Violation of Eigenstate Thermalization Hypothesis in Quantum Field Theories with Higher-Form Symmetry

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We elucidate how the presence of higher-form symmetries affects the dynamics of thermalization in isolated quantum systems. Under reasonable assumptions, we analytically show that a p-form symmetry in a $(d+1)$-dimensional quantum field theory leads to the breakdown of the eigenstate thermalization hypothesis for many nontrivial $(d-p)$-dimensional observables. For discrete higher-form (i.e., $p \geq 1$) symmetry, this indicates the absence of thermalization for observables that are nonlocal but much smaller than the whole system size without any local conserved quantities. We numerically demonstrate this argument for the $(2+1)$-dimensional $\mathbb{Z}_2$ lattice gauge theory. While local observables such as the plaquette operator thermalize even for mixed symmetry sectors, the nonlocal observable exciting a magnetic dipole instead relaxes to the generalized Gibbs ensemble that takes account of the $\mathbb{Z}_2$ one-form symmetry.

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Introduction.—Symmetries play an essential role in statistical mechanics. When the system has local conserved quantities corresponding to symmetries, we should include them in the statistical ensemble. This fact has recently gained much attention in the thermalization of isolated quantum many-body systems [1–10]. While typical nonintegrable systems without any symmetry locally relax to the canonical ensemble, the existence of symmetries can affect its dynamics [11–17]. For example, integrable systems do not thermalize because of many symmetries but relax to the generalized Gibbs ensemble (GGE) [18–27], which takes account of (quasi-)local conserved quantities.

The eigenstate thermalization hypothesis (ETH) [2,3,28] provides a sufficient condition for an isolated quantum system to thermalize and is extensively studied in various fields ranging from condensed matter [8,9,28] to holographic theory [29–36]. It states that every energy eigenstate becomes thermal for an observable of our interest and is verified in various nonintegrable systems [1,28,37–58]. However, local conserved quantities caused by, e.g., continuous global symmetry or integrability, break the ETH for the entire Hilbert space; in this case, the ETH is recovered after fixing conserved quantities [12,16,22]. In contrast, the influence of nonlocal conserved quantities caused by, e.g., discrete symmetries, is more subtle. Indeed, they typically do not violate the (diagonal) ETH and thermalization of local observables [12,59,60] even when we consider mixed symmetry sectors, while they can sometimes be related to nonergodicity [61–79].

Recently, *higher-form symmetry* has been proposed in the context of classifying phase structures of quantum field theories [80–91] (see Ref. [92] for applications to condensed matter) as a generalized concept of conventional global symmetries. It is characterized by topological symmetry operators, whose correlation functions remain invariant under continuous deformations. The generalization is carried out concerning the dimensionality of charged objects and symmetry operators: for p-form symmetries in a $(d+1)$-dimensional spacetime, the charged objects should be p-dimensional, and the symmetry operators are $(d-p)$-dimensional. Conventional global symmetries correspond to zero-form symmetries.

Despite the extensive research on static aspects of higher-form symmetry, its dynamical consequences are little understood. Such higher-form symmetries nest intrinsically in gauge theories (e.g., Yang-Mills theories), which have been actively studied even in the condensed matter and atomic-molecular-optical contexts [93–110], not to mention high energy physics. While a few studies exist to discuss thermalization dynamics in specific models with the generalized symmetries [111–115], general consequences due to the higher-form symmetry have seldom been uncovered.

In this Letter, we analytically show that nontrivial observables break the ETH when higher-form symmetries are present under certain assumptions (Fig. 1). In the case of discrete symmetry groups, the breakdown of the ETH is caused by nonlocal conserved quantities. For a p-form symmetry, such ETH-breaking operators become

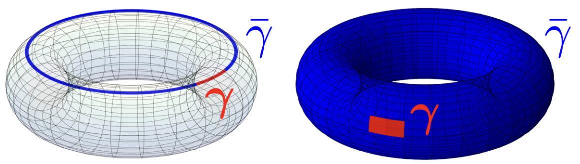

FIG. 1. Schematics of p-form symmetry and its influence on the ETH. The figure is for $\mathcal{M} = T^2$ ($d = 2$). The blue and red regions respectively denote the areas where $U(\bar{\gamma})$ and $U(\gamma)$ nontrivially act, and the symmetry operator is given by $U(\tilde{C}) = U(\bar{\gamma})U(\gamma)$. If $U(\gamma)$ satisfies the ETH with a nonvanishing thermal average, the ETH for $U(\bar{\gamma})$ is broken. Left: the case with higher-form symmetry. The ETH-breaking operator $U(\bar{\gamma})$ is $(d-p)$-dimensional ($p = 1$ is shown) and has a support much smaller than the size of the "bath" $\mathcal{M}\backslash\bar{\gamma}$. Right: the case with the conventional symmetry ($p = 0$), where the support of $U(\bar{\gamma})$ is comparable with the size of $\mathcal{M}$.

$(d-p)$-dimensional, which are nonlocal but have a much smaller size than the entire system for $p \geq 1$. We demonstrate this statement for the two-dimensional $\mathbb{Z}_2$ lattice gauge theory with $\mathbb{Z}_2$ one-form symmetry [116]. Furthermore, while local observables relax to the canonical ensemble, the nonlocal operator exciting a magnetic dipole instead relaxes to the GGE that considers the higher-form symmetry. Our results indicate that symmetries cause nontrivial thermalization processes revealed by nonlocal observables, which go beyond conventional statistical mechanics.

Higher-form symmetry.—We consider quantum field theories in a $(d+1)$-dimensional spacetime, $\mathcal{M} \times \mathbb{R}$, where $\mathcal{M}$ is a connected d-dimensional space manifold. Let G be an Abelian group, and the system is supposed to have a G p-form symmetry, i.e., there exists a $(d-p)$-dimensional topological operator $U_\alpha(\mathcal{C})$ ($\alpha \in G$), where $\mathcal{C} \subset \mathcal{M} \times \mathbb{R}$ denotes a $(d-p)$-dimensional closed surface [126].

We specifically consider the symmetry operator $U_\alpha(C)$ lying in the space for each fixed time, i.e., $C \subset \mathcal{M}$. Then, the topological property of $U_\alpha(C)$ leads to $[U_\alpha(C), e^{-iH\delta t}] = 0$ for an infinitesimal time slice δt, and thus $[H, U_\alpha(C)] = 0$. Importantly, $U_\alpha(C)$ has a $(d-p)$-dimensional support C, which is nonlocal but much smaller than the entire d-dimensional system for $p \geq 1$. This contrasts with conventional zero-form symmetries, whose symmetry operator is d-dimensional, i.e., its support is comparable to the entire system (Fig. 1). While the following discussion holds both for continuous and discrete symmetries, we especially focus on discrete symmetry, which typically entails nonlocal conserved quantities alone.

Under this setting, the Hamiltonian is block diagonalized by $U_\alpha(C)$. Then, the ETH for $U_\alpha(C)$ trivially breaks down for the entire Hilbert space within the energy shell, where symmetry sectors are mixed. However, this does not necessarily indicate the breakdown of the ETH for other nontrivial observables since $U_\alpha(C)$ does not necessarily provide a local conserved quantity. Indeed, there are several evidences [12,59,60] that the ETH holds for local observables despite the existence of the discrete symmetry, especially the zero-form symmetry. In that case, eigenstate expectation values of those observables can be the same even for different symmetry sectors. For example, the transverse-field Ising model $H_{\mathrm{TFIM}} = \sum_{\mathbf{R},\mathbf{R}' \in \mathcal{M}} J_{\mathbf{R},\mathbf{R}'} \sigma^3(\mathbf{R}) \sigma^3(\mathbf{R}') + \sum_{\mathbf{R} \in \mathcal{M}} g_{\mathbf{R}} \sigma^1(\mathbf{R})$ has a $\mathbb{Z}_2$ zero-form symmetry $U(C = \mathcal{M}) = \prod_{\mathbf{R} \in \mathcal{M}} \sigma^1(\mathbf{R})$, where $\sigma^{1,2,3}(\mathbf{R})$ denote the Pauli matrices acting on the vertices $\mathbf{R}$. While we have two symmetry sectors with $U = \pm 1$, they will not lead to distinct eigenstate expectation values for typical local observables, say $\sigma^1(\mathbf{R})$, in the thermodynamic limit.

Breakdown of the ETH for nontrivial operators.—We now state our main result: higher-form symmetry of a nondegenerate Hamiltonian leads to the breakdown of the ETH even for many *nontrivial* $(d-p)$-dimensional operators. For this purpose, we require the following reasonable assumptions: (i) the operator $U_\alpha(\tilde{C})$ can be decomposed as $U_\alpha(\tilde{C}) = U_\alpha(\gamma) U_\alpha(\bar{\gamma})$, where we have introduced a $(d-p)$-dimensional submanifold $\gamma (\subset \tilde{C})$ and its complement $\bar{\gamma} \coloneqq \tilde{C}\backslash\gamma$, both of which have boundaries. (ii) For at least one nontrivial closed surface, say $\tilde{C} (\subset \mathcal{M})$, the energy shell contains eigenstates in different symmetry sectors defined by $U_\alpha(\tilde{C})$. (iii) The microcanonical average $\langle U_\alpha(\gamma) \rangle_{\mathrm{mc}}^{\Delta E}(E)$ defined from the energy shell $[E, E + \Delta E]$ takes a nonzero value in the thermodynamic limit.

Under the above assumptions, we show that either $U_\alpha(\gamma)$ or $U_\alpha(\bar{\gamma})$ necessarily breaks the ETH [127] within the energy shell $[E, E + \Delta E]$ (see Supplemental Material [128] for a proof). Our result indicates that, while the discrete symmetry and topology may not affect the thermalization of conventional local observables, their effect significantly emerges in the dynamics of $(d-p)$-dimensional nonlocal objects. We stress that, while such nonlocal observables go beyond conventional statistical mechanics, they have actively been studied since they play an essential physical role in gauge theory [129,130]. Furthermore, nonlocal operators have become accessible in state-of-the-art experiments using artificial quantum systems [131–133].

Let us point out the notable aspect of our results for higher-form symmetry with $p \geq 1$, although our results provide a hitherto unknown consequence even for the conventional symmetry $p = 0$. For $p = 0$, the ETH-breaking operators [say, $U_\alpha(\bar{\gamma})$] are d-dimensional, and the volume of their support, $V_{\bar{\gamma}}$, is comparable with the volume of the "bath" $V_{\mathcal{M}\backslash\bar{\gamma}}$ for a large system-size limit, i.e., $V_{\bar{\gamma}}/V_{\mathcal{M}\backslash\bar{\gamma}} \to$ finite (see Fig. 1, right). For the example of H_{TFIM}, $\prod_{\mathbf{R} \in \mathcal{M}\backslash\gamma} \sigma^1(\mathbf{R})$ breaks the ETH if $\prod_{\mathbf{R} \in \gamma} \sigma^1(\mathbf{R})$ satisfies it. Thus, the breakdown of the ETH might also be attributed to the smallness of the bath. In contrast, for higher-form symmetry with $p \geq 1$, we have $V_{\bar{\gamma}}/V_{\mathcal{M}\backslash\bar{\gamma}} \to 0$ in the thermodynamic limit (Fig. 1, left) . Thus, the higher-form symmetry hinders thermalization even when the bath is regarded as much larger than the support of the observable of our interest.

Our main claim is generalized to an operator $U(\bar{\gamma})A(g)^\dagger$, where $A(g)$ is an operator defined on an arbitrary region $g(\subset \mathcal{M})$ satisfying $g \cap \bar{\gamma} = \phi$. That is, $A(g)U(\gamma)$ or $A(g)^\dagger U(\bar{\gamma})$ violates the ETH if we impose an assumption (iii)' $\langle A(g)U(\gamma)\rangle^{\Delta E}_{\mathrm{mc}}(E) \neq 0$ instead of (iii). This generalization indicates that for a fixed γ, we have many ETH-violating operators corresponding to the choice of g and $A(g)$. Note that, while $g \subseteq \gamma$ for zero-form symmetries, g may not be included in (or have even larger dimension than) γ for higher-form symmetries.

Finally, the symmetry can, in turn, *ensure* the ETH [134] for certain operators. Indeed, the so-called charged operators W, for which $U_\alpha(C)WU_\alpha^{-1}(C) = e^{i\alpha w}W$ holds with some charge w, satisfy $\langle E_n|W|E_n\rangle = 0$ for all n when $e^{i\alpha w} \neq 1$. Then, W satisfies the ETH, and the long-time average of $\langle W(t)\rangle$ becomes zero. For H_{TFIM} with zero-form symmetry, $W = \sigma^3(\mathbf{R})$ satisfies this condition. For the higher-form symmetry, the Wilson line in the lattice gauge theory discussed later satisfies this condition.

$\mathbb{Z}_2$ lattice gauge theory.—We demonstrate the general discussion above using the $(2+1)$-dimensional $\mathbb{Z}_2$ lattice gauge theory on a square lattice forming a two-torus $\mathcal{M} = T^2$. The Hamiltonian $H_{\mathbb{Z}_2}$ is given by [125,136,137]

$$-\sum_{\boldsymbol{r}} J_{\boldsymbol{r},xy}\sigma^3_{\boldsymbol{r},x}\sigma^3_{\boldsymbol{r}+\boldsymbol{e}_x,y}\sigma^3_{\boldsymbol{r}+\boldsymbol{e}_y,x}\sigma^3_{\boldsymbol{r},y} - \sum_{\boldsymbol{r},j}\sigma^1_{\boldsymbol{r},j}, \tag{1}$$

where $\sigma^{1,2,3}_{\boldsymbol{r},j}$ denote the Pauli matrices acting on the link $(\boldsymbol{r}, j)$, which is specified by the coordinate of vertices $\boldsymbol{r}$ and the direction $j = x, y$. This system has a $\mathbb{Z}_2$ one-form symmetry, and the spatial symmetry operators are characterized by $H_1(T^2, \mathbb{Z}_2) = \mathbb{Z}_2 \oplus \mathbb{Z}_2$ [138]. Indeed, the system has two independent symmetry operators corresponding to the x cycle and y cycle.

To remove the residual gauge redundancies after the temporal gauge fixing [125], we project the entire Hilbert space onto the physical one. Here, spatial gauge transformation is generated by the local operator $Q_v \coloneqq \prod_{b:\ \text{spatial link}, b\ni v}\sigma^1_b$, where v denotes the vertex. The operator Q_v satisfies $Q_v^2 = 1$, $[H_{\mathbb{Z}_2}, Q_v] = 0$. Then, the physical Hilbert space is given by [125] $\mathrm{span}\{|\psi\rangle | Q_v|\psi\rangle = +|\psi\rangle, {}^\forall v\text{: vertices}\}$, where the constraint can be regarded as the $\mathbb{Z}_2$ analog of the Gauss law. After this projection, the expectation value of a nongauge invariant operator for physical states $|\psi\rangle$ always vanishes.

We next define the 't Hooft and Wilson operators on the spatial directions as [139–141]

$$U(C^*) = \prod_{b^* \in C^*}\sigma^1_{b^*} = U^{-1}(C^*), \tag{2}$$

and $W(C) = \prod_{b\in C}\sigma^3_b = W^{-1}(C)$. Here, C and C^* are closed loops on the lattice and dual lattice, respectively

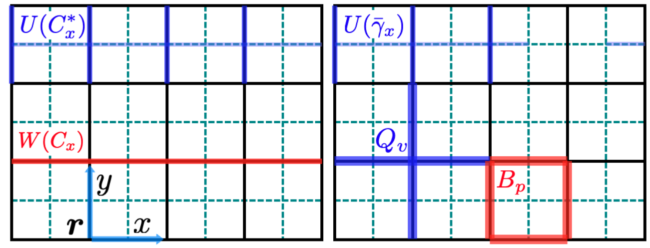

FIG. 2. Schematic diagram of our lattice model ($N_x = 4$ and $N_y = 3$ are shown) under the periodic boundary condition. The solid and green lines denote the lattice and the dual lattice, respectively. We illustrate examples of the operators Q_v, $U(C_x^*)$, $W(C_x)$, $U(\bar{\gamma}_x)$, and B_p, where the Pauli matrices σ^3 and σ^1 respectively act on the red and blue links.

(see Fig. 2). Both $U(C^*)$ and $W(C)$ commute with the operator Q_v and thus are gauge invariant.

The 't Hooft operator $U(C^*)$ serves as the $\mathbb{Z}_2$ one-form symmetry operator of this model, satisfying $[H_{\mathbb{Z}_2}, U(C^*)] = 0$. This operator is topological since it satisfies $U(C_1^*)|\psi\rangle = U(C_2^*)|\psi\rangle$ if C_1^* and C_2^* are homotopically equivalent. It follows that $U(C^*)|\psi\rangle = |\psi\rangle$ if the dual closed loop C^* is topologically trivial, i.e., it can be continuously deformed to a point.

The 't Hooft operator $U(C^*)$ measures the "electric" charge of the Wilson operator. We define closed loops on the lattice winding around the x/y cycle by C_x and C_y (and similarly the loops on the dual lattice by C_x^* and C_y^*). Then, the operators W and U satisfy $U(C_i^*)W(C_j)U^{-1}(C_i^*) = (-1)^{\delta_{ij}+1}W(C_j)$, which is the operator realization of the electric $\mathbb{Z}_2$ one-form symmetry [95].

ETH breaking by $\mathbb{Z}_2$ one-form symmetry.—Let us demonstrate the violation of the ETH for the $\mathbb{Z}_2$ lattice gauge theory. We take the coupling constants $J_{\mathbf{r},jk}$ in (1) to be weakly random (i.e., $J_{\boldsymbol{r},xy}$ is uniformly chosen from $[0.7, 0.8]$) to avoid unwanted degeneracy and integrability. We consider the $N_x \times N_y$ two torus and define x/y cycles for the lattice and dual lattice as $C_{x/y}$ and $C^*_{x/y}$, respectively (Fig. 2).

We calculate the eigenstate expectation values of local operators $U(\gamma_x)$, B_p, and nonlocal one-dimensional observable $U(\bar{\gamma}_x)$. Here, γ_x is just one link included in C_x^*, and $\bar{\gamma}_x \coloneqq C_x^* \backslash \gamma_x$. The operators $U(\gamma_x)$ and $U(\bar{\gamma}_x)$ represent a magnetic dipole excitation residing at the endpoints of γ_x [125]. The plaquette operator B_p is defined by $B_p \coloneqq \prod_{b\in \text{plaquette } p}\sigma^3_b$. Figure 3(a) shows that the local observable $U(\gamma_x)$ satisfies the ETH. In contrast, Fig. 3(b) demonstrates that the nonlocal one-dimensional observable $U(\bar{\gamma}_x)$ has two branches of the eigenstate-expectation values, indicating the breakdown of the ETH owing to the general mechanism explained above. The ETH is recovered when we consider eigenstates within the symmetry sector for $U(C_x^*) = 1(-1)$, even without separating the sector for $U(C_y^*)$. The result for B_p is given in the Supplemental Material [128].

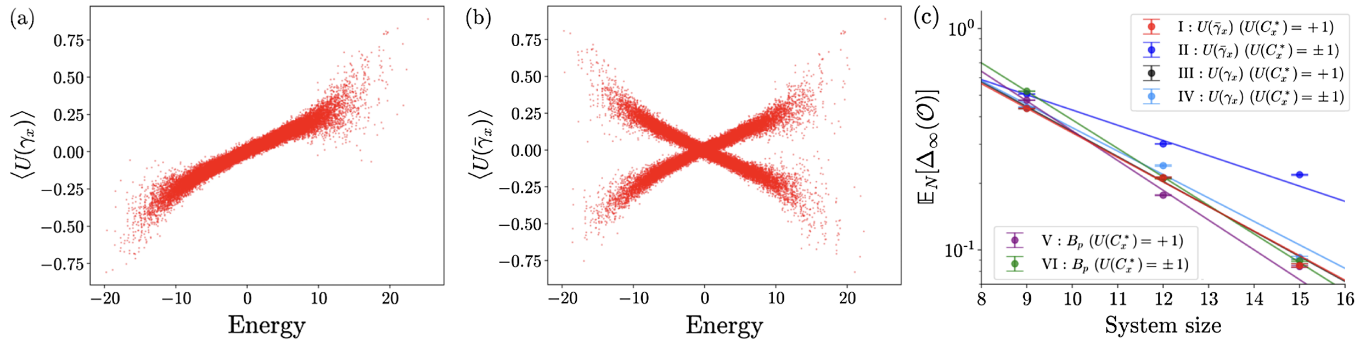

FIG. 3. Expectation values for energy eigenstates in the 5×3 lattice. (a) The local observable $U(\gamma_x)$ satisfies the ETH. (b) The observable with a one-dimensional support $U(\bar{\gamma}_x)$ violates the ETH. The expectation values are separated into two sectors classified by the value of one-form symmetry, i.e., $U(C_x^*) = \pm 1$. (c) System-size dependence of the ETH measure Δ_∞. The decay for $U(\bar{\gamma}_x)$ with the total symmetry sectors (case II) is much slower than the other cases, which indicates that the ETH is hindered. The fitting parameters are shown in the Supplemental Material [128].

To test the ETH more quantitatively, we perform the finite-size scaling analysis. We define the deviation measure for an observable $\mathcal{O}$ [51] by $\Delta_\infty(\mathcal{O}) \coloneqq \max_{n, E_n \in [E, E+\delta E]} |\langle E_n|\mathcal{O}|E_n\rangle - \langle \mathcal{O}\rangle_{\mathrm{mc}}^{\delta E}(E_n)|$, where $E_n = \langle E_n|H_{\mathbb{Z}_2}|E_n\rangle$ is an energy eigenvalue. The strong ETH corresponds to $\Delta_\infty \to 0$ in the thermodynamic limit. Furthermore, Δ_∞ is expected to decay exponentially $\sim e^{-s(E)N/2}$ for a fully chaotic system, where $s(E)$ is the entropy density at energy E [8,51]. Figure 3(c) shows the system-size dependence of the disorder-averaged measure $\mathbb{E}[\Delta_\infty(\mathcal{O})]$, which is fitted with a function e^{-aN+b}. First, the local observables B_p and $U(\gamma_x)$ (irrespective of whether we resolve the symmetry sector) and the nonlocal observable $U(\bar{\gamma}_x)$ after resolving the symmetry sector exhibit sufficiently fast exponential decay with a relatively similar rate. This indicates the ETH for these observables. In contrast, $U(\bar{\gamma}_x)$ for the total symmetry sector decays much slower than the other cases, though it keeps decreasing due to the finite-size effect. Combining the general argument and the ETH for $U(\gamma_x)$, we conclude that the ETH for $U(\bar{\gamma}_x)$ breaks down due to the higher-form symmetry.

Note that the Wilson line always satisfies $\langle E_n|W(C_{x/y})|E_n\rangle = 0$ for all n, because of the general discussion for the charged operator discussed previously. Consequently, the long-time average of the Wilson operator always vanishes.

GGE with $\mathbb{Z}_2$ one-form symmetry.—We next argue that the stationary value of an observable that is nonlocal in the x direction but local in the y direction [e.g., $U(\bar{\gamma}_x)$] is described by the GGE that takes account of the $\mathbb{Z}_2$ one-form symmetry. That is, we have $\langle \mathcal{O}\rangle_{\mathrm{GGE}} = \mathrm{Tr}[\mathcal{O}\rho_{\mathrm{GGE}}(\beta, \lambda_x, \mu_x)]$ with

$$\rho_{\mathrm{GGE}} = \frac{1}{Z_{\mathrm{GGE}}} e^{-\beta H_{\mathbb{Z}_2} - \lambda_x U(C_x^*) - \mu_x U(C_x^*) H_{\mathbb{Z}_2}}, \qquad (3)$$

where "chemical potentials" λ_x and μ_x are uniquely determined from the initial values of the conserved quantities $H_{\mathbb{Z}_2}$, $U(C_x^*)$, and $U(C_x^*)H_{\mathbb{Z}_2}$, and Z_{GGE} is the normalization constant. Our GGE is justified as the stationary state if we assume the restricted ETH for each $U(C_x^*)$-symmetry sector for the observable $\mathcal{O}$ (See the Supplemental Material [128]).

Figure 4 shows time evolutions of $\mathcal{O} = U(\gamma_x)$ and $U(\bar{\gamma}_x)$. For $U(\gamma_x)$, the stationary value is well described by the canonical ensemble $\rho_{\mathrm{can}} = Z_{\mathrm{can}}^{-1} e^{-\beta_{\mathrm{can}} H_{\mathbb{Z}_2}}$, where $\mathrm{Tr}[U(\gamma_x)\rho(t)] \simeq \mathrm{Tr}[U(\gamma_x)\rho_{\mathrm{can}}]$ holds most of the time. In contrast, the canonical ensemble fails for $U(\bar{\gamma}_x)$. Instead, $\mathrm{Tr}[U(\bar{\gamma}_x)\rho(t)] \simeq \mathrm{Tr}[U(\bar{\gamma}_x)\rho_{\mathrm{GGE}}]$ holds most of the time. We stress that ρ_{GGE} works well even though we do not consider the effect of $U(C_y^*)$, probably because $U(\bar{\gamma}_x)$ is local in the y direction.

Note that the GGE suitable for a general finite Abelian group G is obtained by assuming the restricted ETH for each symmetry sector as $\rho_{\mathrm{GGE}}^G = e^{-\beta^G H - \sum_j \lambda_j^G P_j - \sum_j \mu_j^G P_j H}$, where P_i are the projections to each symmetry sector.

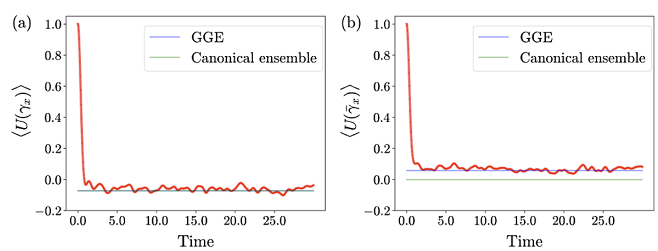

FIG. 4. Time evolution of the expectation values of $U(\gamma_x)$ and $U(\bar{\gamma}_x)$ for the 4×3 lattice. The blue and green lines indicate the prediction of the GGE in Eq. (3) and the standard canonical ensemble, respectively. For the local observable $U(\gamma_x)$ (left), the stationary state is described by the canonical ensemble, which is almost overlapping with the GGE result. In contrast, the stationary value of $U(\bar{\gamma}_x)$ (right) differs from the canonical ensemble and is described by the GGE. The initial states are random superpositions of eigenstates of $U(\gamma_x)$ (left) or $U(\bar{\gamma}_x)$ (right) with the eigenvalue $+1$, whose energy expectations lie within $[-5.0, -3.0]$.

For symmetry sectors defined by $U(C_x^*)$ with $G = \mathbb{Z}_2$, this ensemble is indeed equivalent to (3) after redefinitions of the chemical potentials.

Conclusion and outlook.—We analytically show that the existence of a p-form symmetry leads to the ETH violation of many $(d-p)$-dimensional observables in the form of $U_\alpha(\bar{\gamma})$ under certain assumptions. A significant feature of this statement is that the ETH-violating observable has a nonlocal but lower-dimensional support rather than the whole d-dimensional space manifold for $p \geq 1$. This implies that such objects can be described by the suitable GGE instead of the canonical ensemble. We use the $\mathbb{Z}_2$ lattice gauge theory to demonstrate the above statements. The discussion on the breakdown of the ETH can be applied to systems with p-form symmetries, e.g., various quantum field theories such as $\mathrm{SU}(N)$ Yang-Mills theory with center symmetries.

Our results indicate that symmetries cause nontrivial thermalization dynamics for nonlocal observables, which go beyond conventional statistical mechanics. We stress that this ETH violation stably holds even under local perturbations to the Hamiltonians because the higher-form symmetry is robust against them. For future direction, higher-form symmetry may impact entanglement structures of certain subsystems, including entanglement-entropy dynamics, which is a nonlocal quantity. As entanglement entropy is vital in holography, exploring it may illuminate black-hole dynamics through gauge-gravity duality.

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[127] Note that $U_\alpha(\gamma)$ and $U_\alpha(\bar{\gamma})$ are in general non-Hermitian. However, the breakdown of the ETH for these operators leads to that of certain Hermitian operators as well. Indeed, if $U_\alpha(\bar{\gamma})$ breaks the ETH, we can show that either of the Hermitian operators $U_\alpha(\bar{\gamma}) + U_\alpha^\dagger(\bar{\gamma})$ or $i[U_\alpha(\bar{\gamma}) - U_\alpha^\dagger(\bar{\gamma})]$ breaks the ETH.
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