Quadratic Twist-Noncommutative Gauge Theory

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Studies of noncommutative gauge theory have mainly focused on noncommutative spacetimes with constant noncommutative structure, with little known about actions for noncommutative 4D Yang-Mills theory beyond this case. We construct an action for Yang-Mills theory on a quadratically noncommutative spacetime, i.e., of quantum-plane type, obtained from a Drinfeld twist, with star-gauge symmetry. Applied to supersymmetric Yang-Mills theory, this gives a candidate AdS/CFT dual of string theory on a related deformation of $\mathrm{AdS}_5 \times \mathrm{S}^5$, which is expected to be integrable in the planar limit.

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Noncommutativity between spacetime coordinates is a likely feature of quantum gravity [1], actively studied from numerous angles [2,3]. In string theory, noncommutative gauge theory appears in the low energy dynamics of open strings [4], and thereby the AdS/CFT correspondence [5–7]. Despite the formal and phenomenological relevance of noncommutative gauge theory, it is not clear how to write actions to all orders in the noncommutativity when going beyond the case of constant noncommutativity. In this Letter, we consider a noncommutative Drinfeld-twist deformation of Minkowski space, with quadratically coordinate-dependent noncommutativity, and construct an all-order action for Yang-Mills theory with star-gauge symmetry. Beyond providing a first example of a noncommutative Yang-Mills theory action with quadratic noncommutativity, our choice of deformation is motivated by the AdS/CFT correspondence and integrability. Applied to maximally supersymmetric Yang-Mills theory, our noncommutative deformation provides a concrete candidate gauge theory dual of a particular Yang-Baxter deformation [8–10] of the famously integrable $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring [11,12], as conjectured in [13]. This opens the door to investigating integrability for a range of novel planar noncommutative gauge theories.

We consider noncommutative field theory in the usual spirit of Weyl quantization, trading noncommuting field operators for a noncommutative product—the star product—between commutative fields [14,15]. Our star product is obtained from a Drinfeld twist, whereby it automatically comes with clear algebraic properties and a natural differential calculus [16], and twists rather than plainly breaks Poincaré symmetry [17,18]. The original Groenewold-Moyal noncommutative deformation can be viewed as a twist, and it is well-known how to construct a Yang-Mills action in this case [15]. For general twists, however, it is not clear how to define a suitable dual field strength tensor and construct an action for noncommutative Yang-Mills theory. For example, the approaches of [19,20] for κ-Minkowski space, were necessarily perturbative, and only solved to leading order in the noncommutativity. To our knowledge, the only nonconstant case known to all orders is the $U(1)$ Yang-Mills theory studied in [21] for a particular twist with linear noncommutativity, where standard Hodge duality suffices.

We show how a twisted version of Hodge duality allows us to define a noncommutative Yang-Mills action for a twist based on two commuting Lorentz generators, with a noncommutative structure with quadratic coordinate dependence. Our construction moreover provides a broader framework that covers all Poincaré-based twist deformations of Minkowski space, including non-Abelian ones, whose r matrices are unimodular [22].

Given our motivations in AdS/CFT, we also discuss how to couple our theory to (adjoint) matter, and define quadratically noncommutative maximally supersymmetric Yang-Mills theory. We then discuss its possible AdS/CFT interpretation, as a would-be amalgamation of the famous Groenewold-Moyal [6,7] and real-β Lunin-Maldacena deformations [23], with its gravity dual similarly given by a particular T-duality shift–T-duality (TsT) transformation of the superstring on $\mathrm{AdS}_5 \times \mathrm{S}^5$. This comes with caveats in the form of completely broken supersymmetry and a potentially unbounded string coupling, deserving further investigation, but we expect a notion of duality to survive in the planar limit at the very least. Moreover, both sides of this potential duality admit a nontrivial infinite boost limit, where these caveats do not apply.

Lorentz-deformed Minkowski space.—In deformation quantization, a noncommutative spacetime is described via a regular spacetime whose function algebra is equipped with a noncommutative (star) product. The noncommutative product we consider for functions on $\mathbb{R}^{1,3}$ is based on the Drinfeld twist

$$\mathcal{F} = e^{\frac{i\lambda}{2}(M_{01}\otimes M_{23} - M_{23}\otimes M_{01})} \tag{1}$$

built from two commuting Lorentz generators, M_{01} and M_{23}, with $M_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu)$. It defines our noncommutative star product via

$$f(x) \star g(x) \equiv \mu[\mathcal{F}^{-1}(f(x), g(x))], \tag{2}$$

where $\mu[h(x), z(x)] = h(x)z(x)$ is the ordinary pointwise product of functions, and λ is our deformation parameter. We will refer to this as the Lorentz twist, or deformation. In contrast to the familiar Groenewold-Moyal star product associated to

$$\mathcal{F}_{GM} = e^{-\frac{i\theta^{\mu\nu}}{4}(\partial_\mu\otimes\partial_\nu - \partial_\nu\otimes\partial_\mu)},$$

where θ is a constant antisymmetric matrix, the bidifferential operator appearing in the exponent of our twist has nontrivial coordinate dependence [24].

Our star product is associative because M_{01} and M_{23} commute [25]. Its noncommutative structure is

$$x^\mu \star x^\nu = R_\sigma{}^\mu{}_\rho{}^\nu x^\rho \star x^\sigma, \tag{3}$$

with 24 nonzero components for R,

$$R_0{}^0{}_0{}^0 = 1, R_0{}^0{}_2{}^2 = \cosh\lambda, R_0{}^1{}_2{}^3 = -i\sinh\lambda, \tag{4}$$

and others obtained by index permutation symmetries of the twist [Eq. (1)], namely $2 \leftrightarrow 3$ or $(0,1) \leftrightarrow (2,3)$ combined with a sign change of λ, and $0 \leftrightarrow 1$.

As our goal is gauge theory, we need differential calculus, now suitably twisted [16]. Using standard differential calculus on Minkowski space we define

$$dx^\mu \star f = \mu[\mathcal{F}^{-1}(dx^\mu, f)], \tag{5}$$

where the vector fields M_{01} and M_{23} act via Lie derivatives, summarized as

$$\mathrm{d}x^\mu \star f = \mathrm{d}x^\nu \bar{F}{}_\nu{}^\mu(f), \tag{6}$$

with

$$\bar{F}{}_\nu{}^\mu = \begin{pmatrix} \cosh\frac{\lambda M_{23}}{2} & -\sinh\frac{\lambda M_{23}}{2} & 0 & 0 \\ -\sinh\frac{\lambda M_{23}}{2} & \cosh\frac{\lambda M_{23}}{2} & 0 & 0 \\ 0 & 0 & \cos\frac{\lambda M_{01}}{2} & -\sin\frac{\lambda M_{01}}{2} \\ 0 & 0 & \sin\frac{\lambda M_{01}}{2} & \cos\frac{\lambda M_{01}}{2} \end{pmatrix}{}^{\nu}_{\mu}.$$

Commuting functions through forms gives rise to

$$\mathrm{d}x^\mu \star f = R_\nu{}^\mu(f) \star \mathrm{d}x^\nu, \tag{7}$$

with the R matrix $R_\nu{}^\mu = \bar{F}_\nu{}^\rho \bar{F}_\rho{}^\nu$. Both R and $\bar{F}$ are vector-field-valued elements of the Lorentz group, in the sense that, raising and lowering indices with the usual Minkowski metric,

$$R_\mu{}^\nu R^\rho{}_\nu = \bar{F}_\mu{}^\nu \bar{F}^\rho{}_\nu = \delta^\rho_\mu. \tag{8}$$

We also define a star-wedge product,

$$dx^\mu \wedge_\star dx^\nu = \hat{\mu}[\mathcal{F}^{-1}(dx^\mu, dx^\nu)], \tag{9}$$

where $\hat{\mu}[a,b] = a \wedge b$ is the regular wedge product. Concretely,

$$\mathrm{d}x^\mu \wedge_\star \mathrm{d}x^\nu = \bar{F}_\sigma{}^\mu{}_\rho{}^\nu \mathrm{d}x^\sigma \wedge \mathrm{d}x^\rho, \tag{10}$$

with $\bar{F}_\sigma{}^\mu{}_\rho{}^\nu = R_\sigma{}^\mu{}_\rho{}^\nu|_{\lambda\to\lambda/2}$. We will mostly work with star forms

$$\omega = \omega^\star_{\mu\nu\ldots\rho} \star dx^\mu \wedge_\star dx^\nu \wedge_\star \ldots \wedge_\star dx^\rho, \tag{11}$$

but occasionally will also express them as regular forms:

$$\omega = \omega_{\mu\nu\ldots\rho} dx^\mu \wedge dx^\nu \wedge \ldots \wedge dx^\rho. \tag{12}$$

Our star forms are totally R-antisymmetric, e.g.,

$$\mathrm{d}x^\mu \wedge_\star \mathrm{d}x^\nu = -R_\rho{}^\mu{}_\sigma{}^\nu \mathrm{d}x^\sigma \wedge_\star \mathrm{d}x^\rho. \tag{13}$$

We will use the ordinary exterior derivative, which has the desired product rule

$$d(\omega \wedge_\star \chi) = d\omega \wedge_\star \chi + (-1)^p \omega \wedge_\star d\chi. \tag{14}$$

for p and q forms ω and χ, respectively, as it commutes with Lie derivatives. Under (conventional) conjugation we have $\overline{\omega \wedge_\star \chi} = (-1)^{pq} \bar{\chi} \wedge_\star \bar{\omega}$.

Our star product is graded cyclic under an integral,

$$\int \omega \wedge_\star \chi = (-1)^p \int \chi \wedge_\star \omega, \tag{15}$$

when $\chi \wedge_\star \omega$ is a top form, upon integration by parts [26].

Hodge duality.—To define our twisted Hodge star we take a natural generalization of the Levi-Civita symbol,

$$dx^\mu \wedge_\star dx^\nu \wedge_\star dx^\rho \wedge_\star dx^\sigma = \epsilon^{\mu\nu\rho\sigma} d^4x, \tag{16}$$

where the volume form $d^4x = dx^0 \wedge_\star dx^1 \wedge_\star dx^2 \wedge_\star dx^3 = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$ is not deformed. By explicit evaluation of the star-wedge products, we find that ϵ is graded cyclic, and has 32 nonzero components given by

$$\begin{aligned} &\epsilon^{0123} = -\epsilon^{0132} = \epsilon^{0231} = -\epsilon^{0321} = 1, \\ &\epsilon^{1212} = -\epsilon^{0202} = \epsilon^{1313} = -\epsilon^{0303} = i\sinh\lambda, \\ &\epsilon^{0312} = -\epsilon^{0213} = \cosh\lambda, \end{aligned} \tag{17}$$

plus others related by graded cyclicity.

In regular Hodge duality we can freely permute indices on the Levi-Civita symbol for signs, giving many equivalent definitions of a dual form. The appropriate choice in our twisted setting is

$$* \, \mathrm{d}x^{\mu_1} \wedge_\star \dots \wedge_\star \mathrm{d}x^{\mu_k} = \frac{(-1)^{\sigma(k)}}{(4-k)!} \varepsilon_{\mu_{k+1}\dots\mu_4}{}^{\mu_1\dots\mu_k} \mathrm{d}x^{\mu_4} \wedge_\star \dots \wedge_\star \mathrm{d}x^{\mu_{k+1}}, \tag{18}$$

where $\sigma(p)$ denotes the signature of the reversal of p objects, i.e., $\sigma(1) = \sigma(4) = 0$, $\sigma(2) = \sigma(3) = 1$. The reversed index contraction in the dual form is essential. Restricted to basis star forms, this twisted Hodge star commutes with Lie derivatives along vector fields in the Poincaré algebra, and hence with our star product. This allows us to consistently extend it star linearly to arbitrary forms, where it continues to commute with Poincaré Lie derivatives and our star product.

Our Hodge star has all typical properties, appropriately twisted [22]. It preserves R antisymmetry and reality of star forms, and for a p form ω we have

$$* * \omega = -(-1)^p \omega. \tag{19}$$

For equal-degree p forms ω and χ we also have

$$\omega \wedge_\star *\chi = (-1)^p * \omega \wedge_\star \chi = (-1)^{\sigma(p)+1} p! \omega^\star_{\mu\dots\nu} \star R_\kappa{}^\mu \dots R_\rho{}^\nu \chi^{\star\rho\dots\kappa} \mathrm{d}^4 x,$$

so that

$$\int \omega \wedge_\star *\chi = \int \chi \wedge_\star *\omega. \tag{20}$$

Related to Hodge duality commuting with star products,

$$[dx^\mu \wedge_\star dx^\nu \wedge_\star dx^\rho \wedge_\star dx^\sigma \overset{\star}{,} f] = 0, \tag{21}$$

for any f, following from star commutativity of ϵ and the volume form, these being constant and Lorentz invariant respectively. This implies that ϵ is an invariant of the R matrix,

$$\epsilon^{\tau\kappa\zeta\phi} R_\tau{}^\mu R_\kappa{}^\nu R_\zeta{}^\rho R_\phi{}^\sigma = \epsilon^{\mu\nu\rho\sigma}. \tag{22}$$

A similar form of Hodge duality was discussed for q-Minkowski space in [27]; see also [28].

Yang-Mills theory.—Coming to Yang-Mills theory, as gauge transformations are functions, they are affected by the star product, and it is natural to consider star-gauge transformations [15]. A fundamental field Φ then transforms as

$$\delta_\varepsilon \Phi(x) = i\varepsilon(x) \star \Phi(x),$$

under a gauge transformation by $\varepsilon \in \mathfrak{h}$, where $\mathfrak{h}$ is the Lie algebra of the gauge group H [29].

Working in terms of forms, we have

$$d(\delta_\varepsilon \Phi) = d(i\varepsilon \star \Phi) = i d\varepsilon \star \Phi + i\varepsilon \star d\Phi,$$

and we can define the covariant derivative

$$D\Phi = d\Phi + iA \star \Phi, \tag{23}$$

with

$$\delta_\varepsilon A = d\varepsilon + i[\varepsilon \overset{\star}{,} A], \qquad \delta_\varepsilon(D\Phi) = i\varepsilon \star D\Phi.$$

Next we define the field strength tensor

$$G = dA - iA \wedge_\star A, \tag{24}$$

which transforms star-covariantly

$$\delta_\varepsilon G = i[\varepsilon \overset{\star}{,} G].$$

We now consider a natural deformation of the commutative Yang-Mills action,

$$S_{\text{NC-YM}} = \int \mathrm{Tr}\, G \wedge_\star *G. \tag{25}$$

Since our Hodge dual commutes with star products, $*G$ transforms star covariantly. Since our star product is cyclic under integration, this action is gauge invariant.

To illustrate this nontrivial point, let us derive the transformation of $*G$ in components. Starting from

$$G = G^\star_{\mu\nu} \star dx^\mu \wedge_\star dx^\nu,$$

using Eq. (7), we find

$$\delta_\varepsilon G^\star_{\mu\nu} = i\varepsilon \star G^\star_{\mu\nu} - iG^\star_{\rho\sigma} \star R_\mu{}^\rho R_\nu{}^\sigma \varepsilon. \tag{26}$$

The transformation of $*G$ is then

$$\delta_\varepsilon(*G) = i\varepsilon \star (*G) - iG^\star_{\mu\nu} \epsilon^{\tau\lambda}_{\xi\kappa} \star dx^\rho \wedge_\star dx^\sigma \star R^\xi_\sigma R^\kappa_\rho R^\mu_\tau R^\nu_\lambda \varepsilon = i[\varepsilon \overset{\star}{,} *G], \tag{27}$$

where we used Eqs. (8) and (22).

In star components our action reads

$$S_{\text{NC-YM}} = \int \mathrm{Tr} G^\star_{\mu\nu} \star R_\rho{}^\mu R_\sigma{}^\nu G^{\star\sigma\rho} d^4 x. \tag{28}$$

Expressed in unstarred components, repeated integration by parts gives

$$S_{\text{NC-YM}} = \int \mathrm{Tr} G_{\mu\nu} G^{\nu\mu} d^4 x, \tag{29}$$

where

$$G_{\mu\nu} = \partial_{[\mu} A_{\nu]} - i\bar{F}_\rho{}^\kappa{}_\sigma{}^\tau \bar{F}_{[\nu|}{}^\sigma (A_\kappa) \star \bar{F}^\rho{}_{|\mu]}(A_\tau), \tag{30}$$

showing that the kinetic term for the gauge field is undeformed, while the interaction terms are deformed.

Our action has twisted Poincaré symmetry in the spirit of [17,18], meaning the following. In the commutative setting, the Poincaré algebra acts on individual fields via

Lie derivatives, which by the product rule combine to Lie derivatives of the Lagrangian [30]. For Poincaré generators these are total derivatives, leaving the action invariant. Introducing a coproduct $\Delta(\xi) = \xi \otimes 1 + 1 \otimes \xi$ for generators ξ, the product rule takes the form

$$\xi[\mu(f,g)] = \mu[\Delta(\xi)(f,g)],$$

with multiple coproducts extending this to products involving more fields. Our twisted product is similarly compatible with a twisted coproduct

$$\xi(f \star g) = \xi\{\mu[\mathcal{F}^{-1}(f,g)]\} = \mu[\mathcal{F}^{-1}\Delta_{\mathcal{F}}(\xi)(f,g)],$$

where $\Delta_{\mathcal{F}} = \mathcal{F}\Delta\mathcal{F}^{-1}$. Since every product in our action is a star product, letting the Poincaré algebra act (nonlocally) on products of fields by this twisted coproduct, still result in a total derivative, and an invariant action. The twisted Poincaré algebra for our Lorentz twist is discussed in [31].

Matter fields and supersymmetric Yang-Mills theory.— We can readily couple our theory to matter. For adjoint scalars for instance, we can write

$$S_{\text{NC-}\phi} = \int \mathrm{Tr} D\phi^\dagger \wedge_\star *D\phi + \int \mathrm{Tr}(\phi^\dagger \star \phi)^{\star n} d^4x. \qquad (31)$$

where $D\phi = d\phi - i[A \overset{\star}{,} \phi]$. Gauge invariance follows as for star-Yang-Mills theory.

Working with forms allows us to straightforwardly define actions, while guessing, e.g., the component forms of Eqs. (28)–(30) would be difficult. To tackle fermions in similar spirit, focusing on massless ones for concreteness, we combine left- and right-handed Weyl spinors ψ_α and $\bar{\psi}_{\dot{\alpha}}$ with Grassmann-valued basis spinors s^α and $\bar{s}^{\dot{\alpha}}$ to form the Grassmann-even $\psi = \psi_\alpha s^\alpha$ and $\bar{\psi} = \bar{\psi}_{\dot{\alpha}} s^{\dot{\alpha}}$. We then take our twist to act via the left- and right-handed Weyl representation of the Poincaré algebra on s^α and $\bar{s}^{\dot{\alpha}}$ respectively. These spinors play an analogous role to forms in components resulting in spinor analogues of Eqs. (5)–(10).

We now assemble the usual γ matrices into a convenient object, taking the Pauli matrices σ_i, $i = 1, 2, 3$, and $\sigma_0 = 1_{2\times 2}$ to form

$$\sigma = \sigma_{\mu\alpha\dot{\alpha}} s^\alpha \bar{s}^{\dot{\alpha}} dx^\mu = R^\star{}_{\mu\alpha\dot{\alpha}} s^\alpha \star \bar{s}^{\dot{\alpha}} \star dx^\mu.$$

Coupled by the Pauli matrices, the transformation properties of the spinors and one form cancel, making σ Lorentz invariant, hence star commutative. For adjoint Weyl fermions we then define the kinetic action

$$S_{\text{NC-}\psi} = \int\int\int d^2s d^2\bar{s} \mathrm{Tr}\bar{\psi} \star \sigma \wedge_\star *D\psi, \qquad (32)$$

where $D\psi = d\psi - i[A \overset{\star}{,} \psi]$, and the Grassmann integrals over the basis spinors extract the appropriate components. Gauge invariance of this action follows as before, since σ is star commutative. Combined with an adjoint scalar ϕ, we can form gauge-invariant Yukawa-like interactions such as

$$\int\int d^2s \mathrm{Tr}\psi \star \phi \star \psi \, d^4x.$$

We use these ingredients to define the action for maximally supersymmetric Yang-Mills theory (SYM) on Lorentz-deformed $\mathbb{R}^{1,3}$ as

$$\begin{aligned} S_{\text{NC-SYM}} = & \frac{1}{4g^2}\mathrm{Tr}\int G \wedge_\star *G + \mathrm{Tr}\int D\phi^{IJ} \wedge_\star *D\phi_{IJ} - \frac{g^2}{16}\mathrm{Tr}\int d^4x[\phi^{IJ} \overset{\star}{,} \phi^{KL}] \star [\phi_{IJ} \overset{\star}{,} \phi_{KL}] \\ & + \mathrm{Tr}\int d^2s d^2\bar{s}\int \bar{\psi}^I \star \sigma \wedge_\star *D\psi_I + \frac{ig}{2}\mathrm{Tr}\int d^2s \int d^4x \psi_I \star [\phi^{IJ} \overset{\star}{,} \psi_J] - \frac{ig}{2}\mathrm{Tr}\int d^2\bar{s}\int d^4x \bar{\psi}^I \star [\phi_{IJ} \overset{\star}{,} \bar{\psi}^J], \end{aligned} \qquad (33)$$

where ψ^I, $I = 1, 2, 3, 4$, are the four fermions of SYM, and the $\phi^{IJ} = -\phi^{JI}$ contain the six real scalars. This deformation of SYM classically has twisted superconformal symmetry [22]. As the dilatation generator commutes with our twist, this action is conventionally scale invariant.

AdS/CFT.—It is well-known that Groenewold-Moyal noncommutative SYM has an AdS/CFT dual—the Maldacena-Russo-Hashimoto-Ithzaki background [6,7]—obtained by performing a TsT transformation in the relevant Cartesian directions in the Poincaré patch of AdS_5. It is moreover possible to consider TsT transformations of S^5 to give an AdS/CFT dual—the Lunin Maldacena background [23]—of the real-β deformation of SYM, which can be viewed a noncommutative deformation of SYM in its (global) R-symmetry directions in line with the corresponding TsT transformation. By analogy, we can consider the commuting boost and a rotation associated to our Lorentz deformation, and use the corresponding TsT transformation to deform $\mathrm{AdS}_5 \times \mathrm{S}^5$, with the aim of generating an AdS/CFT dual for our Lorentz-deformed SYM. This results in the background

$$\begin{aligned} ds^2 &= \frac{-d\rho^2 + dr^2 + dz^2}{z^2} + \frac{\rho^2 d\alpha^2 + r^2 d\theta^2}{z^2 + \tilde{\lambda}^2\rho^2 r^2/z^2}, \\ B &= -\tilde{\lambda}\frac{\rho^2 r^2}{z^4 + \tilde{\lambda}^2\rho^2 r^2} d\alpha \wedge d\theta, \quad e^{2(\phi-\phi_0)} = \frac{z^4}{z^4 + \tilde{\lambda}^2\rho^2 r^2}, \end{aligned} \qquad (34)$$

in (the analog of) Rindler coordinates (ρ, α) inside the light cone of the (x^0, x^1) plane, and polar coordinates (r, θ) in the (x^2, x^3) plane, of AdS_5 in the Poincaré patch. It is further supported by nontrivial Ramond-Ramond forms. This background has the usual scale symmetry of AdS, as well as boost and rotational symmetries in the (x^0, x^1) and (x^2, x^3) plane respectively, but the rest of the conformal symmetry of AdS is broken by the deformation.

All such TsT backgrounds can be viewed as Yang-Baxter deformations of the $AdS_5 \times S^5$ superstring [8–10]; see Ref. [32]. From this perspective, the possibility that our Lorentz-deformed SYM is a dual to string theory in the background (34) is part of the broader conjecture of [13] on the AdS/CFT interpretation of Yang-Baxter deformed strings; see also [33]. In particular, the string sigma model for Eq. (34) has the same twisted symmetries as our Lorentz-deformed SYM, adding support in favor of a duality, at least in the planar limit.

While the TsT analogy and the matching of planar symmetry structures between gauge and string theory are certainly promising, the question of whether our Lorentz-deformed SYM is truly dual to string theory in the background (34), in particular beyond the planar limit, deserves further investigation. For instance, while we can readily use a TsT transformation to give a brane geometry that naively supports the desired duality, the physical details of the decoupling limit can be subtle. For example, for constant noncommutativity, while the spacelike and lightlike cases are fine [34], timelike noncommutativity results in a noncommutative open string, rather than gauge theory [35,36]. Our Lorentz deformation mixes these cases, appearing spacelike inside the light cone in the (0,1) plane of the brane geometry, but timelike outside it, and this decoupling limit needs careful analysis. Related to this, while the dilaton (string coupling) of Eq. (34) is bounded in the presented patch, i.e., inside the light cone in the (x^0, x^1) plane of the Poincaré patch, it can blow up in its complement, the region of the Poincaré patch obtained by the analytic continuation $\rho \to i\rho$ of Eq. (34). Another important point is that the above background preserves no supersymmetry, which may well allow for instabilities in the full string theory. A related and relevant example here is the three-parameter generalization of the Lunin-Maldacena background of [37] without supersymmetry, where a full duality to the corresponding deformation of SYM is not clear. This case, however, also nicely illustrates how remnants of a correspondence definitely survive at the planar level; see, e.g., [38]. We similarly expect remnants of a duality to our type of noncommutative gauge theory to survive at the planar level, at least.

Finally, let us note that our deformation admits an interesting limit on both sides of the proposed duality, where the above concerns disappear. Namely, by combining an infinite Lorentz boost in the (x^0, x^3) plane with an appropriate scaling of the deformation parameter(s) and coordinates along the lines of [39], the background (34) becomes one which preserves 16 supercharges, has a bounded string coupling, and arises from a decoupling limit with an everywhere nontimelike B field. On the SYM side, the noncommutative product is modified to one associated to the light-cone version of our twist [Eq. (1)], with the 0 and 3 directions replaced by a common null direction, as will be described in more detail in [22]. This of course matches the structure predicted by the open string side of the decoupling limit. In this Letter, we focused on the Lorentz deformation as the overarching example, with its pleasing algebraic structure based on the Cartan generators of the Lorentz algebra.

Outlook.—We have constructed an action for noncommutative Yang-Mills theory with star-gauge symmetry for the Lorentz twist with quadratic noncommutativity. Our construction relies on properties of the twist and R matrix, combined with our nontrivial twisted Hodge duality, and, for SYM, on our fermionic extension of twisted differential calculus.

There are various open questions surrounding our deformation at the quantum level—for instance, regarding UV/IR mixing and its presumable absence in SYM, and the fate of twisted symmetry. At the classical level, noncommutative gauge theories admit an underlying L_∞ algebraic structure [40,41], and it would be interesting to investigate this for our deformation and contrast it with the braided noncommutative gauge theories of [41,42].

Applied to SYM, the Lorentz deformation gives a natural candidate for an AdS/CFT dual of a related Yang-Baxter deformation of the AdS_5 string. Our construction in fact extends to all noncommutative spacetimes described by Drinfeld twists based on the Poincaré algebra, with unimodular r matrix, providing candidate gauge theory duals for a large class of Yang-Baxter deformations of the AdS_5 string [22]. There are important open questions regarding the status of this general conjectured duality, in particular in cases without supersymmetry or an unbounded dilaton, beyond the planar limit.

We expect planar Lorentz-deformed SYM to be integrable, based on the integrability of its proposed string dual, and its formal similarity to the real β deformation. At the classical level this should take the form of Yangian invariance [43,44], now twisted similarly to [45]. At the quantum level, we should find a spectral problem described by an integrable spin chain, similar to the famous dilatation operator of undeformed SYM [46]. The Lorentz deformation is particularly natural in this regard, as it preserves dilatation symmetry. We have defined a suitable related spectral problem in planar Lorentz-deformed SYM, and are in the process of extracting its integrable structure [47], which we expect to relate to the twisted spin chain of [48], building on a planar equivalence theorem [22] in the spirit of Filk [49]. We hope this will pave the way to integrable AdS/CFT for general (homogeneous) Yang-Baxter deformations of the AdS_5 string and its lower dimensional cousins.

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