

## Non-Hermitian Waveguide Cavity QED with Tunable Atomic Mirrors

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(Received 11 April 2023; accepted 25 July 2023; published 7 September 2023)

Optical mirrors determine cavity properties by means of light reflection. Imperfect reflection gives rise to open cavities with photon loss. We study an open cavity made of atom-dimer mirrors with a tunable reflection spectrum. We find that the atomic cavity shows anti- $\mathcal{PT}$  symmetry. The anti- $\mathcal{PT}$  phase transition controlled by atomic couplings in mirrors indicates the emergence of two degenerate cavity supermodes. Interestingly, a threshold of mirror reflection is identified for realizing strong coherent cavity-atom coupling. This reflection threshold reveals the criterion of atomic mirrors to produce a good cavity. Moreover, cavity quantum electrodynamics with a probe atom shows mirror-tuned properties, including reflection-dependent polaritons formed by the cavity and probe atom. Our Letter presents a non-Hermitian theory of an anti- $\mathcal{PT}$  atomic cavity, which may have applications in quantum optics and quantum computation.

DOI: [10.1103/PhysRevLett.131.103602](https://doi.org/10.1103/PhysRevLett.131.103602)

*Introduction.*—Quantum cavities are the cornerstone of quantum optics for interfacing light-matter interaction [1–4]. The Fabry-Perot interferometer is a cavity that consists of two parallel optical mirrors. A Fabry-Perot cavity can be realized in many quantum systems with varieties of light reflectors [5]. Meanwhile, Fabry-Perot cavities can be synthesized with quantum mirrors, e.g., atoms or artificial atoms [6–12], based on the remarkable photon scattering effects of single atoms in one-dimensional (1D) waveguides [13–21]. Recently, experiments have realized strong coupling in cavity quantum electrodynamics (QED) with single-atom mirrors [22]. Large reflection of atomic mirrors at the central frequency gives rise to a dark mode, i.e., the effective cavity mode [6,9]. Atomic cavities receive much attention for studying quantum optics with tailored light-matter interaction [23–29]. However, due to the limitation in tuning mirror reflection, atomic cavities are far from being thoroughly understood.

Generally speaking, photon loss is unavoidable in quantum cavities [30], because of imperfect mirror reflections that lead to the coupling between cavity modes and continuum in free space [31–34]. Indeed, open cavities have substantial applications in quantum computation [35,36] and quantum networks [37–39]. Knowing how mirrors alter photon reflections is central to design novel quantum devices [40–42]. For example, tailored reflection (transmission) of cavities is useful for practical quantum technologies, such as single-photon resources [43,44] and

Fano lasers [45,46]. A puzzle naturally arises: what is the fate of an open cavity if the mirror reflection is strongly modified? Different from previous theories of open cavities based on conventional light reflectors [31–34,43–46], waveguide-interfaced quantum light-matter interactions [47,48] make atomic cavities an excellent platform to study cavity QED [6–12,22–29] and might shed new light on open cavities.

In this Letter, we study a cavity consisting of atomic mirrors with two-coupled atoms, i.e., atom dimer, coupled to a 1D coplanar waveguide in superconducting quantum circuits [49]. We find that the atom-dimer cavity has anti-parity-time (anti- $\mathcal{PT}$ ) symmetry. Different from previous works that tune anti- $\mathcal{PT}$  symmetric systems via frequency detunings [50–58], here atomic couplings in mirrors alter light reflection and induce non-Hermitian phase transitions of the cavity. Therefore, two degenerate cavity supermodes are created. We propose a non-Hermitian theory for atomic cavity QED.

*Tunable reflection by anti-Bragg atom-dimer mirror.*—Figure 1(a) shows the schematics of a Fabry-Perot cavity with a probe atom. The mirror can be realized, e.g., with a single two-level atom coupled to a waveguide [14–16]. The single-atom reflection spectrum has a Lorentzian line shape [14]  $R(\Delta) = |\Gamma/(\Delta + i\Gamma)|^2$ , with atomic decay rate  $\Gamma$  and detuning  $\Delta = \omega - \omega_0$ . Here,  $\omega$  and  $\omega_0$  are the frequencies of the driving field and atom, respectively. Because of this single-peak reflection, a dark cavity mode can be generated in an atomic cavity [22].

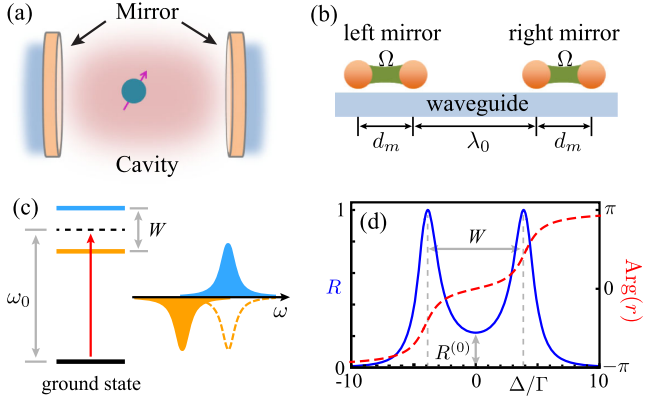


FIG. 1. (a) Schematic of a Fabry-Perot cavity with two parallel mirrors. (b) Atomic cavity consisting of two atom-dimer mirrors, separated by a distance  $\lambda_0$ , coupled to a waveguide. For each mirror, we consider a direct atomic coupling  $\Omega$  and anti-Bragg condition  $d_m = \lambda_0/4$ . (c) Energy levels of scattering states in the anti-Bragg atom-dimer mirror with  $W = 2(\Omega + \Gamma)$  (left) and corresponding reflected photon amplitudes (right). (d) Reflection  $R = |r|^2$  (blue solid) and photon phase shift  $\text{Arg}(r)$  (red dashed) produced by the atom-dimer mirror.

Two atoms produce a scattering state with radiation rate  $2\Gamma$  [20,59] for an atomic spacing  $n\lambda_0/4$ , with an even number  $n$  and single-photon wavelength  $\lambda_0 = 2\pi c/\omega_0$ , i.e., the Bragg condition [60,61]. Such collectively enhanced scattering state (superradiant state) leads to broadband reflection with a Lorentzian profile, useful for high-finesse cavities [9]. However, the scenario is different for spacing  $m\lambda_0/4$  (with  $m$  an odd number), i.e., the anti-Bragg condition [62,63]. We study an atom dimer with  $d_m = \lambda_0/4$  [49], as shown in Fig. 1(b). The master equation is ( $\hbar = 1$ )  $\dot{\rho}(t) = -i[H_m, \rho(t)] + \mathcal{D}[\rho]$ , where the Hamiltonian is  $H_m = \sum_{j=1,2} \omega_0 \sigma_j^+ \sigma_j^- + (\Omega + \Gamma)(\sigma_1^+ \sigma_2^- + \text{H.c.})$ , with a direct atomic coupling  $\Omega$  and waveguide-mediated dispersive coupling  $\Gamma$ . Here,  $\sigma_j^+ = |e_j\rangle\langle g_j|$  with the ground (excited) state  $|g\rangle$  ( $|e\rangle$ ). The Lindblad operator is  $\mathcal{D}[\rho] = \sum_j \Gamma(2\sigma_j^- \rho \sigma_j^+ - \sigma_j^+ \sigma_j^- \rho - \rho \sigma_j^+ \sigma_j^-)$ . In this mirror, there are two scattering states  $\Phi_{\pm} = (1/\sqrt{2})(1, \pm 1)^T$  with equal decay rate  $\Gamma$ , as shown in Fig. 1(c). The photon amplitude reflected by the anti-Bragg atom-dimer mirror is [64]

$$r(\Delta) = \sum_{n=\pm} (-1)^n \frac{\Gamma}{(\Delta - \delta_n + i\Gamma)}, \quad (1)$$

where  $\delta_{\pm}$  are frequencies of the scattering states with respect to  $\omega_0$ . Quantum interference between reflected photons is determined by the frequency difference  $W = 2(\Omega + \Gamma)$  between two scattering states. Figure 1(d) shows the intensity and phase shift of the reflected photon. At the central frequency  $\Delta = 0$ , phase shifts 0 and  $\pi$  are produced, respectively, for anti-Bragg atom-dimer mirrors with  $W > 0$

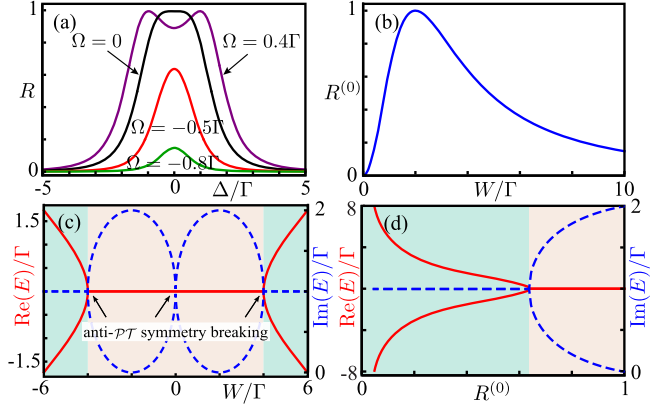


FIG. 2. (a) Reflection of the atom-dimer mirror for various values of atomic coupling. (b) Reflection  $R^{(0)}$  at  $\Delta = 0$  versus the frequency difference  $W$  between two scattering states in the mirror. (c) Anti- $\mathcal{PT}$  phase transitions in the anti-Bragg atom-dimer cavity. Red solid and blue dashed curves correspond to real and imaginary parts of two supermodes. (d) Reflection-dependent atom-dimer cavity. A low-loss cavity supermode is produced by large mirror reflection.

and  $W < 0$ , due to swapping of scattering states. Without loss of generality, we focus on the regime  $W \geq 0$ .

In Fig. 2(a), we show details of the reflection spectra altered by the atomic coupling  $\Omega$ . At  $\Omega = -\Gamma$  ( $W = 0$ ), atoms in the mirror have no coupling [49]. Interestingly, degenerate scattering states produce out-of-phase photon components. Because of destructive quantum interference, the anti-Bragg atom-dimer mirror becomes transparent for incident photons with various frequencies, i.e., trivial mirror. Single- and two-peak reflection spectra are obtained for  $-\Gamma < \Omega \leq 0$  and  $\Omega > 0$ , respectively, enabling the study of reflection-tuned atomic cavities. From Eq. (1), we obtain the reflection at  $\Delta = 0$ ,

$$R^{(0)} = \frac{\Gamma^2 W^2}{(\Gamma^2 + \frac{W^2}{4})^2}. \quad (2)$$

The reflection  $R^{(0)}$  responsible for cavity losses [9,45] is nontrivially changed by  $W$ . As shown in Fig. 2(b),  $R^{(0)}$  increases with  $W$  for single-peak reflection. After reaching unity at  $W = 2\Gamma$ ,  $R^{(0)}$  is reduced, producing a two-peak reflection spectrum. Therefore,  $W$  plays an important role in controlling the reflection spectrum of the atom-dimer mirror.

*Atomic cavity protected by anti- $\mathcal{PT}$  symmetry.*—To explore the relation between atomic mirrors and cavity, we consider two atom-dimer mirrors with Hamiltonian  $H_0 = \sum_i \Omega(\sigma_{\mathcal{M}_i, \mathcal{A}_1}^+ \sigma_{\mathcal{M}_i, \mathcal{A}_2}^- + \text{H.c.})$ , where  $i = l, r$  denotes the left and right atomic mirror, and  $\mathcal{A}_1, \mathcal{A}_2$  represent mirror atoms. Considering the cavity architecture in Fig. 1(b), we trace out the degrees of freedom of photons in the waveguide [66–72] and obtain an effective

non-Hermitian Hamiltonian in the single-excitation subspace  $\{|\psi_j\rangle = \sigma_j^+ |g_1 g_2 g_3 g_4\rangle\}$  of four mirror atoms [64],

$$H_c = (\Omega + \Gamma)s_0 \otimes \tau_x + \Gamma s_x \otimes \tau_0 - i\Gamma s_y \otimes \tau_y - i\Gamma s_0 \otimes \tau_0, \quad (3)$$

where  $s_n$  and  $\tau_n$  ( $n = x, y, z$ ) are Pauli matrices in the space  $\{\mathcal{M}_l, \mathcal{M}_r\}$  of two mirrors and the subspace  $\{\mathcal{A}_1, \mathcal{A}_2\}$  of mirror atoms, respectively. We have assumed that two atomic mirrors are separated by  $\lambda_0$ . Equation (3) describes the quantum light-mirror interaction in the atomic cavity. To clarify the mechanism of the cavity, we make a unitary transformation to Eq. (3) (see Supplemental Material [64]) and simplify the system as two decoupled subsystems  $\mathcal{H}_1 \oplus \mathcal{H}_2$  with

$$\mathcal{H}_1 = \begin{pmatrix} -\Omega - i\Gamma & -i\Gamma \\ -i\Gamma & \Omega - i\Gamma \end{pmatrix}, \quad (4)$$

$$\mathcal{H}_2 = \begin{pmatrix} -\Omega - 2\Gamma - i\Gamma & i\Gamma \\ i\Gamma & \Omega + 2\Gamma - i\Gamma \end{pmatrix}. \quad (5)$$

$\mathcal{H}_1$  and  $\mathcal{H}_2$  are protected by anti- $\mathcal{PT}$  symmetry  $(\mathcal{PT})\mathcal{H}_i(\mathcal{PT})^{-1} = -\mathcal{H}_i$  [73–76]. Importantly, without using frequency detunings [50–58], here anti- $\mathcal{PT}$  phase transitions are produced by atomic couplings, which can uncover novel properties of anti- $\mathcal{PT}$  symmetric systems. Moreover, we avoid the influence of frequency-dependent couplings [77,78] on the atom-dimer cavity.

The anti- $\mathcal{PT}$  symmetry inspires us to study the atom-dimer cavity using non-Hermitian theory [79–82]. We diagonalize the system as  $H_c = \sum_n E_n |\Psi_n^R\rangle \langle \Psi_n^L|$ , with the biorthogonal basis  $\langle \Psi_n^L | \Psi_m^R \rangle = \delta_{nm}$ . The index  $j$  labels supermodes of the atomic cavity. In Fig. 2(c), we show real and imaginary parts of the eigenvalues for two supermodes in the anti- $\mathcal{PT}$ -symmetry-protected regimes  $W \in (-4\Gamma, 0) \cup (0, 4\Gamma)$ . Anti- $\mathcal{PT}$  phase transitions take place at second-order exceptional points  $W = 0, \pm 4\Gamma$ . For  $0 \leq W \leq 4\Gamma$ , the eigenvalues of  $\mathcal{H}_1$  correspond to two degenerate supermodes  $\Psi_{\pm}$  with decay rates

$$\Gamma_{\pm} = \Gamma \pm \sqrt{\Gamma^2 - \Omega^2}. \quad (6)$$

At  $\Omega = 0$ , a long-living supermode exists in the atom-dimer cavity with mirrors having complete reflection  $R^{(0)}$  at the central frequency. This supermode plays the role of a cavity mode [22] and becomes dissipative for reduced  $R^{(0)}$ . By solving Eq. (2) for  $\Omega$  and substituting the solutions to Eq. (6), we obtain reflection-tuned energy levels and decay rates of the supermodes  $\Psi_{\pm}$  for  $W \geq 2\Gamma$ , as shown in Fig. 2(d). For weak reflection, these two supermodes have equal decay rate  $\Gamma$ , the same as individual mirror atoms. However, large reflection leads to an anti- $\mathcal{PT}$  phase transition, producing cavity supermodes with controlled loss.

*Reflection threshold for strong cavity-atom coupling.*— In addition to photon loss, the mode profile is a fundamental property of cavities [83,84]. In optical cavities, the cavity-atom coupling is proportional to the cavity mode electric field, which is related to boundary conditions imposed by mirrors [43,44]. To study cavity fields affected by atomic mirrors, we consider a probe atom [see Fig. 1(a)]. The atom-cavity interaction is described by  $H_{\text{int}} = (\delta\omega - i\gamma)\sigma_p^+ \sigma_p^- - i\sqrt{\gamma}\Gamma \sum_{j=1}^4 e^{i\phi_j} (\sigma_j^+ \sigma_p^- + \sigma_p^+ \sigma_j^-)$ . Here,  $\delta\omega = \omega_p - \omega_0$  is the detuning between the probe atom and mirror atoms,  $\gamma$  is decay rate of the probe atom,  $\sigma_j^{\pm}$  are operators of the  $j$ th mirror atoms, and  $\phi_j = 2\pi|x_j - x_p|/\lambda_0$  [85–87]. The probe atom is placed at  $x_p = \lambda_0/4$  in the cavity, such that the couplings vanish between the probe atom and supermodes unprotected by anti- $\mathcal{PT}$  symmetry [64]. By writing the whole Hamiltonian  $\tilde{H} = H_c + H_{\text{int}}$  in terms of two supermodes  $\Psi_{\pm}$ , we obtain

$$\tilde{H} = \begin{pmatrix} -i\Gamma_- & 0 & G_L \\ 0 & -i\Gamma_+ & V_L \\ G_R & V_R & \delta\omega - i\gamma \end{pmatrix}, \quad (7)$$

where the couplings are  $G_L = -i\sqrt{\gamma}\Gamma \sum_j e^{i\phi_j} \langle \Psi_-^L | \psi_j \rangle$ ,  $G_R = -i\sqrt{\gamma}\Gamma \sum_j e^{i\phi_j} \langle \psi_j | \Psi_-^R \rangle$ , and similar for  $V_{L,R}$  by considering the supermode  $\Psi_+$ . With the protection of anti- $\mathcal{PT}$  symmetry, the probe atom is coherently and dissipatively coupled to the slow- and fast-decay supermodes with the same coupling strength, i.e.,  $\text{Im}[G_R] = 0$  and  $V_R = iG_R$ . As shown in Fig. 3(a), the probe atom is

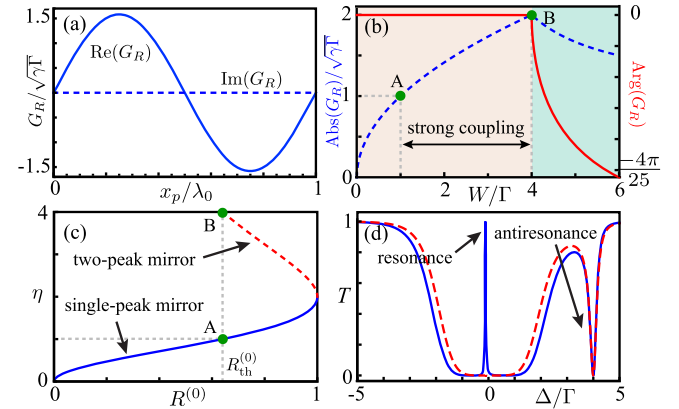


FIG. 3. (a) Coupling between probe atom and the slow-decay supermode. The solid curve and dashed line denote the real and imaginary parts of the coupling  $G_R$ . The horizontal axis represents the position of the probe atom in the cavity. (b) Absolute value and argument of cavity-atom coupling  $G_R$  for the slow-decay supermode. The probe atom is located at  $x_p = 0.25\lambda_0$ . (c) Atomic cavity field changed by the mirror reflection. (d) Transmission spectrum of the far-detuned probe atom. Red dashed and blue solid curves correspond to  $\Omega = 0$  and  $0.2\Gamma$ , respectively.

maximally coupled to the supermode  $\Psi_-$  at  $x_p = \lambda_0/4$ . We find  $G_L = G_R/\sqrt{1 - \Omega^2/\Gamma^2}$  and  $V_L = V_R/\sqrt{1 - \Omega^2/\Gamma^2}$ , which diverge at exceptional points. Hence,  $G_R$  and  $V_R$  characterize the effective fields of the atomic cavity.

Figure 3(b) presents the absolute value and argument of the coupling  $G_R$  between the atom and the slow-decay supermode. Indeed, the coupling is coherent for  $0 \leq W \leq 4\Gamma$ , and its concise form is

$$G_R = \sqrt{\gamma W}. \quad (8)$$

This equation uncovers an intrinsic relation between mirror reflection and atom-dimer cavity: the frequency difference  $W$  between mirror's scattering states determines the atom-cavity coupling. At point A,  $W = \Gamma$  indicates the emergence of enhanced cavity-atom coupling with respect to the couplings between probe atom and mirror atoms. At point B with  $W = 4\Gamma$ , the cavity undergoes a phase transition and the coherent cavity-atom coupling reaches its maximum. Therefore, the parameter space between A and B is the strong-coupling regime.

To gain further insight into the relation between mirror reflection and the atom-dimer cavity, we define a coupling factor  $\eta = G_R^2/\gamma\Gamma$  in terms of  $R^{(0)}$  via Eq. (2). The coupling factor corresponding to atomic mirrors with single-peak reflection spectra is  $\eta = 2(1 - \sqrt{1 - R^{(0)}})/\sqrt{R^{(0)}}$ . As shown in Fig. 3(c),  $\eta$  monotonically increases with  $R^{(0)}$ , showing a reflection-controlled cavity field [33]. For two-peak mirrors, the coupling factor becomes  $\eta = 2\sqrt{R^{(0)}}/(1 - \sqrt{1 - R^{(0)}})$ . It increases with growing  $W$  even though  $R^{(0)}$  declines. However, when  $R^{(0)}$  reduces to a critical value at point B, the coupling  $G_R$  becomes dissipative. Interestingly, we find that points A and B correspond to a reflection threshold  $R_{\text{th}}^{(0)} = 0.64$ . The strong coherent coupling requires  $R^{(0)} > R_{\text{th}}^{(0)}$ .

In Fig. 3(d), we show the transmission spectrum of the detuned cavity-atom system. The antiresonance in the transmission is due to the probe atom [14–16]. Owing to the anti-Bragg scattering, dissipative supermodes in the atomic cavity produce a transmissionless spectrum for  $\Omega = 0$ . Weak atomic coupling leads to a photon transmission amplitude  $t \approx \Gamma_-/[i(\Delta + \Delta_p) + \Gamma_-]$ , where  $\Delta_p$  is the frequency shift induced by the probe atom. In contrast to conventional subradiant states that inhibit photon transmission [88], the slow-decay supermode  $\Psi_-$  enhances photon transmission. This cavitylike behavior [89] makes it useful to detect atomic cavity QED via photon transport in a waveguide.

*Mirror-controlled cavity-atom polaritons.*—In cavity QED, polaritons can be produced with interacting photons and atoms or excitons [90,91]. In our system, non-Hermitian cavity-atom interactions in Eq. (7) control the formation of cavity-atom polaritons. Solutions of the equation  $\det(\tilde{H} - E) = 0$  can be derived using Cardano's

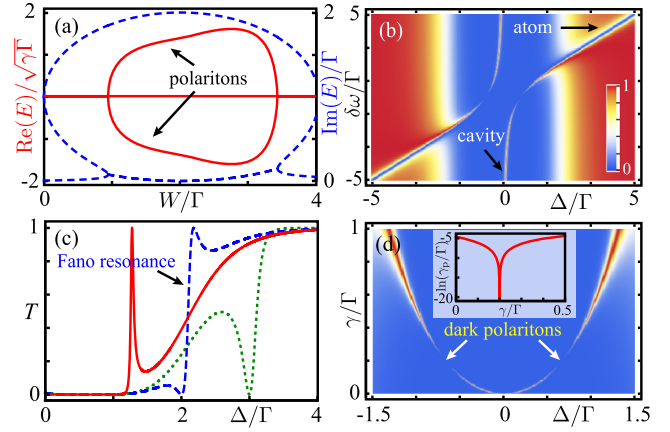


FIG. 4. (a) Cavity-atom polaritons are formed with  $\gamma = 0.005\Gamma$ . (b) Transmission detection of polaritons for  $\gamma = 0.2\Gamma$ . (c) Polariton-induced Fano resonance. Red solid, blue dashed, and green dotted curves correspond to  $\delta\omega/\Gamma = 1, 2,$  and  $3,$  respectively. (d)  $\gamma$ -dependent polaritons. The inset shows  $\ln(\gamma_p/\Gamma)$  versus  $\gamma$ , where  $\gamma_p$  denotes decay rate of polaritons. We consider  $\Omega = 0.2\Gamma$  in (b)–(d),  $\gamma = 0.2\Gamma$  in (b),(c), and  $\delta\omega = 0$  in (a),(d).

formula [64,92]. In Fig. 4(a), we show the eigenspectrum of the cavity-atom system. In the strong-coupling regime, the probe atom hybridizes intensely with the slow-decay supermode, giving rise to two equally decaying polaritons. We find that, at the condition  $\Omega = \gamma$ , the eigenvalues are  $E_{1,2} = \pm\sqrt{\Omega^2 + 2\Gamma\gamma}$  and  $E_3 = -i(2\Gamma + \gamma)$ . Here,  $E_{1,2}$  correspond to two polaritons without dissipation, i.e., dark polaritons; while  $E_3$  contains the whole dissipation. Therefore, the fast-decay supermode is crucial for tuning decay rates of polaritons. Moreover, the dark polaritons are only generated in cavities with two-peak atom-dimer mirrors, because  $\gamma > 0$ . As shown in Fig. 4(b), the cavity-atom polaritons can be detected by cavity transmission. The vanishing signals at resonance represent dark polaritons. The dark polaritons studied here emerge from the novel non-Hermitian interaction between the probe atom and the two-mode atomic cavity and cannot be produced by single-mode optical cavities [93].

In Fig. 4(c), we show the transmission of one polariton for various atom-cavity frequency detunings. A Fano resonance is found around  $\delta\omega = 2\Gamma$ . Considering distinct spectroscopic signatures of the probe atom and the cavity supermode  $\Psi_-$  shown in Fig. 3(d), the Fano resonance reveals the half-matter half-light nature of polaritons, useful for optical switching and sensing [94]. Figure 4(d) displays the transmission spectrum of  $\gamma$ -dependent polaritons. The frequencies of polaritons agree with Eq. (8). This confirms the effective cavity-atom coupling represented by  $G_R$ . The inset shows the linewidth of polaritons versus  $\gamma$  with a minimum at  $\gamma = \Omega$ .

*Applications of the atom-dimer cavity.*—Slow-decay states, or subradiant states, are useful for quantum

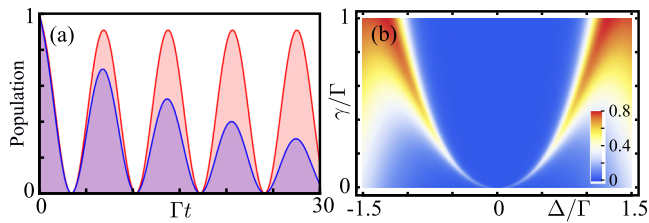


FIG. 5. (a) Rabi oscillations of a probe atom due to the dark polaritons. We consider an atomic coupling  $\Omega = 0.1\Gamma$  and decay rate  $\gamma = 0.1\Gamma$  of probe atom. Red and blue curves respond to free-space loss  $\gamma' = 0$  and  $\gamma' = 0.02\Gamma$ , respectively. (b) Transmission spectrum of cavity-atom polaritons for  $\Omega = -0.2\Gamma$  and free-space loss  $\gamma' = 0.02\Gamma$ .

information storage [95,96]. However, it is challenging to access and manipulate these subtle many-body states [97–99]. The atom-dimer cavity provides an interface between a single atom and a subradiant state. We show the persistent Rabi oscillations of the probe atom in Fig. 5(a), produced by dark polaritons. After considering the free-space loss [22], the population transfer is still efficient. Because of this cavity-atom interface, quantum information can be flexibly stored in and retrieved from the atom-dimer cavity. Figure 5(b) shows the transmission spectrum of polaritons for a cavity with single-peak mirrors. Different from the two-peak atomic cavity, only bright polaritons are formed.

*Conclusions.*—In this Letter, we study an open cavity with tunable atom-dimer mirrors. Atomic couplings in mirrors nontrivially control the anti- $\mathcal{PT}$ -symmetry-protected cavity and produce two reflection-dependent degenerate supermodes. The coherent coupling between the slow-decay supermode and the probe atom is related to frequency difference between mirror’s scattering states. We propose a non-Hermitian cavity QED theory and identify a reflection threshold for strong cavity-atom coupling. The roles played by the slow- and fast-decay supermodes are clarified for realizing dark polaritons. Our Letter presents a novel cavity in a coupling-controlled anti- $\mathcal{PT}$  symmetric system, which allows one to study non-Hermitian light-matter interaction.

The authors thank Stephen Hughes and Zongping Gong for critical readings. W.N. is supported by the National Natural Science Foundation of China (Grant No. 12105025). T.S. acknowledges support from National Key Research and Development Program of China (Grant No. 2017YFA0718304) and the NSFC (Grants No. 11974363, No. 12135018, and No. 12047503). Y.X.L. is supported by the NSFC under Grant No. 11874037, the Key-Area Research and Development Program of Guangdong Province under Grant No. 2018B030326001, and the Innovation Program for Quantum Science and Technology (Grant No. 2021ZD0300200). F.N. is supported in part by Nippon Telegraph and Telephone Corporation (NTT)

Research, the Japan Science and Technology Agency (JST) [via the Quantum Leap Flagship Program (Q-LEAP) and the Moonshot R&D Grant No. JPMJMS2061], the Asian Office of Aerospace Research and Development (AOARD) (via Grant No. FA2386-20-1-4069), and the Foundational Questions Institute Fund (FQXi) via Grant No. FQXi-IAF19-06.

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- [1] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).
- [2] G. S. Agarwal, *Quantum Optics* (Cambridge University Press, Cambridge, England, 2012).
- [3] S. Haroche and J.-M. Raimond, *Exploring the Quantum: Atoms, Cavities, and Photons* (Oxford University Press, New York, 2006).
- [4] I. Carusotto and C. Ciuti, Quantum fluids of light, *Rev. Mod. Phys.* **85**, 299 (2013).
- [5] A. V. Kavokin, J. J. Baumberg, G. Malpuech, and F. P. Laussy, *Microcavities* (Oxford University Press, New York, 2017), 10.1093/oso/9780198782995.001.0001.
- [6] L. Zhou, H. Dong, Y.-X. Liu, C. P. Sun, and F. Nori, Quantum supercavity with atomic mirrors, *Phys. Rev. A* **78**, 063827 (2008).
- [7] Y. Chang, Z. R. Gong, and C. P. Sun, Multiatomic mirror for perfect reflection of single photons in a wide band of frequency, *Phys. Rev. A* **83**, 013825 (2011).
- [8] G. Hétet, L. Slodička, M. Hennrich, and R. Blatt, Single Atom as a Mirror of an Optical Cavity, *Phys. Rev. Lett.* **107**, 133002 (2011).
- [9] D. E. Chang, L. Jiang, A. Gorshkov, and H. Kimble, Cavity QED with atomic mirrors, *New J. Phys.* **14**, 063003 (2012).
- [10] F. Fratini, E. Mascarenhas, L. Safari, J.-P. Poizat, D. Valente, A. Auffèves, D. Gerace, and M. F. Santos, Fabry-Perot Interferometer with Quantum Mirrors: Nonlinear Light Transport and Rectification, *Phys. Rev. Lett.* **113**, 243601 (2014).
- [11] I.-C. Hoi, A. F. Kockum, L. Tornberg, A. Pourkabirian, G. Johansson, P. Delsing, and C. M. Wilson, Probing the quantum vacuum with an artificial atom in front of a mirror, *Nat. Phys.* **11**, 1045 (2015).
- [12] P. Y. Wen, K.-T. Lin, A. F. Kockum, B. Suri, H. Ian, J. C. Chen, S. Y. Mao, C. C. Chiu, P. Delsing, F. Nori, G.-D. Lin, and I.-C. Hoi, Large Collective Lamb Shift of Two Distant Superconducting Artificial Atoms, *Phys. Rev. Lett.* **123**, 233602 (2019).
- [13] P. Domokos, P. Horak, and H. Ritsch, Quantum description of light-pulse scattering on a single atom in waveguides, *Phys. Rev. A* **65**, 033832 (2002).
- [14] J.-T. Shen and S. Fan, Coherent Single Photon Transport in a One-Dimensional Waveguide Coupled with Superconducting Quantum Bits, *Phys. Rev. Lett.* **95**, 213001 (2005).
- [15] D. E. Chang, A. S. Sørensen, E. A. Demler, and M. D. Lukin, A single-photon transistor using nanoscale surface plasmons, *Nat. Phys.* **3**, 807 (2007).
- [16] L. Zhou, Z. R. Gong, Y.-X. Liu, C. P. Sun, and F. Nori, Controllable Scattering of a Single Photon inside a One-Dimensional Resonator Waveguide, *Phys. Rev. Lett.* **101**, 100501 (2008).

- [17] L. Zhou, S. Yang, Y.-X. Liu, C.P. Sun, and F. Nori, Quantum Zeno switch for single-photon coherent transport, *Phys. Rev. A* **80**, 062109 (2009).
- [18] O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov, Y. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai, Resonance fluorescence of a single artificial atom, *Science* **327**, 840 (2010).
- [19] B. Peropadre, J. Lindkvist, I.-C. Hoi, C. M. Wilson, J. J. Garcia-Ripoll, P. Delsing, and G. Johansson, Scattering of coherent states on a single artificial atom, *New J. Phys.* **15**, 035009 (2013).
- [20] A. F. Van Loo, A. Fedorov, K. Lalumiere, B. C. Sanders, A. Blais, and A. Wallraff, Photon-mediated interactions between distant artificial atoms, *Science* **342**, 1494 (2013).
- [21] P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, Chiral quantum optics, *Nature (London)* **541**, 473 (2017).
- [22] M. Mirhosseini, E. Kim, X. Zhang, A. Sipahigil, P. B. Dieterle, A. J. Keller, A. Asenjo-Garcia, D. E. Chang, and O. Painter, Cavity quantum electrodynamics with atom-like mirrors, *Nature (London)* **569**, 692 (2019).
- [23] Z. R. Gong, H. Ian, L. Zhou, and C. P. Sun, Controlling quasibound states in a one-dimensional continuum through an electromagnetically-induced-transparency mechanism, *Phys. Rev. A* **78**, 053806 (2008).
- [24] H. Dong, Z. R. Gong, H. Ian, L. Zhou, and C. P. Sun, Intrinsic cavity QED and emergent quasinormal modes for a single photon, *Phys. Rev. A* **79**, 063847 (2009).
- [25] P.-O. Guimond, A. Roulet, H. N. Le, and V. Scarani, Rabi oscillation in a quantum cavity: Markovian and non-Markovian dynamics, *Phys. Rev. A* **93**, 023808 (2016).
- [26] G. Calajó, Y.-L. L. Fang, H. U. Baranger, and F. Ciccarello, Exciting a Bound State in the Continuum through Multiphoton Scattering Plus Delayed Quantum Feedback, *Phys. Rev. Lett.* **122**, 073601 (2019).
- [27] G.-Z. Song, J.-L. Guo, W. Nie, L.-C. Kwek, and G.-L. Long, Optical properties of a waveguide-mediated chain of randomly positioned atoms, *Opt. Express* **29**, 1903 (2021).
- [28] S. Arranz Regidor and S. Hughes, Cavitylike strong coupling in macroscopic waveguide QED using three coupled qubits in the deep non-Markovian regime, *Phys. Rev. A* **104**, L031701 (2021).
- [29] C. Zhou, Z. Liao, and M. S. Zubairy, Decay of a single photon in a cavity with atomic mirrors, *Phys. Rev. A* **105**, 033705 (2022).
- [30] S. M. Dutra, *Cavity Quantum Electrodynamics: The Strange Theory of Light in a Box* (Wiley, Hoboken, New Jersey, 2005).
- [31] C. Lamprecht and H. Ritsch, Quantized Atom-Field Dynamics in Unstable Cavities, *Phys. Rev. Lett.* **82**, 3787 (1999).
- [32] P. Lalanne, C. Sauvan, and J. P. Hugonin, Photon confinement in photonic crystal nanocavities, *Laser Photonics Rev.* **2**, 514 (2008).
- [33] T. Hümmer, F. J. García-Vidal, L. Martín-Moreno, and D. Zueco, Weak and strong coupling regimes in plasmonic QED, *Phys. Rev. B* **87**, 115419 (2013).
- [34] P. T. Kristensen, R.-C. Ge, and S. Hughes, Normalization of quasinormal modes in leaky optical cavities and plasmonic resonators, *Phys. Rev. A* **92**, 053810 (2015).
- [35] T. Pellizzari, S. A. Gardiner, J. I. Cirac, and P. Zoller, Decoherence, Continuous Observation, and Quantum Computing: A Cavity QED Model, *Phys. Rev. Lett.* **75**, 3788 (1995).
- [36] X. Gu, A. F. Kockum, A. Miranowicz, Y.-X. Liu, and F. Nori, Microwave photonics with superconducting quantum circuits, *Phys. Rep.* **718–719**, 1 (2017).
- [37] H. J. Kimble, The quantum internet, *Nature (London)* **453**, 1023 (2008).
- [38] A. Reiserer and G. Rempe, Cavity-based quantum networks with single atoms and optical photons, *Rev. Mod. Phys.* **87**, 1379 (2015).
- [39] Z. Tian, P. Zhang, and X.-W. Chen, Static Hybrid Quantum Nodes: Toward Perfect State Transfer on a Photonic Chip, *Phys. Rev. Applied* **15**, 054042 (2021).
- [40] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, *Rev. Mod. Phys.* **86**, 1391 (2014).
- [41] X. Li and L. F. Wei, Designable single-photon quantum routings with atomic mirrors, *Phys. Rev. A* **92**, 063836 (2015).
- [42] Z. Qian, L. Shan, X. Zhang, Q. Liu, Y. Ma, Q. Gong, and Y. Gu, Spontaneous emission in micro- or nanophotonic structures, *Photonix* **2**, 21 (2021).
- [43] I. Friedler, C. Sauvan, J. P. Hugonin, P. Lalanne, J. Claudon, and J. M. Gérard, Solid-state single photon sources: The nanowire antenna, *Opt. Express* **17**, 2095 (2009).
- [44] E. V. Denning, J. Iles-Smith, A. D. Osterkryger, N. Gregersen, and J. Mork, Cavity-waveguide interplay in optical resonators and its role in optimal single-photon sources, *Phys. Rev. B* **98**, 121306(R) (2018).
- [45] J. Mork, Y. Chen, and M. Heuck, Photonic Crystal Fano Laser: Terahertz Modulation and Ultrashort Pulse Generation, *Phys. Rev. Lett.* **113**, 163901 (2014).
- [46] Y. Yu, W. Xue, E. Semenova, K. Yvind, and J. Mork, Demonstration of a self-pulsing photonic crystal Fano laser, *Nat. Photonics* **11**, 81 (2017).
- [47] D. E. Chang, J. S. Douglas, A. González-Tudela, C.-L. Hung, and H. J. Kimble, Colloquium: Quantum matter built from nanoscopic lattices of atoms and photons, *Rev. Mod. Phys.* **90**, 031002 (2018).
- [48] A. S. Sheremet, M. I. Petrov, I. V. Ivorsh, A. V. Poshakinskiy, and A. N. Poddubny, Waveguide quantum electrodynamics: Collective radiance and photon-photon correlations, *Rev. Mod. Phys.* **95**, 015002 (2023).
- [49] B. Kannan, A. Almanakly, Y. Sung, A. Di Paolo, D. A. Rower, J. Braumüller, A. Melville, B. M. Niedzielski, A. Karamlou, K. Serniak, A. Vepsäläinen, M. E. Schwartz, J. L. Yoder, R. Winik, J. I.-J. Wang, T. P. Orlando, S. Gustavsson, J. A. Grover, and W. D. Oliver, On-demand directional microwave photon emission using waveguide quantum electrodynamics, *Nat. Phys.* **19**, 394 (2023).
- [50] P. Peng, W. Cao, C. Shen, W. Qu, J. Wen, L. Jiang, and Y. Xiao, Anti-parity-time symmetry with flying atoms, *Nat. Phys.* **12**, 1139 (2016).
- [51] Y. Choi, C. Hahn, J. W. Yoon, and S. H. Song, Observation of an anti-PT-symmetric exceptional point and energy-difference conserving dynamics in electrical circuit resonators, *Nat. Commun.* **9**, 2182 (2018).

- [52] Y. Li, Y.-G. Peng, L. Han, M.-A. Miri, W. Li, M. Xiao, X.-F. Zhu, J. Zhao, A. Alù, S. Fan, and C.-W. Qiu, Anti-parity-time symmetry in diffusive systems, *Science* **364**, 170 (2019).
- [53] Y. Jiang, Y. Mei, Y. Zuo, Y. Zhai, J. Li, J. Wen, and S. Du, Anti-Parity-Time Symmetric Optical Four-Wave Mixing in Cold Atoms, *Phys. Rev. Lett.* **123**, 193604 (2019).
- [54] J. Zhao, Y. Liu, L. Wu, C.-K. Duan, Y.-X. Liu, and J. Du, Observation of Anti- $\mathcal{PT}$ -Symmetry Phase Transition in the Magnon-Cavity-Magnon Coupled System, *Phys. Rev. Appl.* **13**, 014053 (2020).
- [55] H. Zhang, R. Huang, S.-D. Zhang, Y. Li, C.-W. Qiu, F. Nori, and H. Jing, Breaking anti- $\mathcal{PT}$  symmetry by spinning a resonator, *Nano Lett.* **20**, 7594 (2020).
- [56] Y. Yang, Y.-P. Wang, J. W. Rao, Y. S. Gui, B. M. Yao, W. Lu, and C.-M. Hu, Unconventional Singularity in Anti-Parity-Time Symmetric Cavity Magnonics, *Phys. Rev. Lett.* **125**, 147202 (2020).
- [57] J. M. P. Nair, D. Mukhopadhyay, and G. S. Agarwal, Enhanced Sensing of Weak Anharmonicities through Coherences in Dissipatively Coupled Anti- $\mathcal{PT}$  Symmetric Systems, *Phys. Rev. Lett.* **126**, 180401 (2021).
- [58] Y. Yang, X. Xie, Y. Li, Z. Zhang, Y. Peng, C. Wang, E. Li, Y. Li, H. Chen, and F. Gao, Radiative anti-parity-time plasmonics, *Nat. Commun.* **13**, 7678 (2022).
- [59] A. Tiranov, V. Angelopoulos, C. J. van Diepen, B. Schirnski, O. A. D. Sandberg, Y. Wang, L. Midolo, S. Scholz, A. D. Wieck, A. Ludwig, A. S. Sørensen, and P. Lodahl, Collective super- and subradiant dynamics between distant optical quantum emitters, *Science* **379**, 389 (2023).
- [60] N. V. Corzo, B. Gouraud, A. Chandra, A. Goban, A. S. Sheremet, D. V. Kupriyanov, and J. Laurat, Large Bragg Reflection from One-Dimensional Chains of Trapped Atoms Near a Nanoscale Waveguide, *Phys. Rev. Lett.* **117**, 133603 (2016).
- [61] H. L. Sørensen, J.-B. Béguin, K. W. Kluge, I. Iakoupov, A. S. Sørensen, J. H. Müller, E. S. Polzik, and J. Appel, Coherent Backscattering of Light Off One-Dimensional Atomic Strings, *Phys. Rev. Lett.* **117**, 133604 (2016).
- [62] M. Hübner, J. Kuhl, T. Stroucken, A. Knorr, S. W. Koch, R. Hey, and K. Ploog, Collective Effects of Excitons in Multiple-Quantum-Well Bragg and Anti-Bragg Structures, *Phys. Rev. Lett.* **76**, 4199 (1996).
- [63] A. N. Poddubny, Driven anti-Bragg subradiant correlations in waveguide quantum electrodynamics, *Phys. Rev. A* **106**, L031702 (2022).
- [64] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.131.103602> for additional details about the anti-Bragg atom-dimer mirror with tunable reflection, atomic cavity protected by anti-parity-time symmetry, and atomic cavity QED, which includes Ref. [65].
- [65] Y. P. Zhong, H.-S. Chang, K. J. Satzinger, M.-H. Chou, A. Bienfait, C. R. Conner, É. Dumur, J. Grebel, G. A. Peairs, R. G. Povey, D. I. Schuster, and A. N. Cleland, Violating Bell's inequality with remotely connected superconducting qubits, *Nat. Phys.* **15**, 741 (2019).
- [66] F. Le Kien, S. D. Gupta, K. P. Nayak, and K. Hakuta, Nanofiber-mediated radiative transfer between two distant atoms, *Phys. Rev. A* **72**, 063815 (2005).
- [67] A. Gonzalez-Tudela, D. Martin-Cano, E. Moreno, L. Martin-Moreno, C. Tejedor, and F. J. Garcia-Vidal, Entanglement of Two Qubits Mediated by One-Dimensional Plasmonic Waveguides, *Phys. Rev. Lett.* **106**, 020501 (2011).
- [68] G. Díaz-Camacho, D. Porras, and J. J. García-Ripoll, Photon-mediated qubit interactions in one-dimensional discrete and continuous models, *Phys. Rev. A* **91**, 063828 (2015).
- [69] T. Shi, D. E. Chang, and J. I. Cirac, Multiphoton-scattering theory and generalized master equations, *Phys. Rev. A* **92**, 053834 (2015).
- [70] T. Caneva, M. T. Manzoni, T. Shi, J. S. Douglas, J. I. Cirac, and D. E. Chang, Quantum dynamics of propagating photons with strong interactions: a generalized input-output formalism, *New J. Phys.* **17**, 113001 (2015).
- [71] Y. S. Greenberg and A. A. Shtygashev, Non-Hermitian Hamiltonian approach to the microwave transmission through a one-dimensional qubit chain, *Phys. Rev. A* **92**, 063835 (2015).
- [72] G. Calajó, F. Ciccarello, D. Chang, and P. Rabl, Atom-field dressed states in slow-light waveguide QED, *Phys. Rev. A* **93**, 033833 (2016).
- [73] L. Ge and H. E. Türeci, Antisymmetric  $\mathcal{PT}$ -photonic structures with balanced positive- and negative-index materials, *Phys. Rev. A* **88**, 053810 (2013).
- [74] J.-H. Wu, M. Artoni, and G. C. La Rocca, Non-Hermitian Degeneracies and Unidirectional Reflectionless Atomic Lattices, *Phys. Rev. Lett.* **113**, 123004 (2014).
- [75] F. Yang, Y.-C. Liu, and L. You, Anti- $\mathcal{PT}$  symmetry in dissipatively coupled optical systems, *Phys. Rev. A* **96**, 053845 (2017).
- [76] Q. Li, C.-J. Zhang, Z.-D. Cheng, W.-Z. Liu, J.-F. Wang, F.-F. Yan, Z.-H. Lin, Y. Xiao, K. Sun, Y.-T. Wang, J.-S. Tang, J.-S. Xu, C.-F. Li, and G.-C. Guo, Experimental simulation of anti-parity-time symmetric Lorentz dynamics, *Optica* **6**, 67 (2019).
- [77] A. F. Kockum, G. Johansson, and F. Nori, Decoherence-Free Interaction between Giant Atoms in Waveguide Quantum Electrodynamics, *Phys. Rev. Lett.* **120**, 140404 (2018).
- [78] B. Kannan, M. J. Ruckriegel, D. L. Campbell, A. F. Kockum, J. Braumüller, D. K. Kim, M. Kjaergaard, P. Krantz, A. Melville, B. M. Niedzielski, A. Vepsäläinen, R. Winik, J. L. Yoder, F. Nori, T. P. Orlando, S. Gustavsson, and W. D. Oliver, Waveguide quantum electrodynamics with superconducting artificial giant atoms, *Nature (London)* **583**, 775 (2020).
- [79] C. M. Bender and S. Boettcher, Real Spectra in Non-Hermitian Hamiltonians Having  $\mathcal{PT}$  Symmetry, *Phys. Rev. Lett.* **80**, 5243 (1998).
- [80] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, Non-Hermitian physics and  $\mathcal{PT}$  symmetry, *Nat. Phys.* **14**, 11 (2018).
- [81] Ş. K. Özdemir, S. Rotter, F. Nori, and L. Yang, Parity-time symmetry and exceptional points in photonics, *Nat. Mater.* **18**, 783 (2019).
- [82] Y. Ashida, Z. Gong, and M. Ueda, Non-Hermitian physics, *Adv. Phys.* **69**, 249 (2020).
- [83] C. J. Hood, H. J. Kimble, and J. Ye, Characterization of high-finesse mirrors: Loss, phase shifts, and mode structure in an optical cavity, *Phys. Rev. A* **64**, 033804 (2001).

- [84] J. Vučković, M. Lončar, H. Mabuchi, and A. Scherer, Design of photonic crystal microcavities for cavity QED, *Phys. Rev. E* **65**, 016608 (2001).
- [85] A. Albrecht, L. Henriët, A. Asenjo-Garcia, P. B. Dieterle, O. Painter, and D. E. Chang, Subradiant states of quantum bits coupled to a one-dimensional waveguide, *New J. Phys.* **21**, 025003 (2019).
- [86] Y.-X. Zhang and K. Mølmer, Theory of Subradiant States of a One-Dimensional Two-Level Atom Chain, *Phys. Rev. Lett.* **122**, 203605 (2019).
- [87] Y. Ke, A. V. Poshakinskiy, C. Lee, Y. S. Kivshar, and A. N. Poddubny, Inelastic Scattering of Photon Pairs in Qubit Arrays with Subradiant States, *Phys. Rev. Lett.* **123**, 253601 (2019).
- [88] W. Nie, T. Shi, F. Nori, and Y.-X. Liu, Topology-Enhanced Nonreciprocal Scattering and Photon Absorption in a Waveguide, *Phys. Rev. Appl.* **15**, 044041 (2021).
- [89] D. Plankensteiner, C. Sommer, H. Ritsch, and C. Genes, Cavity Antiresonance Spectroscopy of Dipole Coupled Subradiant Arrays, *Phys. Rev. Lett.* **119**, 093601 (2017).
- [90] D. N. Basov, A. Asenjo-Garcia, P. J. Schuck, X. Zhu, and A. Rubio, Polariton panorama, *Nanophotonics* **10**, 549 (2021).
- [91] S. Ghosh, R. Su, J. Zhao, A. Fieramosca, J. Wu, T. Li, Q. Zhang, F. Li, Z. Chen, T. Liew, D. Sanvitto, and Q. Xiong, Microcavity exciton polaritons at room temperature, *Photonics Insights* **1**, R04 (2022).
- [92] H. Jing, Ş. K. Özdemir, H. Lü, and F. Nori, High-order exceptional points in optomechanics, *Sci. Rep.* **7**, 3386 (2017).
- [93] K. Hennessy, A. Badolato, M. Winger, D. Gerace, M. Atatüre, S. Gulde, S. Fält, E. L. Hu, and A. Imamoglu, Quantum nature of a strongly coupled single quantum dot-cavity system, *Nature (London)* **445**, 896 (2007).
- [94] M. F. Limonov, M. V. Rybin, A. N. Poddubny, and Y. S. Kivshar, Fano resonances in photonics, *Nat. Photonics* **11**, 543 (2017).
- [95] M. O. Scully, Single Photon Subradiance: Quantum Control of Spontaneous Emission and Ultrafast Readout, *Phys. Rev. Lett.* **115**, 243602 (2015).
- [96] A. Asenjo-Garcia, M. Moreno-Cardoner, A. Albrecht, H. J. Kimble, and D. E. Chang, Exponential Improvement in Photon Storage Fidelities Using Subradiance and “Selective Radiance” in Atomic Arrays, *Phys. Rev. X* **7**, 031024 (2017).
- [97] G. Ferioli, A. Glicenstein, L. Henriët, I. Ferrier-Barbut, and A. Browaeys, Storage and Release of Subradiant Excitations in a Dense Atomic Cloud, *Phys. Rev. X* **11**, 021031 (2021).
- [98] O. Rubies-Bigorda, V. Walther, T. L. Patti, and S. F. Yelin, Photon control and coherent interactions via lattice dark states in atomic arrays, *Phys. Rev. Res.* **4**, 013110 (2022).
- [99] R. Holzinger, R. Gutiérrez-Jáuregui, T. Hönigl-Decrinis, G. Kirchmair, A. Asenjo-Garcia, and H. Ritsch, Control of Localized Single- and Many-Body Dark States in Waveguide QED, *Phys. Rev. Lett.* **129**, 253601 (2022).