Extremal Kerr Black Holes as Amplifiers of New Physics

Gary T. Horowitz,[1] Maciej Kolanowski[©],[2] Grant N. Remmen[©],[1,3] and Jorge E. Santos[4]

[1]*Department of Physics, University of California, Santa Barbara, California 93106, USA*
[2]*Institute of Theoretical Physics, Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*
[3]*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA*
[4]*Department of Applied Mathematics and Theoretical Physics, University of Cambridge,*
Wilberforce Road, Cambridge, CB3 0WA, United Kingdom

We show that extremal Kerr black holes are sensitive probes of new physics. Stringy or quantum corrections to general relativity are expected to generate higher-curvature terms in the gravitational action. We show that in the presence of these terms, asymptotically flat extremal rotating black holes have curvature singularities on their horizon. Furthermore, near-extremal black holes can have large yet finite tidal forces for infalling observers. In addition, we consider five-dimensional extremal charged black holes and show that higher-curvature terms can have a large effect on the horizon geometry.

Introduction.—Various arguments suggest that general relativity is a low-energy approximation to a more complete theory. When the effective action is written just in terms of the metric (and possibly other light fields), it includes higher-curvature terms in addition to the usual Einstein-Hilbert term. This occurs whenever there are new classical high-energy degrees of freedom, such as in string theory, or when quantum corrections are included from integrating out massive states. It is often assumed that these higher-order terms are negligible in regions of low curvature and can be ignored. We show that this is not always the case. Extremal Kerr black holes turn out to be very sensitive to these small corrections.

Although the near-horizon geometry of an extremal black hole receives only a small correction from these higher-derivative terms, generic stationary, axisymmetric perturbations generate singularities on the horizon. When one extends the near-horizon geometry to a full asymptotically flat solution, one necessarily introduces these singular perturbations. So the full solution has a singular horizon. Infalling timelike or null geodesics have infinite tidal forces crossing the horizon [1]. This singularity is very similar to that found recently for generic extremal black holes in anti-de Sitter space [4]. As in that case, near-extremal black holes have large (finite) tidal forces, but curvature scalars always remain small. (See Ref. [5] for an earlier example of higher-curvature terms leading to singular extremal horizons in a theory with a scalar dilaton).

In a purely gravitational effective field theory (EFT)—or even in a theory containing matter, but in the case where we are interested in vacuum solutions—we can build the action as a functional of the Riemann tensor alone. Any terms containing the Ricci tensor or scalar curvature can be removed order by order in the derivative expansion via a field redefinition of the graviton, e.g., $h_{ab} \rightarrow h_{ab} + cR_{ab}$. Modulo various tensor identities [6], the only curvature invariant at fourth order in derivatives that is independent of the Ricci tensor is $R_{abcd}R^{abcd}$. This is field redefinition equivalent to the Gauss-Bonnet term $R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$, which in four spacetime dimensions is topological and therefore cannot affect the bulk equations of motion. At the next order, there are two independent operators in four dimensions constructible from the Riemann tensor, $\nabla_a R_{bcde} \nabla^a R^{bcde}$ and $R_{ab}^{cd} R_{cd}^{ef} R_{ef}^{ab}$. The former can be rearranged, up to a total derivative, into a Riemann-cubed term and operators involving the Ricci tensor, so we can drop it without loss of generality. Similarly, there are two independent $(R_{abcd})^4$ terms [7].

We are thus left with the leading effective theory [8]

$$\mathcal{L} = \frac{1}{2\kappa^2}(R + \eta\kappa^4 \mathcal{R}^3 + \lambda\kappa^6 \mathcal{C}^2 + \tilde{\lambda}\kappa^6 \tilde{\mathcal{C}}^2), \qquad (1)$$

where we write Newton's constant in terms of $\kappa^2 = 8\pi G$ and define $\mathcal{R}^3 \equiv R_{ab}^{cd} R_{cd}^{ef} R_{ef}^{ab}$, $\mathcal{C} \equiv R_{abcd}R^{abcd}$, and $\tilde{\mathcal{C}} \equiv \tilde{R}_{abcd}R^{abcd}$, with the dual tensor $\tilde{R}_{abcd} = \epsilon_{ab}^{pq}R_{pqcd}$ [9]. In the above, η, λ, and $\tilde{\lambda}$ are dimensionless.

The equations of motion that follow from Eq. (1), around background solutions satisfying $R_{ab} = 0$, can be written as [8,10,11]

$$R_{ab} - \frac{1}{2}Rg_{ab} = T_{ab}^{\text{cubic}} + T_{ab}^{\text{quartic}} \qquad (2)$$

with

$$
T_{ab}^{\text{cubic}} = \eta\kappa^4 \left[3R_a^{cde} R_{de}^{gh} R_{ghcb} + \frac{1}{2} g_{ab} R_{gh}^{cd} R_{cd}^{ef} R_{ef}^{gh} \right.
$$
$$
\left. - 6\nabla^c\nabla^d (R_{acgh} R_{bd}^{gh}) \right]
$$
$$
T_{ab}^{\text{quartic}} = -\lambda\kappa^6 \left(8R_{acbd}\nabla^c\nabla^d \mathcal{C} + \frac{g_{ab}}{2}\mathcal{C}^2 \right)
$$
$$
- \tilde{\lambda}\kappa^6 \left(8\tilde{R}_{acbd}\nabla^c\nabla^d \tilde{\mathcal{C}} + \frac{g_{ab}}{2}\tilde{\mathcal{C}}^2 \right), \tag{3}
$$

which should be regarded as providing the leading corrections to vacuum solutions to linear order in η, λ, and $\tilde{\lambda}$; see also Ref. [12]. We will be interested in computing how these corrections affect, in a surprisingly dramatic fashion, the behavior of perturbations to extremal black holes.

Extremal Kerr.—We expand the metric as

$$
g_{ab} = g_{ab}^{(0)} + \eta h_{ab}^{(6)} + \lambda h_{ab}^{(8)} + \tilde{\lambda}\tilde{h}_{ab}^{(8)}, \tag{4}
$$

with $g^{(0)}$ satisfying the vacuum Einstein equation, and solve Eqs. (2) and (3) to linear order in the EFT coefficients $\{\eta, \lambda, \tilde{\lambda}\}$ [13]. To study tidal force singularities, we are interested in the leading behavior of the Weyl tensor associated with the EFT-corrected metric. The effects of interest in this Letter only become important near the horizon of a near-extremal black hole, since they arise from unusual scaling in the near-horizon region when the horizon becomes arbitrarily far away (in spacelike directions). For this reason we can focus mainly on the near-horizon geometry. We proceed in two steps: (i) we EFT-correct the near-horizon extreme Kerr (NHEK) geometry [15], and (ii) we then determine how metric perturbations fall off near the horizon by computing the so-called scaling dimensions γ.

We focus on extremal black hole solutions that are stationary and axisymmetric with respect to $k = \partial/\partial t$ and $m = \partial/\partial\phi$, respectively. Furthermore, we will impose the symmetry $(t, \phi) \to -(t, \phi)$. The most general ansatz compatible with the symmetries above can be written as

$$
ds^2 = 2J\Omega^2 \left[-\rho^2 dt^2 + \frac{F_1}{\rho^2} (d\rho + \rho F_2 dx)^2 \right.
$$
$$
\left. + \frac{dx^2}{A} + B^2 (d\phi + \rho\omega dt)^2 \right], \tag{5}
$$

where the factors of ρ are adjusted so that $\rho = 0$ is a Killing extremal horizon; $\phi \sim \phi + 2\pi$; and Ω, ω, A, B, F_1, and F_2 are functions of ρ and x, with $(\rho, x) \in \mathbb{R}^+ \times [-1, 1]$. To fix the gauge, we further impose $F_1 = 1$ and $F_2 = 0$. In the above, all coordinates are dimensionless, and in the case of the NHEK solution, J is simply the Kerr angular

momentum (in geometric units of length squared given by the size of the black hole).

EFT-corrected near-horizon geometries: In order to find the near-horizon geometry, we start by imposing $O(2, 1) \times U(1)$ symmetry, in which case we take $\Omega = \Omega_{\text{NH}}(x)$ and $B = B_{\text{NH}}(x)$ in Eq. (5) to be functions of x only, as well as taking $A = A_{\text{NH}} = (1 - x^2)/\Gamma_{\text{NH}}^2$, with Γ_{NH} and $\omega = \omega_{\text{NH}}$ constants. The resulting line element reads:

$$
ds_{\text{NH}}^2 = 2J\Omega_{\text{NH}}^2 \left[-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + \frac{\Gamma_{\text{NH}}^2 dx^2}{1 - x^2} \right.
$$
$$
\left. + B_{\text{NH}}^2 (d\phi + \rho\omega_{\text{NH}} dt)^2 \right]. \tag{6}
$$

The first two terms in brackets correspond to the metric on a two-dimensional unit-radius AdS_2, with $\rho = 0$ being the location of the AdS_2 Poincaré horizon.

We expand all quantities appearing in Eq. (6) as

$$
\Omega_{\text{NH}} = \Omega^{(0)}(x)[1 + \eta\Omega^{(6)}(x) + \lambda\Omega^{(8)}(x) + \tilde{\lambda}\tilde{\Omega}^{(8)}(x)]
$$
$$
B_{\text{NH}} = B^{(0)}(x)[1 + \eta B^{(6)}(x) + \lambda B^{(8)}(x) + \tilde{\lambda}\tilde{B}^{(8)}(x)]
$$
$$
\Gamma_{\text{NH}} = \Gamma^{(0)}[1 + \eta\Gamma^{(6)} + \lambda\Gamma^{(8)} + \tilde{\lambda}\tilde{\Gamma}^{(8)}]
$$
$$
\omega_{\text{NH}} = \omega^{(0)}[1 + \eta\omega^{(6)} + \lambda\omega^{(8)} + \tilde{\lambda}\tilde{\omega}^{(8)}]. \tag{7}
$$

For $\eta = \lambda = \tilde{\lambda} = 0$ we recover the NHEK geometry, for which

$$
\Omega^{(0)}(x) = \sqrt{\frac{1 + x^2}{2}}, \quad B^{(0)}(x) = \frac{2\sqrt{1 - x^2}}{1 + x^2}, \quad \Gamma^{(0)} = \omega^{(0)} = 1. \tag{8}
$$

Proceeding to the next order, we determine the corresponding EFT corrections. For instance, we find

$$
\Gamma^{(6)} = -\frac{15\kappa^4}{32\sqrt{2}J^2}, \qquad \omega^{(6)} = \frac{\kappa^4}{7J^2},
$$
$$
\Gamma^{(8)} = -\frac{366435\kappa^6}{256\sqrt{2}J^3}, \qquad \omega^{(8)} = \frac{(4864 + 1575\pi)\kappa^6}{20J^3},
$$
$$
\tilde{\Gamma}^{(8)} = -\frac{368829\kappa^6}{64\sqrt{2}J^3}, \qquad \tilde{\omega}^{(8)} = \frac{(4736 + 1575\pi)\kappa^6}{5J^3}, \tag{9}
$$

where some of these constants were chosen to ensure the absence of conical singularities near the poles $x = \pm 1$. Explicit expressions for $\Omega^{(I)}(x)$ and $B^{(I)}(x)$ can be found in the Supplemental Material [16] (Sec. V A).

Deforming the EFT-corrected NHEK geometries: Having found the new near-horizon geometries, we now study how they respond to external tidal deformations. These deformations are generated by simply having the near-horizon geometry connected to an asymptotically flat region. We expect such deformations to become arbitrarily

small near the extremal horizon, and as such we can use perturbation theory to study their properties.

We again start with Eq. (5) with $F_1 = 1$ and $F_2 = 0$, but now Ω, B, A, and ω can be functions of both x and ρ. Since we are perturbing around an $O(2,1) \times U(1)$-symmetric solution and we are restricting our attention to stationary and axisymmetric perturbations, we decompose our perturbations in terms of harmonics on AdS_2 with dependence on ρ only. These turn out to be power-law solutions of the form ρ^γ. We thus set

$$
\begin{aligned}
A(\rho, x) &= A_{\mathrm{NH}}(x)[1 + \varepsilon\rho^\gamma Q_1(x)] \\
B(\rho, x) &= B_{\mathrm{NH}}(x)[1 + \varepsilon\rho^\gamma Q_2(x)] \\
\Omega(\rho, x) &= \Omega_{\mathrm{NH}}(x)[1 + \varepsilon\rho^\gamma Q_3(x)] \\
\omega(\rho, x) &= \omega_{\mathrm{NH}}[1 + \varepsilon\rho^\gamma Q_4(x)],
\end{aligned}
\tag{10}
$$

with ε being a bookkeeping parameter we take to be infinitesimally small. All the Q_i, with $i = 1, 2, 3, 4$, and γ have an expansion in η, λ, and $\tilde{\lambda}$ of the form [17]

$$
\begin{aligned}
Q_i(x) &= Q_i^{(0)}(x) + \eta Q_i^{(6)}(x) + \lambda Q_i^{(8)}(x) + \tilde{\lambda}\tilde{Q}_i^{(8)}(x) \\
\gamma &= \gamma^{(0)} + \eta\gamma^{(6)} + \lambda\gamma^{(8)} + \tilde{\lambda}\tilde{\gamma}^{(8)}.
\end{aligned}
\tag{11}
$$

The main goal of this Letter is to determine the corrections to the scaling dimensions $\gamma^{(6)}$, $\gamma^{(8)}$, and $\tilde{\gamma}^{(8)}$, since these control the deviation from the standard Kerr result. At each order, we find that $Q_2^{(I)}$ and $Q_3^{(I)}$ can be expressed in terms of $Q_1^{(I)}$ and $Q_4^{(I)}$ and their first derivatives. We are thus left with two second-order differential equations for $Q_1^{(I)}$ and $Q_4^{(I)}$.

For $I = 0$, i.e., deviations away from NHEK, the equations for $Q_1^{(0)}$ and $Q_4^{(0)}$ are second-order homogeneous equations of the Sturm-Liouville type where $\gamma^{(0)}$ appears as an eigenvalue. These can be solved analytically in terms of standard Legendre polynomials P_ℓ of degree ℓ (see Sec. V B of the Supplemental Material [16]). We find two classes of solutions, labeled by $\gamma_\pm^{(0)}$, with critical exponents given by [18]

$$
\begin{aligned}
\gamma_+^{(0)}(\ell) &= \ell \quad \text{with} \quad \ell \in \mathbb{N} \geq 2, \\
\gamma_-^{(0)}(\ell) &= \ell + 1 \quad \text{with} \quad \ell \in \mathbb{N} \geq 1.
\end{aligned}
\tag{12}
$$

Since the scaling exponents are positive integers, the perturbation decays, and the horizon remains perfectly smooth. In other words, without higher-derivative corrections, the extremal Kerr horizon is unaffected by these deformations. It should be emphasized that these modes are already present for the asymptotically flat black holes.

For fixed values of ℓ, there are two *distinct* values of $\gamma^{(0)}$, thus yielding a nondegenerate spectrum at fixed ℓ at zeroth order. We can thus proceed using standard perturbation theory. The resulting equations can again be solved

analytically for each value of ℓ, and after some algebra one finds

$$
\begin{aligned}
\gamma_+^{(6)}(2) &= \gamma_-^{(6)}(1) = \frac{24\kappa^4}{7J^2} \\
\gamma_+^{(8)}(2) &= -\frac{21(32 + 45\pi)\kappa^6}{5J^3} \\
\gamma_-^{(8)}(1) &= -\frac{9(8576 + 3045\pi)\kappa^6}{20J^3} \\
\tilde{\gamma}_+^{(8)}(2) &= -\frac{12(736 + 315\pi)\kappa^6}{5J^3} \\
\tilde{\gamma}_-^{(8)}(1) &= -\frac{189(384 + 145\pi)\kappa^6}{5J^3}.
\end{aligned}
\tag{13}
$$

At the moment we have no understanding of why $\gamma_-^{(6)}(1) = \gamma_+^{(6)}(2)$. Furthermore, $\gamma_-^{(I)}(\ell - 1) \leq \gamma_+^{(I)}(\ell)$ for fixed values of ℓ.

Even though these are small corrections, the scaling exponent is no longer an integer, so when $\gamma < 2$ the curvature will diverge. The reader might worry that we have done this calculation using coordinates that are not regular at the extremal horizon. In the Supplemental Material [16] in Sec. V C, we give an explicit map between the Kerr-like coordinates of Eq. (5) and Bondi-Sachs coordinates, which are manifestly regular at $\rho = 0$. It is in Bondi-Sachs coordinates that we compute the relevant components of the Weyl tensor and find diverging tidal force singularities at the future (and past) extremal event horizon so long as $\gamma < 2$. Since these modes were already present for the Kerr black hole, they are inevitably present if we simply assume that the full solution (beyond the near-horizon region) is also asymptotically flat. It should be noted that singular tidal forces at extremality can be linked to high curvature for near-extremal black holes. In particular, it was shown in Ref. [4] that the curvature will diverge as $T^{\gamma-2}$ in the limit $T \to 0$.

Since the metric is C^1 but not C^2, Einstein's equations remain well-defined in a distributional sense. However, since the event horizon is a Cauchy horizon for a constant-t hypersurface, the evolution is not unique. We now turn to a situation where higher-curvature terms can have an even more dramatic effect.

Extremal Reissner-Nordström.—The analogous calculation can be done for Reissner-Nordström (RN) black holes. The leading effective action now includes the following terms at fourth order in derivatives:

$$
\begin{aligned}
\mathcal{L}_4 = {}&d_1 R^2 + d_2 R_{ab}R^{ab} + d_3 R_{abcd}R^{abcd} \\
&+ \kappa^2(d_4 R F^2 + d_5 R^{ab}F_{ab}^2 + d_6 R^{abcd}F_{ab}F_{cd}) \\
&+ \kappa^4[d_7(F^2)^2 + d_8 F_{ab}^2 F^{2ab}],
\end{aligned}
\tag{14}
$$

where all d_i are dimensionless and where we write $F^2 \equiv F_{ab}F^{ab}$ and $F_{ab}^2 \equiv F_{ac}F_b^c$. As before, it suffices to focus on

the near-horizon geometry, which is the Bertotti-Robinson solution. Since this is a well-defined and highly symmetrical solution in any spacetime dimension D, we will not restrict ourselves to $D = 4$ [19]. In the same way as in the previous section, we start by finding an EFT-corrected Bertotti-Robinson solution, keeping $O(2,1) \times SO(D-1)$ symmetry. The most general solution is still $AdS_2 \times S^{D-2}$, with shifted radii of curvature L_{AdS_2} and r_+ depending on Q and d_i,

$$ds_{BR}^2 = L_{AdS_2}^2 \left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} \right) + r_+^2 d\Omega_{D-2}^2$$

$$F = \frac{Q L_{AdS_2}}{r_+^2} dt \wedge dr. \qquad (15)$$

Next, we consider the (EFT-corrected) linearized equation for static, nonspherical perturbations, which could be sourced by distant matter. We may use the large symmetry at hand to decompose the perturbations into AdS_2 harmonics and spherical harmonics. The former translates into simple power-law behavior, $\delta g \sim \rho^\gamma$. The scalar-derived perturbations are given by

$$\delta g_{IJ} = s \mathbb{S}_\ell \rho^\gamma g_{IJ}$$

$$\delta g_{AB} = \rho^\gamma (h_L \mathbb{S}_\ell g_{AB} + h_T \mathbb{S}_{AB}^\ell)$$

$$\delta F = d(q \mathbb{S}_\ell \rho^{\gamma+1}) \wedge dt, \qquad (16)$$

where indices I, J and A, B correspond to AdS_2 and S^{D-2}, respectively; $\mathbb{S}_\ell$ is the ℓth spherical harmonic; $\mathbb{S}_{AB}^\ell$ is the traceless part of its second derivative; and s, h_L, h_T, q are constants. The linearized equations then become an algebraic system for the coefficients that has solutions only for specific values of the exponent γ. The scaling dimension (for a given ℓ) may take four values. Two of them are always negative and can be removed by boundary conditions at the horizon. For the smaller of the remaining two, we find

$$\gamma(\ell) = \frac{\ell}{D-3} - 1 - d_0 \frac{8\kappa^2(D-4)\ell(\ell+D-3)}{(D-2)(D-3)^2(2\ell-D+3)r_+^2},$$

$$\qquad (17)$$

where

$$d_0 = \frac{(D-3)(D-4)^2}{4} d_1 + \frac{(D-3)(2D^2-11D+16)}{4} d_2$$

$$+ \frac{(2D^3-16D^2+45D-44)}{2} d_3$$

$$+ \frac{(D-3)(D-2)(D-4)}{2} d_4$$

$$+ \frac{(D-2)(D-3)^2}{2} (d_5 + d_6)$$

$$+ \frac{(D-2)^2(D-3)}{2} (2d_7 + d_8) \qquad (18)$$

is a combination of Wilson coefficients invariant under arbitrary field redefinitions $g_{ab} \to g_{ab} + r_1 R_{ab} + r_2 R g_{ab} + r_3 F_{ab}^2 + r_4 F^2 g_{ab}$ [21]. We see that the EFT correction in Eq. (17) vanishes in four dimensions and, if $d_0 > 0$ as predicted by Ref. [21], becomes negative in higher dimensions. In particular, the $\ell = 2$ mode in $D = 5$ leads to a negative exponent, so the perturbation blows up at the horizon. This is a sign of the breakdown of perturbation theory, and the near-horizon geometry will no longer be $AdS_2 \times S^3$. This is analogous to what happens to nonspherical extremal black holes in AdS_5 [22]. As in that case, perturbations of the new horizon will likely still have noninteger scaling dimensions $0 < \gamma < 2$, so there will still be curvature singularities.

Discussion.—The fate of the black hole horizon depends sensitively on the signs of the coefficients of the higher-dimension operators. From our calculation of the scaling dimensions, we see that extremal Kerr black hole horizons develop singularities if η is negative, or if λ or $\tilde{\lambda}$ are positive. Further, if $d_0 > 0$, then the extremal RN black hole in five dimensions develops a singular horizon [23]. The signs of the various Wilson coefficients in the EFT are therefore of paramount importance in determining the nature of the horizon.

Fortuitously, there exists an infrared consistency program of bounding the coefficients of EFTs from first principles using the tools of analytic dispersion relations in quantum field theory, in a manner agnostic of the details of the ultraviolet theory [24]. Using either analyticity and unitarity of scattering amplitudes [25] or causality of graviton propagation [26], one can prove that, in any consistent theory of quantum gravity, the coefficients of the quartic Riemann operators λ and $\tilde{\lambda}$ must be positive. This is borne out in string theory [25], where calculations of the low-energy EFT yield $\lambda = \alpha'^3[13 + \zeta(3)]/256\kappa^6$ and $\tilde{\lambda} = \alpha'^3[1 + \zeta(3)]/1024\kappa^6$ in the bosonic case [27,28], $\lambda = 4\tilde{\lambda} = \alpha'^3\zeta(3)/256\kappa^6$ in the type II case [29–31], and $\lambda = 4\tilde{\lambda} = \alpha'^3[1 + 2\zeta(3)]/512\kappa^6$ in the heterotic [30,31] (equivalently, type I [32]) case.

However, the Riemann-cubed operator is not amenable to such a general theory-independent bound on the basis of extant dispersion relations. While vanishing on general grounds in any supersymmetric theory [33], the η coefficient receives finite contributions at one loop [34] from massive states in the theory [35],

$$\eta = \frac{1}{15120(4\pi)^2\kappa^2} \sum \left(\frac{1}{m_s^2} - \frac{4}{m_f^2} + \frac{3}{m_v^2} \right), \qquad (19)$$

where m_s, m_f, and m_v label the masses of the heavy real scalars, Dirac fermions, and vectors in the theory. As expected, this combination vanishes for supersymmetric theories. In the standard model, where the lightest massive states are fermions, we therefore will have $\eta < 0$, leading to singular horizons for rapidly spinning Kerr black holes.

Conversely, if the near-horizon region of a rapidly spinning black hole were observed to be regular, this would constitute evidence for new light hidden-sector bosons.

Further, the signs of Wilson coefficients in the Einstein-Maxwell EFT are of great importance for the weak gravity conjecture (WGC) [21,38–41] and black hole thermodynamics [12,21,40–42]. The WGC is the statement that, when an Abelian gauge theory is consistently coupled to quantum gravity, there should always be a state in the spectrum for which $Q/M > 1$ in Planck units, with 1 corresponding to the charge-to-mass ratio of an extremal black hole [38]. In the presence of higher-dimension operators in the EFT, the black hole solutions are deformed, and thus the maximum allowed charge-to-mass ratio of black holes is also shifted. For electric black holes, the particular combination d_0 defined in Eq. (18) dictates this shift, with $\Delta(Q/M) \propto d_0$ [21], so that $d_0 > 0$ allows the black holes themselves to satisfy the WGC. In this case, there are expected to be nonperturbative decay processes, i.e., a black hole version of Schwinger pair production, by which large extremal black holes decay to smaller ones with Q/M slightly > 1. Interestingly, the shift in the black hole entropy and on-shell action are also $\propto d_0$ [21,41,42], and it was argued via black hole thermodynamics in Ref. [21] that $d_0 > 0$ in tree-level completions. It is noteworthy that, for precisely this same situation $d_0 > 0$, we also find an instability toward singular extremal charged black holes in $D = 5$, along with a negative shift in the associated scaling dimension for all $D \geq 5$. We leave the question of whether this effect is somehow connected with the WGC-mandated black hole decay to future work.

When the curvature diverges on the horizon of an extremal black hole, a near-extremal black hole has curvature that diverges as one approaches extremality [4]. Since we observe rapidly spinning black holes in nature (see, e.g., Refs. [43–45]), one might wonder if the effect described here could be observable. That is possible, but detecting tidal forces much larger than the ambient curvature would require black holes much closer to extremality than have been observed to date. However, we take our results as a proof of principle that black hole horizons can serve to amplify the effects of higher-derivative terms in the action, leading to a surprising breakdown of the EFT in the near-horizon region, and we leave the broader question of possibly realizing this effect astrophysically to future work.

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[9] We work with the sign conventions $\epsilon_{0123} = \sqrt{-g}$, $R_{ab} = R^c_{acb}$, $R^a_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^a_{ce} \Gamma^e_{bd} - \Gamma^a_{de} \Gamma^e_{bc}$, $\Gamma^a_{bc} = g^{ad}(\partial_c g_{db} + \partial_b g_{dc} - \partial_d g_{bc})/2$, and mostly plus metric signature.

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[13] We note that the second-order perturbation $\delta h^{(6)}_{ab}$ from $\mathcal{R}^3$ at $O(\eta^2)$ is smaller than the linear-order quartic-Riemann perturbation $h^{(8)}_{ab}$ via the following reasoning. The linear perturbation from the cubic-Riemann term, by Eqs. (2) and (3), is given schematically by $\Box h^{(6)}_{ab} \sim \eta \mathcal{R}^3$, so $h^{(6)}_{ab} \sim \eta \kappa^4/J^2$. Then the $O(\eta^2)$ backreaction effect is generated via the equation of motion as $\Box \delta h^{(6)}_{ab} \sim (\nabla h^{(6)}_{ab})^2 + \eta \mathcal{R}^2 \delta \mathcal{R} \sim \eta^2 \kappa^8/J^5$, so $\delta h^{(6)}_{ab} \sim \eta^2 \kappa^8/J^4$. In comparison, the linear-order metric perturbation $h^{(8)}_{ab}$ from the quartic-Riemann operator is generated schematically via $\Box h^{(8)}_{ab} \sim \lambda \mathcal{R}^4$, so $h^{(8)}_{ab} \sim \lambda \kappa^6/J^3$ and analogously for $\tilde{\lambda}$. Hence, $(\delta h^{(6)}_{ab})/(h^{(8)}_{ab}) \sim (\eta^2/\lambda)(\kappa^2/J)$, which we should be able to make arbitrarily small for sufficiently large black holes. Indeed, by the dispersion relation arguments of Ref. [14], one has $\eta^2/\lambda \lesssim 1/(\kappa^2 \Lambda^2_{\text{UV}})$, for Λ_{UV} the energy scale of new physics, so as long as the size of the black hole is larger than the Compton wavelength of the ultraviolet states generating the higher-dimension operators

(which must be the case to apply the EFT in the first place), we indeed find that $O(\eta^2)$ effects are negligible.

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