \neq 1$. Yet these $z \neq 1$ theories include many interesting models of strange metals [1]. In contrast, we can more naturally generalize this argument to holographic models of a quantum critical point in d spatial dimensions, at finite density ρ of a conserved U(1) charge. We take the exponents $z > \max(1 + \theta/d, \theta)$ and $\theta \leq d - 1$, so that the holographic model obeys bulk energy conditions [14]. We then add Harris-relevant disorder through (1), satisfying (2) and (4) with $1 \gg \nu > 0$. The system flows to a disordered IR fixed point characterized by a new set of scaling exponents z^*, θ^*:

$$z^* \approx z + \frac{2\nu}{d}(z - \theta), \qquad \theta^* = \theta. \tag{9}$$

While the hyperscaling violation θ remains the same as that in the disorder-free critical point for any ν, the dynamical exponent z will increase linearly in ν at the leading order.

We have calculated the ac electrical conductivity $\sigma(\omega)$ at finite density IR fixed points. We find (schematically) that

$$\sigma(\omega) \sim \frac{K T^{-\frac{2+d-\theta^*}{z^*}}}{1 - i\omega\tau} + F(\omega/T)\omega^{2+\frac{d-\theta^*-2}{z^*}}, \tag{10}$$

where $K \sim \rho^2/D^*$ is a temperature-independent constant, and F is a scaling function. When $z^* < 2 + d - \theta^*$, we find that τT scales anomalously (diverges) as $T \to 0$: see (31). If $\omega \ll T$, therefore, there is a sharp Drude peak, and the first term in (10) dominates. The physical reason for this Drude peak is that the IR fixed point has perturbatively weak disorder ($D^* \sim \nu$), so the low frequency conductivity will be dominated by slow momentum relaxation: this is called a "coherent" contribution to transport [26]. The lifetime of momentum τ can be calculated using established methods [27], and we argue that it can be sensitive to UV thermodynamic data. Hence, although the *static* properties of the IR fixed point are universal, the width of any Drude peak is not. If $z^* \geq 2 + d - \theta^*$, $\tau \lesssim 1/T$ would naively be sub-Planckian, so our conclusion is that there is no well-defined Drude peak: the frequency dependence of the second term in (10) is more important. When $\omega \gg T$, the second term in (10) dominates. This is called the "incoherent" conductivity, and is associated with current-relaxing dynamics decoupled from momentum relaxation. The incoherent conductivity of the IR fixed point theory is universal and exhibits Planckian ω/T scaling; the function F is insensitive to UV physics.

Holography.—Having summarized the physics of the disordered fixed points, let us explain the holographic models we studied. In general, holography ("AdS/CMT") [14] is a powerful framework for building toy models of quantum matter without quasiparticles by mapping the physics onto a gravitational theory in one higher dimension. Fields in the higher-dimensional "bulk" theory correspond to low-dimension operators in the quantum field theory (QFT). All QFTs have a stress tensor, which is dual to the spacetime metric g_{ab} in the bulk. A finite density system requires a conserved U(1) current, dual to a bulk gauge field A_a. A scalar field (dilaton) Φ in the bulk represents a scalar (spin-0) operator in QFT. Following [16,28,29], we consider specifically the Einstein-Maxwell-Dilaton (EMD) action in $d + 2$-dimensional spacetime

$$S_0 = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[\left(R - 2(\partial\Phi)^2 - V(\Phi)\right) - \frac{Z(\Phi)}{4}F^2\right], \tag{11}$$

with coordinates $(r, t, \boldsymbol{x})$. The bulk coordinate r can intuitively be thought of as encoding energy scale in the QFT: the UV corresponds to $r \to 0$, while the IR is $r \to \infty$. These EMD models are a standard holographic model capable of realizing fixed points for generic z, θ. To study Harris-relevant disorder, we introduce a bulk scalar field ψ, dual to the disorder operator $\mathcal{O}$ in the QFT, and consider bulk action $S = S_0 + S_\psi$, with

$$S_\psi = -\int \mathrm{d}^{d+2}x\sqrt{-g}\left[\frac{1}{2}(\partial\psi)^2 + \frac{B(\Phi)}{2}\psi^2\right]. \tag{12}$$

We emphasize that this differs from the usual strategy of studying disordered QFTs by introducing replicas [3]: here, we study a single realization of the disorder, which is encoded by holographic duality in the *boundary conditions*: $\psi(r \to 0, t, \boldsymbol{x}) \sim r^c h(\boldsymbol{x})$ for a constant c. Note that the disordered boundary condition is random in x, but static in t.

We will reveal the emergent IR fixed point by solving the nonlinear bulk equations of gravity, subject to these boundary conditions. Details of the construction, including precise functional forms for V, Z, B, etc., are in the Supplemental Material [23]. In the absence of disorder, the metric is given by

$$\mathrm{d}s^2 = \frac{1}{r^2}\left[\frac{a(r)}{b(r)}\mathrm{d}r^2 - a(r)b(r)\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2\right], \quad (13)$$

while the dilaton and gauge fields are

$$\Phi = \Phi(r), \qquad A = p(r)\mathrm{d}t. \quad (14)$$

The scaling exponents z, θ are captured by the constants a_0 and b_0 in $a(r) \sim r^{a_0}$ and $b(r) \sim r^{b_0}$. To study a finite density black hole, we can identify the charge density with

$$\rho = -\frac{Zp'}{ar^{d-2}}. \quad (15)$$

In the presence of spatially inhomogeneous ψ, an analytical solution of the classical bulk equations cannot be found. Indeed, with hindsight, (9) shows that a_0 and b_0 will get linear corrections in ν, which are nonperturbative corrections in ν to the actual bulk fields. To understand how to solve these complicated bulk equations, let us begin with a physical picture for the radial evolution of the geometry from UV ($r = 0$) to IR ($r = \infty$). If the disorder is self-averaging (the geometry is, at leading order, independent of disorder realization), then the geometry must be approximately homogeneous in x: after averaging over disorder realizations, translation invariance is restored. The bulk geometry is constructed holographically by varying the action (11) and solving the equation of motion for each field; e.g., for the metric, we obtain

$$R_{ab} - \frac{R}{2}g_{ab} = \frac{1}{2}\left(T^A_{ab} + T^\Phi_{ab} + \overline{T^\psi_{ab}}\right), \quad (16)$$

where $T^{A,\Phi,\psi}_{ab}$ denote the *bulk* stress tensors associated with each of these fields, and $\overline{\cdots}$ denotes disorder averaging. We then solve for the bulk fields a, b, p, Φ nonperturbatively, assuming that they are sourced by the homogeneous $\overline{T^\psi_{ab}}$. We make the general ansatz

$$a(r) \approx \alpha_0 r^{a_0 - \gamma_a(r)}, \quad (17a)$$

$$b(r) \approx \beta_0 r^{b_0 - \gamma_b(r)}, \quad (17b)$$

$$\Phi(r) \approx c_\Phi(r)\log r, \quad (17c)$$

$$p(r) \approx \pi_0 r^{p_0 - \gamma_p(r)}, \quad (17d)$$

which readily suggests a physical interpretation: $\gamma_{a,b,p}$ will encode the flow of critical exponents from the UV to IR fixed points.

Plugging in (17) into the homogenized bulk equation of motions, we obtain equations to solve for $\gamma_{a,b,p}$ and c_Φ. Together with the equation of motion for each Fourier mode $\psi(r,\boldsymbol{k})$, we can then solve for all bulk fields and obtain a self-consistent solution to (16). While we leave most details of this calculation to the Supplemental Material [23], let us describe the critical part of the calculation. The bulk equations of motion imply that c_Φ remains constant and $\gamma_a \approx \gamma_b \approx \gamma_p = \gamma$, which in turn obeys

$$\gamma + r\log r\gamma' - ADr^{\frac{2d\nu}{d-\theta} - \frac{d}{z-\theta}\gamma} = \frac{(d-\theta)r}{d(d+z-\theta)}\partial_r(\gamma + r\log r\gamma'). \quad (18)$$

A is a constant depending on z and θ. Applying dominant balance to (18), the right-hand side is negligible, and

$$\gamma(r) \approx \frac{z-\theta}{d\log r}\log\left[1 + AD\frac{(d-\theta)}{2\nu(z-\theta)}r^{\frac{2d\nu}{d-\theta}}\right]. \quad (19)$$

The bulk geometry locally looks like a scaling geometry, with z varying extremely slowly; this enables us to analytically solve for the eventual fixed point. Numerical solutions confirm that this fixed point is the only one consistent with an approximately homogeneous bulk geometry [23].

To illustrate what (19) implies, we define a *dimensionless* effective disorder strength

$$D_{\mathrm{eff}} \equiv Dr^{\frac{2d\nu}{d-\theta} - \frac{d}{z-\theta}\gamma} = \frac{Dr^{\frac{2d\nu}{d-\theta}}}{1 + DA\frac{(d-\theta)}{2\nu(z-\theta)}r^{\frac{2d\nu}{d-\theta}}}. \quad (20)$$

Notice that $D_{\mathrm{eff}} \to 0$ as $r \to 0$, since disorder is Harris relevant. In the IR,

$$D_{\mathrm{eff}} \to D^* = \frac{2\nu(z-\theta)}{A(d-\theta)} \quad (21)$$

approaches a universal constant. This is the disorder strength of *exactly Harris marginal disorder* that supports the IR fixed point! Since (19) implies that $\gamma = AD^*$ at the IR fixed point, we can solve for the IR critical exponents z^*, θ^*, and we find (9). The crossover energy scale E_c between the UV and IR fixed points occurs at the nonperturbatively large scale

$$E_c \sim \left(\frac{D}{\nu}\right)^{\frac{z^*}{2\nu}}, \quad (22)$$

emphasizing the nonperturbative nature of our (approximate) solution to the nonlinear bulk equations. It is interesting that such a detailed analysis of the bulk equations is needed to reproduce what, in a field theoretic language (6), is a perturbative one-loop effect.

It remains to explain why the geometry is self-averaging [20]. While at $O(D)$ the disorder contributed to a homogeneous source $\overline{T^{\psi}_{ab}}$ for gravity in (16), there will also be inhomogeneous source terms proportional to $h(\boldsymbol{k})h(\boldsymbol{q})$ with $\boldsymbol{k}+\boldsymbol{q}\neq\mathbf{0}$. These inhomogeneous source terms would not matter if the left-hand side of (16) was linear; since it is nonlinear in g_{ab}, such source terms do feed back and correct the metric beyond our ansatz. However, to correct the disorder averaged metric, we will need at least two such powers of the source term, meaning that there are four factors of h. Thus, the corrections to our approximation are $O(D^2)=O(\nu^2)$. Since at the IR fixed point, disorder remains perturbatively small, this correction can be neglected at leading nontrivial order, thus justifying that the geometry is self-averaging at the perturbatively accessible fixed point.

We studied a charge-neutral critical point with a nontrivial hyperscaling violation $\theta\neq 0$. This is done by turning off the bulk gauge field ($A_a=0$); Lorentz invariance in the boundary directions demands $z=1$. The dilaton field will get renormalized (c_Φ is no longer constant), and the disordered IR fixed point has critical exponents [23]

$$z^*=1+\frac{6\nu(1-\theta)(d-\theta)}{d[3d+(\theta-5)\theta]},\tag{23a}$$

$$\theta^*=\theta+\frac{2\nu(\theta-1)(d-\theta)}{d[3d+(\theta-5)\theta]}\theta.\tag{23b}$$

We see that θ is renormalized. Interestingly, as long as $\theta\neq 0$, we have a different fixed point from (9) by taking $z\to 1$ there, and this is because when $z\neq 1$, θ is not renormalized. Nevertheless, (9) and (23) agree in the CFT limit: $z=1$ and $\theta=0$.

Observe that (9) and (23) are consistent with the general expectation that disorder should become exactly marginal at the IR fixed point: if it was relevant, it would drive us to a new fixed point; if it was irrelevant, then the IR would not have finite disorder D^*! To confirm that the disorder is exactly Harris marginal at the IR fixed point, we compute its scaling dimension Δ_{IR}. In anti–de-Sitter (AdS) space, the mass of a bulk field determines the dual operator's scaling dimension; for us, Δ_{IR} is fixed by $B(\Phi)$. Calculating Δ_{IR} from $B(\Phi)$ and demanding that it is Harris marginal ($\Delta_{\mathrm{IR}}=[(d-\theta^*)/2]+z^*$), we find the condition that

$$\frac{d}{z-\theta}(z^*-z)+\frac{2dz-d\theta}{(z-\theta)(d-\theta)}(\theta^*-\theta)=2\nu.\tag{24}$$

Obviously, (9) and (23) satisfy the above equation.

Previous literature [30,31] has studied theories with $z/(-\theta)=\eta>0$ fixed, while $z\to\infty$. Such theories are analyzed in the Supplemental Material [23].

Conductivities.—We now discuss the thermoelectric transport properties of the disordered IR fixed point. We study the theory at temperatures $T\ll E_c$, whereby the geometry is approximately that of the IR fixed point, but contains a black hole horizon $r=r_+$ with Hawking temperature T. This corresponds to modifying the geometry found in (17) via [14]

$$b(r)\to b(r)\left(1-\left(\frac{r}{r_+}\right)^{d+\frac{dz^*}{d-\theta^*}}\right),\tag{25}$$

where $T\sim r_+^{-[dz^*/(d-\theta^*)]}$. At the horizon, the entropy density s scales $s\sim r_+^{-d}\sim T^{[(d-\theta^*)/z^*]}$.

In general, if we apply a temperature gradient $-\nabla Te^{-i\omega t}$ and electric field $\mathbf{E}e^{-i\omega t}$, the charge current $\mathbf{J}e^{-i\omega t}$ and heat current $\mathbf{Q}e^{-i\omega t}$ are proportional to these sources:

$$\begin{pmatrix}\mathbf{J}\\ \mathbf{Q}\end{pmatrix}=\begin{pmatrix}\sigma(\omega) & \alpha(\omega)\\ T\alpha(\omega) & \bar{\kappa}(\omega)\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ -\nabla T\end{pmatrix}.\tag{26}$$

Let us first discuss the dc ($\omega=0$) conductivities. Via the membrane paradigm [32,33], we can evaluate them by analyzing the geometry at the horizon: see the Supplemental Material [23]. We find that the thermoelectric conductivities are all approximated by a Drude-like form, signifying that the transport coefficients are dominated by slow momentum relaxation: [14]

$$\sigma_{\mathrm{dc}}\approx\frac{\rho^2}{\Gamma},\qquad\alpha_{\mathrm{dc}}\approx\frac{\rho s}{\Gamma},\qquad\bar{\kappa}_{\mathrm{dc}}\approx\frac{Ts^2}{\Gamma},\tag{27}$$

where

$$\Gamma\sim D^*T^{\frac{d-\theta^*+2}{z^*}}.\tag{28}$$

Remarkably, (27) agrees with the perturbative result in [16] with Harris-marginal disorder (in the IR), again confirming the criterion in (24).

Following [34–36], we now analyze the subleading (in D_*) corrections to transport coefficients that describe transport decoupled from momentum relaxation. As we show in the Supplemental Material [23], in this holographic model such corrections to thermoelectric transport coefficients are captured by the open-circuit thermal conductivity

$$\kappa_{\mathrm{dc}}\equiv\bar{\kappa}_{\mathrm{dc}}-T\alpha_{\mathrm{dc}}^2\sigma_{\mathrm{dc}}^{-1}\sim T^{\frac{z^*+d-\theta^*-2}{z^*}}.\tag{29}$$

In ordinary metals, one finds that $\kappa_{\mathrm{dc}}\sim T\sigma_{\mathrm{dc}}$ as $T\to 0$ with a precise prefactor (this is called the Wiedemann-Franz law) [14]; clearly, this is badly violated at these disordered fixed points, since

$$\mathcal{L} \equiv \frac{\kappa_{dc}}{T\sigma_{dc}} \sim D^* T^{2\frac{d-\theta^*}{z^*}} \quad (30)$$

vanishes as $T \to 0$. Anomalous scaling of $\mathcal{L}$ is not too surprising given that the leading order results (27) exactly cancel in κ_{dc}; indeed, it is the subleading corrections to σ_{dc} that are responsible for nonvanishing κ_{dc}. One calls such contributions to thermoelectric transport "incoherent" [26] as they are decoupled from slow momentum relaxation.

Let us now extend the discussion to ac ($\omega > 0$) conductivity; for simplicity, we focus only on the electrical conductivity $\sigma(\omega)$. Following [27], we find that there can be a Drude peak at low frequency $\omega \ll T$: $\sigma(\omega) \sim \sigma_{dc}/(1 - i\omega\tau)$, where $\tau = \mathcal{M}/\Gamma$. We argue in the Supplemental Material [23] that $\mathcal{M} \sim T^0$ is a UV-sensitive quantity, implying that τ is *not universal*, and exhibits anomalous temperature dependence:

$$\tau \sim T^{-\frac{2+d-\theta^*}{z^*}}. \quad (31)$$

The holographic calculation of τ is only accurate if $\tau \gg 1/T$, so there is a sharp Drude peak only when $2 + d - \theta^* \geq z^*$. For theories that violate this inequality, we expect no sharp features in $\sigma(\omega)$ until the scale $\omega \sim T$. For frequencies $\omega \gg T$, we find that the incoherent conductivity dominates the response function:

$$\sigma(\omega) \sim \omega^{2+\frac{d-\theta^*-2}{z^*}}. \quad (32)$$

The various power laws found above are consistent with recent holographic scaling theories for IR fixed points at finite density [36,37]. Following [38], we assign the charge density operator an anomalous dimension Φ_ρ:

$$[\rho] = d - \theta^* + \Phi_\rho. \quad (33)$$

Scaling analysis shows that $[\sigma_{dc}] = d - \theta^* - 2 + 2\Phi_\rho$ [36]. In order to match with (28), we find $\Phi_\rho = -d + \theta^*$, which implies $[\rho] = 0$. It has previously been observed [37] that $[\rho] = 0$ ensures the IR fixed point thermodynamics is consistent with scaling theories, and thus (28) is consistent with this expectation. A more careful analysis reveals that the incoherent conductivity has a *different* IR scaling dimension: $[\sigma_{inc}] = 3(d - \theta^*) - 2 + 2z^* + 2\Phi_\rho$ [37]. This is consistent with (32), and a direct calculation of the dc incoherent conductivity in the Supplemental Material [23].

Outlook.—In this Letter, we have analytically predicted the emergence of a disordered fixed point in a strongly interacting QFT, at either zero or finite density. The exponents z^* and θ^* are independent of UV disorder strength D, as are the dc thermoelectric transport coefficients.

The holographic formalism described here is versatile and could be used to study the emergence of finite disorder fixed points in more general settings, such as in background magnetic fields [13], or in the presence of nontrivial topological effects [39]. It would also be interesting to generalize to models with inhomogeneous charge disorder, where lattice constructions can reveal robust T-linear resistivity [40].

We encourage further numerical work [41] to solve the fully inhomogeneous Einstein equations, and analyze the fixed points described here. The most promising direction may be to focus on one-dimensional disordered systems; prior work [19] constructed black holes with relevant disorder, but their value of $\nu = 3/4$ may be beyond the regime of validity of our perturbation theory. At strong disorder, it may be possible for the horizon to fragment into disconnected pieces, a fascinating phenomenon whose implications for the boundary theory deserve further investigation [42,43].

Our result (9) may extend beyond holographic models. In a (charge-neutral) large-N vector model with nondisordered fixed point with $d = 2$, $z = 1$, $\theta = 0$, the mass disorder at the critical point is relevant with $\nu = (16/3\pi^2 N)$; a recent calculation [11] found that $z^* \approx 1 + \nu$ at the disordered fixed point. This agrees with (9). It would be fascinating if our results can be extended to recent models [44,45] of compressible, disordered non-Fermi liquids based on field theories, including those based on Sachdev-Ye-Kitaev models which display $\sigma \sim \omega^{-1}$.

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