Generalized Symmetries for Generalized Gravitons

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We construct generalized symmetries for linearized Einstein gravity in arbitrary dimensions. First-principle considerations in quantum field theory force generalized symmetries to appear in dual pairs. Verifying this prediction helps us find the full set of nontrivial conserved charges—associated, in equal parts, with 2-form and $(D-2)$-form currents. Their total number is $D(D+1)$. We compute the quantum commutators of pairs of dual charges, showing that they are nonvanishing for regions whose boundaries are nontrivially linked with each other and zero otherwise, as expected on general grounds. We also consider general linearized higher-curvature gravities. These propagate, in addition to the usual graviton, a spin-0 mode as well as a massive ghostlike spin-2 mode. When the latter is absent, the theory is unitary and the dual-pairs principle is respected. In particular, we find that the number and types of charges remain the same as for Einstein gravity, and that they correspond to continuous generalizations of the Einsteinian ones.

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The notion of generalized symmetry can be traced back to 't Hooft's seminal paper [1], but it has been recently put on a broader and firmer ground by Gaiotto *et al.* [2]. The idea is that symmetry operators can live in codimension-one, two, … hypersurfaces. Dually, the charged operators can be local, line, … operators. In this context the Landau paradigm gets extended to include a far larger zoo of theories, most importantly gauge theories. Recent reviews include [3–5].

It is then important to inquire about the interplay between generalized symmetries and gravity. A conservative starting point is to consider gravitons (spin-2 fields) on Minkoswki spacetime [6]. The simplest case corresponds to the Einstein—or Fierz-Pauli—graviton. The generalized symmetries of this theory were studied recently in [7–9], mostly in $D=4$ spacetime dimensions. An insightful outcome is that generalized symmetries are charged under spacetime symmetries, shedding important new light on the theorems of Noether and of Weinberg-Witten [8], and providing intriguing relations between the gravitational interaction and the physics of fractons [7,9,10]. The next-to-simplest scenario is to go beyond $D=4$ and consider general theories of linearized gravity, dubbed here theories of "generalized gravitons." Characterizing the generalized symmetries of these theories is the main goal of this Letter.

There are several motivations for this analysis. First, the study of conserved charges in gravity theories has been a key area in the field, one of its highlights being the Wald formalism [11–13]. If new conserved charges exist, it is important to find them. Second, as we review below, generalized symmetries in quantum field theory (QFT) always come in dual pairs [5,14,15]. This principle has important implications. In the holographic context, it provides an argument against the existence of higher-form symmetries in quantum gravity [5]. It also lies behind the proof of the universal charged density of states in QFT [16–18]. In certain scenarios, such as the ones considered in this Letter, it might predict the existence of new, otherwise unexpected, charges. Conversely, it might suggest that certain naive charges do not generate new symmetries. Third, the principle of the completeness of the spectrum in quantum gravity [19,20] has recently been connected to the absence of generalized symmetries [5,14,21–23]. Studying the generalized symmetries of generalized gravitons we are advancing toward understanding what kind of matter is required to break these symmetries. In fact, for the Einstein graviton, Ref. [7] showed that conventional ways of breaking the generalized symmetries lead to the breaking of Poincaré invariance. One last motivation comes from condensed matter physics and the connection between gravity and fractons [24]. The zoo of theories considered here enlarges the space of potential fractonic systems as well as the one of tensor gauge theories [25].

Generalized symmetries, algebras, and conserved currents.—In D-dimensional flat space, a p-form symmetry current J is a p-form which satisfies the conservation property $d \star J = 0$, where $\star J$ is the $(D-p)$-form defined by the Hodge dual of J and d is the exterior derivative. Conserved currents like J define conserved higher-form charges Q by integrating $\star J$ over closed $(D-p)$ surfaces $\Sigma_{(D-p)}$ embedded in $\mathbb{R}^D$, namely [2],

$$\Phi = \int_{\Sigma_{(D-p)}} \star J. \tag{1}$$

Given Φ, the operator that implements the generalized symmetry reads $U_g = e^{ig\Phi}$. To have a true symmetry, we need $\Phi \neq 0$, and this implies $\star J \neq dG$, where G is a physical field of the theory. The paradigmatic example is the free Maxwell field, which has two conserved currents F and $\star F$, where F is the field strength. Although $F = dA$ and $\star F = d\tilde{A}$, neither A nor $\tilde{A}$ are physical fields.

An algebraic approach to the notion of generalized symmetry appeared in [5,14]. A QFT naturally assigns von Neumann algebras $\mathcal{A}(R)$ to spacetime regions R. A minimal assignation corresponds to the "additive algebra" $\mathcal{A}_{\rm add}(R)$, i.e., the algebra generated by local operators in R. Causality forces

$$\mathcal{A}_{\rm add}(R) \subseteq [\mathcal{A}_{\rm add}(R')]', \tag{2}$$

where R' is the set of points spatially separated from R, and $\mathcal{A}'$ is the algebra of operators that commute with $\mathcal{A}$. If the inclusion (2) is not saturated for certain R, there is a larger algebra associated with R—the "maximal algebra," $\mathcal{A}_{\rm max}(R) \equiv [\mathcal{A}_{\rm add}(R')]'$—which necessarily contains a set $\{a\}$ of nonlocally generated operators in region R such that

$$\mathcal{A}_{\rm max}(R) = \mathcal{A}_{\rm add}(R) \vee \{a\}. \tag{3}$$

For example, in free Maxwell theory, magnetic fluxes over open surfaces are, in this sense, nonlocal operators associated with ringlike regions. Wilson loops are particular examples. Given this algebraic structure, one can define classes of operators $[a]$ by making the quotient of the maximal algebra by the additive algebra. A class is defined by a certain representative a and all operators that arise from it by adding products of local operators in R.

A nontrivial conclusion follows. The inclusion (2) forces a "dual" inclusion in the complementary region R'. This follows from von Neumann's double commutant theorem, see [5,14], and implies the existence of nonlocal operators $\{b\}$ associated with the R':

$$\mathcal{A}_{\rm max}(R') = \mathcal{A}_{\rm add}(R') \vee \{b\}. \tag{4}$$

Hence, nonlocal operator algebras come in dual pairs—a novel realization of this principle for generalized Maxwell fields can be found in the Supplemental Material [26], which includes [27–39]. The "size" of these dual algebras is precisely the same, measured by the so-called Jones index [40–42]; see [15,43,44] for simpler introductions and specific computations in this context.

There are two further consequences from this approach. Consider a QFT with an additive algebra charged under a global symmetry group G. The question is whether this action can change the nonlocal classes of a given region R. The first consequence is that this can only happen in a pointlike manner [8], namely, $U(g)[a]U(g)^{-1} = [b]$. If $[b]$ is different from $[a]$ for certain $[a]$ and certain $U(g)$, we say the classes are charged under the global symmetry. For continuous symmetry groups this implies the nonlocal classes for R must form a continuum. The second consequence is that if the nonlocal classes of R are charged under the symmetry, the nonlocal classes of R' must be charged too [8], forming another continuum of classes.

Going back to the discussion of conserved p-form currents, consider now integrating $\star J$ over an open surface, whose boundary $\partial\Sigma_{D-p}$ is a closed $(D-p-1)$ manifold. The corresponding flux operator Φ—defined as in (1)—only depends on the boundary $\partial\Sigma_{D-p}$, and therefore commutes with local operators outside $\partial\Sigma_{D-p}$. Since $\star J \neq dG$ with G a physical field, Φ cannot be written as a circulation over the boundary of a physical field. Considering a region R enclosing the boundary $\partial\Sigma_{D-p}$ and with the same topology, this region will contain operators—namely, Φ and all operators arising from multiplying it with local operators in R—that commute with all local operators in R', but cannot be locally generated in R, showing that $\mathcal{A}_{\rm add}(R) \subsetneq \mathcal{A}_{\rm max}(R)$.

This forces an analogous strict inclusion in R'. Further, if the original classes of R are charged under a continuum symmetry group, the dual classes in R' form also a continuum, and are generated by a dual conserved $(D-p)$-form current $\tilde{J}$ (assuming a generalized version of Noether's theorem). Hence, in certain scenarios, such as the one of linearized gravities, the existence of a conserved p-form current predicts the existence of a dual conserved $(D-p)$-form current.

Generalized symmetries for linearized Einstein gravity.— A small perturbation $h_{\mu\nu}$ on top of Minkowski spacetime is defined by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \|h_{\mu\nu}\| \ll 1, \quad h_{\mu\nu} = h_{\nu\mu}. \tag{5}$$

The Einstein-Hilbert action reduces, at quadratic order in the perturbation, to the Fierz-Pauli one,

$$S_{\rm FP} = \int d^D x \left[\frac{1}{2}\partial_\lambda h^{\mu\nu}\partial_\nu h_{\mu\lambda} - \frac{1}{2}\partial_\mu h \partial_\nu h^{\mu\nu} + \frac{1}{4}\partial_\mu h \partial^\mu h - \frac{1}{4}\partial^\lambda h^{\mu\nu}\partial_\lambda h_{\mu\nu}\right]. \tag{6}$$

This action has a gaugelike symmetry under linearized diffeomorphisms,

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \tag{7}$$

and its variation with respect to $h_{\mu\nu}$ yields the linearized Einstein equations,

$$R^{(1)}_{\mu\nu} = 0. \tag{8}$$

The p-form conserved currents must be gauge invariant with respect to (7). They must be written in terms of the linearized Riemann tensor $R_{\mu\nu\rho\sigma}$—or, equivalently, the Weyl—which is the generator of the physical local algebra of the theory [7,45]. Naturally, we are also free to use $\eta_{\mu\nu}$ as well as the Levi-Civita symbol $\varepsilon_{\mu_1\ldots\mu_D}$ to build currents. In fact, it is convenient to use the dual of the Riemann tensor [46,47]:

$$R^*_{\mu_1\ldots\mu_{D-2}\alpha\beta} \equiv \frac{1}{2}\varepsilon_{\mu_1\ldots\mu_{D-2}\lambda\sigma} R^{\lambda\sigma}_{\alpha\beta}. \tag{9}$$

Four dimensions.—Let us consider the $D = 4$ case first. One finds the following four families of conserved 2-forms [7–9]:

$$\mathsf{A}_{\mu\nu} \equiv R^{(1)}_{\mu\nu\alpha\beta} a^{\alpha\beta}, \tag{10}$$

$$\mathsf{B}_{\mu\nu} \equiv R^{(1)}_{\mu\nu\alpha\beta}(x^\alpha b^\beta - x^\beta b^\alpha), \tag{11}$$

$$\mathsf{C}_{\mu\nu} \equiv R^{(1)}_{\mu\nu\alpha\beta} c^{\alpha\beta\gamma} x_\gamma, \tag{12}$$

$$\mathsf{D}_{\mu\nu} \equiv R^{(1)}_{\mu\nu\alpha\beta}\left(x^\alpha d^{\beta\gamma} x^\gamma - x^\beta d^{\alpha\gamma} x^\gamma + \frac{1}{2} d^{\alpha\beta} x^2\right). \tag{13}$$

Here, $a^{\alpha\beta}$, b^α, $c^{\alpha\beta\gamma}$, and $d^{\alpha\beta}$ are real skew-symmetric free parameters which label each of the 20 independent 2-forms [48]. The charges satisfy the appropriate conservation equations:

$$d \star \mathsf{A} = d \star \mathsf{B} = d \star \mathsf{C} = d \star \mathsf{D} = 0. \tag{14}$$

The conservation of A uses the fact that the Riemann tensor is itself a conserved current on shell. The conservation of B relies on the Einstein equation. For C, the first Bianchi identity is used. For D we need the Einstein equation and the first Bianchi identity.

Similar conserved currents $\{\tilde{\mathsf{A}}, \tilde{\mathsf{B}}, \tilde{\mathsf{C}}, \tilde{\mathsf{D}}\}$ can be constructed from the dual curvature, by simply replacing $R^{(1)}_{\mu\nu\rho\sigma}$ by $R^{(1)*}_{\mu\nu\rho\sigma}$ and $\{a^{\alpha\beta}, b^\alpha, c^{\alpha\beta\gamma}, d^{\alpha\beta}\}$ by a new set of constant arrays $\{\tilde{a}^{\alpha\beta}, \tilde{b}^\alpha, \tilde{c}^{\alpha\beta\gamma}, \tilde{d}^{\alpha\beta}\}$. However, in $D = 4$ we have a U(1) duality symmetry rotating the Riemann and its dual:

$$\begin{pmatrix} R \\ R^* \end{pmatrix} \to \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} R \\ R^* \end{pmatrix}. \tag{15}$$

This means the algebra generated by $\{\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{D}\}$ is the same as the one generated by $\{\tilde{\mathsf{A}}, \tilde{\mathsf{B}}, \tilde{\mathsf{C}}, \tilde{\mathsf{D}}\}$.

In four dimensions, the fact that symmetries come in dual pairs might appear trivial at first sight since for a ringlike region the complement is also a ring. Still, it is no coincidence that the total number of them is even—namely, 20—and they can be organized in dual pairs when computing commutators [8]. Note that the currents should come in dual pairs since the conserved 2-form currents (10) are charged under spacetime symmetries.

General dimensions.—We now move to $D > 4$ dimensions, where the complementary of a ringlike region is no longer a ring. Charges and dual charges then correspond to regions with different topologies. At first sight, following [9], the families A, B, C, D described above are still conserved in general dimensions, giving rise to $D(D+1)(D+2)/6$ candidates to generate generalized symmetries associated with rings. The principle that generalized symmetries come in dual pairs predicts an equal number of dual conserved $(D-2)$-forms. Natural candidates appear by considering the obvious extension of the families $\tilde{\mathsf{A}}$, $\tilde{\mathsf{B}}$, $\tilde{\mathsf{C}}$, $\tilde{\mathsf{D}}$ to higher dimensions. However, for $D > 4$ we only recover $D(D+1)/2$ conserved $(D-2)$-forms in this way. These are the two families $\tilde{\mathsf{A}}$, $\tilde{\mathsf{B}}$ constructed as

$$\tilde{\mathsf{A}}_{\mu_1\mu_2\ldots\mu_{D-2}} \equiv R^{(1)*}_{\mu_1\mu_2\ldots\mu_{D-2}\alpha\beta} \tilde{a}^{\alpha\beta}, \tag{16}$$

$$\tilde{\mathsf{B}}_{\mu_1\mu_2\ldots\mu_{D-2}} \equiv R^{(1)*}_{\mu_1\mu_2\ldots\mu_{D-2}\alpha\beta}(x^\alpha \tilde{b}^\beta - x^\beta \tilde{b}^\alpha). \tag{17}$$

The problem is that in $D > 4$ we cannot build conserved currents of the $\tilde{\mathsf{C}}_{\mu\nu}$ and $\tilde{\mathsf{D}}_{\mu\nu}$ type since the Bianchi identity of the dual Riemann tensor with only three indices contracted does not hold:

$$\varepsilon^{\mu_1\ldots\mu_{D-3}\alpha\beta\gamma} R^{(1)*}_{\nu_1\ldots\nu_{D-3}\alpha\beta\gamma} = \frac{1}{2}\eta^{\beta\gamma\mu_1\ldots\mu_{D-3}}_{\alpha\delta\nu_1\ldots\nu_{D-3}} R^{(1)\delta\alpha}_{\beta\gamma}. \tag{18}$$

While in $D = 4$ this reduces to a combination of Ricci tensors which vanishes by virtue of the Einstein equation, this is no longer true in $D > 4$.

This mismatch between the number of conserved charges associated with generalized symmetries in complementary regions has two possible origins. The first is that we might be missing charges arising from new conserved $(D-2)$-forms. In this case, the charges have to be of the $\tilde{\mathsf{C}}$ and $\tilde{\mathsf{D}}$ types, because potential nonvanishing commutators with the A's and B's should be dimensionless (a c number)—see below. We argue these charges do not exist in the Supplemental Material [26]. The second possibility is that some of the conserved A, B, C, D

currents become exact in $D > 4$. Although counterintuitive at first sight, this turns out to be the case. Define

$$\mathcal{A}_{\mu\nu\rho} \equiv -\frac{R^{*}_{\mu\nu\rho\alpha_1\ldots\alpha_{D-3}}}{(D-4)!}\tilde{a}^{\alpha_1\ldots\alpha_{D-3}\sigma}x_\sigma, \tag{19}$$

$$\mathcal{C}_{\mu\nu\rho} \equiv \frac{R^{*}_{\mu\nu\rho\alpha_1\ldots\alpha_{D-3}}}{(D-5)!(D-2)}\left(\frac{1}{2}\tilde{c}^{\alpha_1\ldots\alpha_{D-3}}x^2 + \frac{\eta^{\alpha_1\ldots\alpha_{D-3}}_{\beta_1\ldots\beta_{D-3}}}{(D-4)!}c^{\beta_1\ldots\beta_{D-4}\sigma}x^{\beta_{D-3}}x_\sigma\right), \tag{20}$$

where $\tilde{a}^{\alpha_1\ldots\alpha_{D-2}} \equiv \frac{1}{2}\epsilon^{\alpha_1\ldots\alpha_{D-2}\mu\nu}a_{\mu\nu}$ and $\tilde{c}^{\alpha_1\ldots\alpha_{D-3}} \equiv \frac{1}{3!}\epsilon^{\alpha_1\ldots\alpha_{D-3}\mu\nu\rho}c_{\mu\nu\rho}$. By direct computation, we find the corresponding divergences to be given by A and C, respectively; i.e., $\partial^\rho\mathcal{A}_{\mu\nu\rho} = \mathsf{A}_{\mu\nu}$ and $\partial^\rho\mathcal{C}_{\mu\nu\rho} = \mathsf{C}_{\mu\nu}$. Equivalently,

$$d\star\mathcal{A} \propto \star\,\mathsf{A}, \qquad d\star\mathcal{C} \propto \star\,\mathsf{C}. \tag{21}$$

Hence, $\star\,\mathsf{A}$ and $\star\,\mathsf{C}$ are exact in the physical algebra of the theory in $D > 4$. Fluxes constructed from them belong to the additive algebra of the ring and do not generate generalized symmetries. Note that (19) and (20) do not correspond to skew-symmetric differential forms in $D = 4$. One could try to antisymmetrize the free indices, but such a procedure results in both $\mathcal{A}_{\mu\nu\rho}$ and $\mathcal{C}_{\mu\nu\rho}$ vanishing identically.

Summarizing, for $D > 4$ we find two families of conserved 2-forms generating generalized symmetries: the B's and the D's; see (11) and (13). They generate a total of $D(D+1)/2$ conserved charges. In the complementary regions we also find two families of conserved $(D-2)$-forms generating generalized symmetries. These are the $\tilde{\mathsf{A}}$'s and the $\tilde{\mathsf{B}}$'s; see (16) and (17). They generate an equal number of $D(D+1)/2$ conserved charges. The two sets contain the same number of charges and have the right dimensions to produce nonvanishing commutators. Starting from the ADM formalism [49], we have evaluated such commutators explicitly in the Supplemental Material [26]. They read

$$\left[\int_{\Sigma_{(D-2)}}(\star\,\mathsf{B}+\star\,\mathsf{D}), \int_{\Sigma_2}(\star\,\tilde{\mathsf{B}}+\star\,\tilde{\mathsf{A}})\right] = i(D-3)\left(2\eta_{\mu\nu}b^\mu\tilde{b}^\nu + \eta_{\mu\nu}\eta_{\rho\sigma}d^{\mu\rho}\tilde{a}^{\nu\sigma}\right), \tag{22}$$

for regions whose boundaries have nontrivial linkings. This verifies the B's are paired with the $\tilde{\mathsf{B}}$'s and the D's are paired with the $\tilde{\mathsf{A}}$'s, in agreement with the four-dimensional results of [8].

An interesting question is if in $D \geq 6$ one can find conserved p-forms with $p \neq 2$ and $p \neq (D-2)$. This is the case, but all the ones we found were always exact forms, and do not generate new symmetries.

Generalized symmetries for linearized higher-curvature gravities.—We consider now a general gravity action built from the Riemann tensor and the metric

$$S = \frac{1}{16\pi G}\int d^Dx\sqrt{|g|}\mathcal{L}(g^{\alpha\beta}, R^\rho_{\sigma\mu\nu}). \tag{23}$$

As shown in the Supplemental Material (which also includes [50–55]) [26], the most general theory of this form contributing to the linearized equations of motion on Minkowski space is quadratic in the Riemann tensor,

$$\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}, \tag{24}$$

where $\alpha_{1,2,3}$ are arbitrary dimensionful constants. Following [56], we write α_1 and α_2 in terms of two new parameters, m_s and m_g:

$$\alpha_1 \equiv \frac{(D-2)m_g^2 + Dm_s^2}{4(D-1)m_s^2m_g^2} + \alpha_3, \qquad \alpha_2 \equiv \frac{-1}{m_g^2} - 4\alpha_3. \tag{25}$$

The equations read now

$$\mathcal{E}^{(1)}_{\mu\nu} \equiv \left[1 - \frac{\partial^2}{m_g^2}\right]R^{(1)}_{\mu\nu} - \Delta_{\mu\nu}R^{(1)} = 0, \tag{26}$$

where

$$\Delta_{\mu\nu} \equiv \frac{\eta_{\mu\nu}}{2}\left[1 - \frac{\partial^2}{m_g^2}\right] + \frac{(D-2)(m_g^2 - m_s^2)}{2(D-1)m_s^2m_g^2}[\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^2]. \tag{27}$$

Equation (26) reduces to the Einstein one for $m_g^2, m_s^2 \to \infty$. As explained, e.g., in [56], m_g^2 and m_s^2 correspond to the squared masses of two—spin-0 and spin-2, respectively—extra modes which appear in the spectrum.

Since the gauge symmetry (7) remains the same, the Riemann tensor is still the generator of gauge-invariant operators. However, the Ricci tensor does not vanish on shell anymore and, consequently, $R^{(1)}_{\mu\nu\rho\sigma}$ is no longer a conserved current. On the other hand, the properties of the dual Riemann tensor do not change. Hence, we obtain the very same set of $D(D+1)/2$ independent $(D-2)$-form conserved currents corresponding to the families $\tilde{\mathsf{A}}$ and $\tilde{\mathsf{B}}$ as for Einstein gravity; see (16) and (17).

Since these currents are charged under spacetime symmetries, the dual-pairs principle should imply the existence of an equal number of $D(D+1)/2$ dual 2-form conserved currents. Equivalently, this suggests the existence of a generalized tensor playing the role of the Riemann. A candidate is given by

$$\mathcal{W}_{\mu\nu\alpha\beta} \equiv \mathcal{R}_{\mu\nu\alpha\beta} + \frac{2}{(D-2)}\left[\eta_{\nu[\alpha}\mathcal{R}_{\beta]\mu} - \eta_{\mu[\alpha}\mathcal{R}_{\beta]\nu}\right] + \frac{2}{(D-2)(D-1)}\eta_{\mu[\alpha}\eta_{\beta]\nu}\mathcal{R}, \tag{28}$$

where we defined

$$\mathcal{R}_{\mu\nu\alpha\beta} \equiv \left[1 - \frac{\partial^2}{m_g^2}\right] R^{(1)}_{\mu\nu\alpha\beta} + 2\Delta_{\mu[\beta}R^{(1)}_{\alpha]\nu} + 2\Delta_{\nu[\alpha|}R^{(1)}_{\beta]\mu}. \tag{29}$$

The tensor $\mathcal{W}_{\mu\nu\alpha\beta}$ is traceless and satisfies its own Bianchi identity, $\eta^{\mu\alpha}\mathcal{W}_{\mu\nu\alpha\beta} = 0$, $\varepsilon^{\mu_1\cdots\mu_{D-3}\alpha\beta\gamma}\mathcal{W}_{\alpha\beta\gamma\nu} = 0$. One can show that it is a conserved current on shell and that the second Bianchi identity holds only in two cases: (i) when the additional spin-2 mode is absent from the spectrum ($m_g^2 \to \infty$) and (ii) when $m_s^2 = m_g^2$. In those situations $\partial^\mu \mathcal{W}_{\mu\nu\alpha\beta} = 0$ and $\varepsilon^{\mu_1\cdots\mu_{D-3}\alpha\beta\gamma}\partial_\alpha \mathcal{W}_{\beta\gamma\mu\nu} = 0$. In the first case, the quadratic part of the action reduces to a single R^2 term. In the second, we are left with a single Weyl^2 term. In both situations, it follows that $\mathcal{W}_{\mu\nu\alpha\beta}$ generates nontrivial charges of the B and D classes identical to the Einstein gravity ones—see (11) and (13)—by simply replacing $R^{(1)}_{\mu\nu\rho\sigma}$ with $\mathcal{W}_{\mu\nu\rho\sigma}$. Analogously, one can show that the putative A and C ones are exact in a similar way to (19) and (20). Note that $\mathcal{R}_{\mu\nu\alpha\beta}$ does not define additional charges. Similarly, one might define $\tilde{\mathsf{A}}$ and $\tilde{\mathsf{B}}$ charges using the dual of $\mathcal{W}_{\mu\nu\alpha\beta}$. However, one can show that such charges produce the same nonlocal classes as the ones defined in (16) and (17). All these claims are proven in the Supplemental Material [26].

Summarizing, in the absence of the additional spin-2 mode, we find that the higher-curvature theories possess $D(D+1)$ conserved currents, organized in two equal-size dual sets $\{\tilde{A}, \tilde{B}\}$ and $\{B, D\}$. The currents are continuous deformations of the Einsteinian ones.

When m_g^2 is finite and $m_s^2 \neq m_g^2$, this construction fails and we find a violation of the dual-pairs principle. This is likely related to the fact that the additional spin-2 mode is a ghost [56,57], whose presence renders the theory nonunitary [58,59]. It is reasonable to expect generalized symmetries and the dual-pairs principle to be sensitive to such issue. Nonetheless, it is also a logical possibility that a more elusive set of charges exists in this case and ends up saving the day for these theories.

On the other hand, the case $m_g^2 = m_s^2$ has similar unitarity problems [59]. The fact that this does not violate the dual-pairs principle suggests that consistent theories will always respect such principle, but that the opposite implication will not be true in general.

Conclusions and future work.—In this Letter we have found $D(D+1)$ generalized symmetries for linearized Einstein gravity as well as for higher-curvature gravities propagating an additional spin-0 mode in general dimensions. Half of the symmetries are generated by 2-form currents and the other half by $(D-2)$-form currents, which verifies the QFT principle that generalized symmetries always come in dual pairs. In the case of higher-curvature gravities propagating an additional massive spin-2 mode, the theory is nonunitarity, and the dual-pairs principle seems to be violated.

An interesting outcome is that generalized gravitons can be defined by their generalized symmetries, supporting the perspective of [9]; see the Supplemental Material [26], which also includes [60]. More precisely, linearized gravity is a theory of symmetry, characterized by the conservation of its closed-form currents. This parallels the case of the Maxwell field. It is also reminiscent of AdS/CFT [61,62], where gravity is dual to the dynamics of the CFT stress tensor, constrained by its conservation, tracelessness and associated Ward identities.

Further interesting outcomes are that the graviton generalized symmetries are charged under spacetime symmetries. Following [8] this implies the Weinberg-Witten theorem [63] for these theories. It also implies that these theories enlarge the space of so-called tensor gauge theories [25], providing further examples of the proposed connection between gravity and fractonic systems [8–10,64–67].

There are several venues for future work. First, our analysis could be extended to linearized theories of gravity whose Lagrangian is also a functional of the covariant derivative, theories with explicit mass terms in the action, and more general backgrounds. A more difficult question is whether the existence of a nontrivial space of low-energy gravity theories—defined in terms of the spectrum of generalized symmetries—implies the existence of a similar space of UV completions. One expects this not to be the case, and that the absence of generalized symmetries in quantum gravity [19,20] should lead to a unified theory in the UV, where all these different phases are smoothly connected to each other.

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