Rusciano *et al.* Reply: In their Comment [1] to our letter [2], Berthier, Flenner, and Szamel (BFS) point out that, because of the presence of pre-Fickian and pre-Gaussian regimes, Fickianity and Gaussianity in glass formers are attained at infinite time only [3]. We reply that this asymptotic interpretation leads to *a priori* denying the existence of Fickian yet non-Gaussian diffusion (FnGD), and even of standard Brownian diffusion, not only in glass formers, but in any actual system. Indeed, Brownian diffusion is always "gradually" attained after other regimes (e.g., ballistic motion), even in the most simple model systems [4]. In many cases, like for glass formers and a variety of other systems [5–12], these pre-Fickian and/or pre-Gaussian regimes are long-lasting and easily detectable. At longer times, however, Fickianity and/or Gaussianity are observed to be fully-fledged attained in those systems, along similar routes (often consistent with algebraic fits), both in experiments and simulations (within obvious uncertainties inherent to any measurements) [13]. Thus, it is fully legitimate and meaningful to measure a Fickian time τ_F and a Gaussian time τ_G, which we in fact find to be distinct from one another in glass formers [2]: the existence of FnGD in these systems is therefore undisputable.

In earlier papers [15–17], brought into play in the preceding comment [1], BFS themselves wrote about "time and/or length scales of the onset of Fickian diffusion in supercooled liquids" to mark finite time and/or length scales. Conversely, they now claim that the "approach to Fickian behavior is […] scale-free and no characteristic timescale controls the emergence of Fickian behavior." This claim is incorrect. Indeed, the approach to Fickian behavior is not described by a single power law and, therefore, is not scale-free. Rather, it is characterized by a continuous change of the effective mean square displacement (MSD) exponent $[d\log\langle x^2(t)\rangle/d\log t]$, as commonly reported for glass formers and for many other systems [6,10,18,19]. Our time τ_F marks the end of the non-scale-free approach to Fickian behavior (within obvious uncertainties), being therefore fully well defined.

Concerning τ_G, it is defined in the late decay of $\alpha_2(t)$, within an observed master curve, which is always attained well after τ_F. Thus, it is $\tau_G > \tau_F$ systematically. Our τ_G is indicative of the time where the master curve is fully established, within obvious uncertainties. The presence of a master curve—being it compatible with a power-law fit over a certain time range is not crucial—makes the temperature or concentration dependence of τ_G unaffected by the adopted threshold. Hence, our approach draws on a robust "time-temperature (or time-concentration) superposition" with its shift factors, in analogy with many other cases, including de Gennes theory of polymer dynamics [20–22].

Next, we remark that our analysis in [2] deals with quantities specifically targeted to spot out FnGD, whereas the papers mentioned by BFS, published quite earlier than the discovery of FnGD [5], obviously focus on different quantities or scopes [13]. Hence, we reply to the further criticisms in [1] as follows: (i) In [17], the naming Fickian diffusion is explicitly associated with Fickian and Gaussian diffusion, mirroring a common belief before the discovery of FnGD [5]. (ii) Concerning the exponential decay length $l(t)$ of the Van Hove function, BFS explicitly cite their works [23,24] on exponential tails "*at* $t \le \tau_\alpha$." Such a time range is not considered at all in our work, which instead focuses on much longer times, $t\in[\tau_F, \tau_G]$, with $\tau_F > \tau_\alpha$. (iii) Concerning the power law $l(t) \propto t^\alpha$, we recently demonstrated [25] that there are no discrepancies between [2] and [19], provided that exponential fits are performed in the appropriate time range, as in our Letter. (iv) Regarding our ξ_G and the length in [15], they are the root MSDs at $t = \tau_G$ and at $t = \tau_\alpha$, respectively. Since $\tau_G \gg \tau_\alpha$, including a generally different temperature dependence, the two lengths are different. (The further length in [16] is obtained from a multipoint correlation function, hence it is intrinsically different from our ξ_G.) (v) Mermin-Wagner fluctuations in 2D systems are known to affect the short-time (caged) dynamics [26], and are therefore irrelevant here: our results refer to very long-time dynamics, when particles have definitely escaped their original cages. Indeed, we recently demonstrated that FnGD is the same in two- and three-dimensional glass formers [25]. (vi) At variance with what was suggested by BFS [1], long-time diffusion in glass formers is consistent with the picture emerging by popular FnGD models [27]. Indeed, recent papers [19,28,29] by some authors of [27] explicitly focus on FnGD in glass formers.

Overall, we firmly deem that FnGD exists in glass formers and that τ_F and τ_G are not only well defined, but are fundamental timescales for the long-time glassy dynamics.

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[29] For instance, verbatim from [28]: "The connection between BnG diffusion, exponential-tailed distributions and glass-formers […] opens new directions of study since the overall picture of this phenomenology is still far from being completely understood" (Here, BnG stands for Brownian non-Gaussian, a commonly used alternative naming for FnGD).