Universal Scaling Bounds on a Quantum Heat Current

Shunsuke Kamimura,[1,2,*] Kyo Yoshida,[1] Yasuhiro Tokura,[1,†] and Yuichiro Matsuzaki[2,‡]
[1]*Faculty of Pure and Applied Sciences, University of Tsukuba, Tsukuba 305-8571, Japan*
[2]*Research Center for Emerging Computing Technologies, National Institute of Advanced Industrial Science and Technology (AIST), 1-1-1 Umezono, Tsukuba, Ibaraki 305-8568, Japan*

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In this Letter, we derive new bounds on a heat current flowing into a quantum L-particle system coupled with a Markovian environment. By assuming that a system Hamiltonian and a system-environment interaction Hamiltonian are extensive in L, we prove that the absolute value of the heat current scales at most as $\Theta(L^3)$ in a limit of large L. Furthermore, we present an example of noninteracting particles globally coupled with a thermal bath, for which this bound is saturated in terms of scaling. However, the construction of such a system requires many-body interactions induced by the environment, which may be difficult to realize with the existing technology. To consider more feasible cases, we consider a class of the system where any nondiagonal elements of the noise operator (derived from the system-environment interaction Hamiltonian) become zero in the system energy basis, if the energy difference exceeds a certain value ΔE. Then, for $\Delta E = \Theta(L^0)$, we derive another scaling bound $\Theta(L^2)$ on the absolute value of the heat current, and the so-called superradiance belongs to a class for which this bound is saturated. Our results are useful for evaluating the best achievable performance of quantum-enhanced thermodynamic devices, including far-reaching applications such as quantum heat engines, quantum refrigerators, and quantum batteries.

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Quantum mechanics successfully describes the counter-intuitive behaviors of microscopic objects, which cannot be explained by classical theory. Recently, considerable effort has been devoted toward engineering large quantum systems while sustaining quantum effects such as entanglement and coherence. Quantum technology is a new field to seek novel industrial applications using such quantum properties.

To quantify the performance of quantum devices, we typically consider its scaling in a limit of a large number of qubits, and we compare quantum and classical methods with equal resources (e.g., using the same amount of time). Shor demonstrated that a fault-tolerant quantum computer can solve factorization problems exponentially faster than the best-known classical algorithm [1]. Quantum metrology exploits the nonclassical properties of probe qubits to measure external fields with higher sensitivity. The estimation uncertainty is known to decrease by $L^{-1/2}$ with L classical probes; in principle, the uncertainty can decrease by L^{-1} with L entangled qubits [2–4].

Meanwhile, since the Industrial Revolution, thermodynamics has been conventionally adopted to describe the macroscopic behaviors of classical systems. Its extension to fluctuating nonequilibrium classical small systems, referred to as stochastic thermodynamics, has been studied extensively in recent decades [5]. Specifically, stochastic thermodynamics can provide a richer set of limitations on heat engine performances, as exemplified by fluctuation theorems [6–9] and trade-off relations [10,11].

Quantum thermodynamics refers to the extension of thermodynamics to quantum systems [12–15]. Quantum versions of heat engines [16–18] and batteries [19] have been proposed, where the device is typically modeled as an open quantum system. Several experimental demonstrations of such quantum thermodynamic devices have been reported [20–24]. As with other quantum technologies, scaling advantages in the performances of heat engines [25–34] and quantum batteries [35–41] have been demonstrated for specific models. Pioneered in the seminal work for the discovery of superradiance [42], many previous studies regarding a scaling enhancement (with the system size) of a heat current have been reported, which plays a critical role in the advantage of quantum thermodynamic devices. Despite recent discoveries such as a quadratic heat current in a high-temperature environment [25,26] or interacting particles [27,31] and more theoretical studies behind these systems [29,32], universal scaling bounds on a heat current for arbitrary open quantum systems remain unknown.

In this Letter, we derive new bounds on a heat current $J(t)$ flowing into an open quantum system. We consider an L-particle system whose Hamiltonian is extensive with respect to L. A straightforward approach for generating the heat current is the parallel use of L particles, where each particle is individually coupled with the environment. We refer to this approach as a parallel scheme. For this scheme, the heat current scales as $|J(t)| = \Theta(L)$. Throughout this Letter, a function $f(L)$ is written as $f(L) = \Theta(g(L))$ if

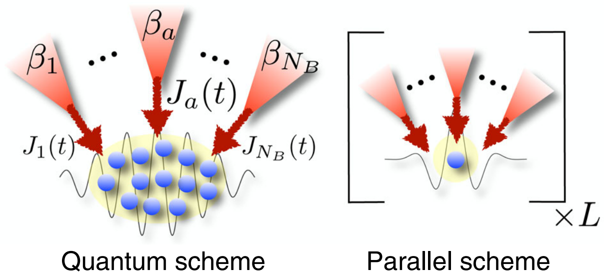

FIG. 1. Schematic for a quantum scheme for heat current generation realized in a system composed of L particles surrounded by N_B baths. For a parallel scheme, the L particles are used in parallel.

there exist constants k_1, k_2, and L_0, such that $k_1 g(L) \leq f(L) \leq k_2 g(L)$ is satisfied for any $L \geq L_0$. Meanwhile, in a quantum scheme, the particles are collectively coupled with the environment (see Fig. 1). For fair comparison of a quantum scheme and a parallel scheme, we assume that a system-environment interaction Hamiltonian is extensive as well. According to the convention of quantum thermodynamics, we specifically focus on an open quantum system that is weakly coupled with the environment. Under these conditions, we derive a scaling bound $|J(t)| \leq \Theta(L^3)$, and we present an example for which it is saturated in terms of scaling. However, the model requires an L-body interaction, which may be difficult to realize with the existing technology. Therefore, to derive another bound for more feasible models, we restrict the class of the system such that the interaction with the environment only induces transitions between system energy eigenstates whose energy difference is smaller than a certain value ΔE. By mathematically imposing this condition, we obtain another scaling bound $|J(t)| \leq \Theta(L^2)$ for $\Delta E = \Theta(L^0)$. Furthermore, we find that in the case of superradiance, which is a well-established collective energy emission process observed in light-matter systems [42], this bound is saturated in terms of scaling. Our derived bounds universally limit how a heat current can scale with the system size, regardless of the choice of the system. Our results are useful for evaluating the best achievable performance of quantum-enhanced thermodynamic devices. As an example, we can also construct a new quantum heat engine with a quantum enhancement.

Model.—We consider a total system composed of a quantum system subject to an environment modeled as N_B heat baths. The total Hamiltonian is expressed as follows:

$$\hat{H}_{\mathrm{tot}} = \hat{H}_S + \hat{H}_B + \hat{H}_{\mathrm{int}}, \tag{1}$$

$$\hat{H}_B = \sum_{a=1}^{N_B} \hat{H}_B^{(a)}, \tag{2}$$

$$\hat{H}_{\mathrm{int}} = \sum_{a=1}^{N_B} \hat{H}_{\mathrm{int}}^{(a)}, \tag{3}$$

where $\hat{H}_S$ ($\hat{H}_B$) is a system (an environmental) Hamiltonian, $\hat{H}_B^{(a)}$ is a free Hamiltonian of the ath bath, and $\hat{H}_{\mathrm{int}}$ is a system-environment interaction Hamiltonian that is defined as a summation of interaction Hamiltonians $\hat{H}_{\mathrm{int}}^{(a)}$ between the system and the ath bath. Further, we assume that these Hamiltonians are time-independent. Let $\hat{\rho}_B^{(a)} = e^{-\beta_a \hat{H}_B^{(a)}}/\mathrm{Tr}_a[e^{-\beta_a \hat{H}_B^{(a)}}]$ denote a thermal equilibrium state of the ath bath with an inverse temperature β_a, and let $\hat{\rho}_B = \bigotimes_{a=1}^{N_B} \hat{\rho}_B^{(a)}$ denote a product state of these thermal equilibrium states. Here, Tr_a is a partial trace over the degrees of freedom of the ath bath. For the interaction Hamiltonian $\hat{H}_{\mathrm{int}}^{(a)}$, we introduce Hermitian noise (bath) operators $\{\hat{A}_k^{(a)}\}_{k=1,2,\ldots,c_a}$ ($\{\hat{B}_k^{(a)}\}_{k=1,2,\ldots,c_a}$) acting on the degrees of freedom of the system (ath bath), where the label k denotes a "channel" through which the system-bath interaction is induced, and c_a denotes the number of channels. Then, the interaction Hamiltonian can be expressed as follows:

$$\hat{H}_{\mathrm{int}}^{(a)} = \sum_{k=1}^{c_a} \hat{A}_k^{(a)} \otimes \hat{B}_k^{(a)}. \tag{4}$$

After applying the Born and Markov approximations, we obtain the Redfield equation as follows ($\hbar = 1$) [43]:

$$\frac{d\hat{\rho}_S(t)}{dt} = -i[\hat{H}_S, \hat{\rho}_S(t)] + \sum_{a=1}^{N_B} \mathcal{D}_a[\hat{\rho}_S(t)], \tag{5}$$

$$\mathcal{D}_a[\hat{\rho}] = \int_0^\infty ds \sum_{k,l=1}^{c_a} C_{kl}^{(a)}(s) \Big[\tilde{A}_l^{(a)}(-s)\hat{\rho}\hat{A}_k^{(a)} - \hat{A}_k^{(a)}\tilde{A}_l^{(a)}(-s)\hat{\rho}\Big] + \mathrm{H.c.}, \tag{6}$$

where $\hat{\rho}_S(t)$ denotes a reduced density operator of the system (in the Schrödinger picture), and $\mathcal{D}_a$ denotes a dissipator due to the ath bath. Here, we define a correlation function as $C_{kl}^{(a)}(s) = \mathrm{Tr}_a[\tilde{B}_k^{(a)}(s)\hat{B}_l^{(a)}\hat{\rho}_B^{(a)}]$, where an operator is transformed as $\hat{O} \mapsto \tilde{O}(t) = e^{i\hat{H}_0 t}\hat{O}e^{-i\hat{H}_0 t}$ with respect to the free Hamiltonian $\hat{H}_0 = \hat{H}_S + \hat{H}_B$. According to the convention of quantum thermodynamics, we define an instantaneous net heat current $J(t)$ from the environment to the system as follows:

$$J(t) = \mathrm{Tr}\left[\hat{H}_S \frac{d\hat{\rho}_S(t)}{dt}\right]. \tag{7}$$

For a multibath environment, we obtain an "additivity" $J(t) = \sum_{a=1}^{N_B} J_a(t)$, where a heat current $J_a(t)$ from the ath bath to the system is defined as follows:

$$J_a(t) = \mathrm{Tr}\{\hat{H}_S \mathcal{D}_a[\hat{\rho}_S(t)]\}. \tag{8}$$

Bound 1.—We can derive a general upper bound on the absolute value of the heat current as follows [44]:

$$|J_a(t)| \leq 4\|\hat{H}_S\| \sum_{k,l=1}^{c_a} \Xi_{kl}^{(a)} \|\hat{A}_k^{(a)}\| \|\hat{A}_l^{(a)}\|, \tag{9}$$

where we introduce a coefficient $\Xi_{kl}^{(a)} = \int_0^\infty ds |C_{kl}^{(a)}(s)|$ and an operator norm for a given operator $\hat{O}$ (induced by a vector norm $\||\psi\rangle\| = \sqrt{\langle\psi|\psi\rangle}$), which is expressed as $\|\hat{O}\| = \sup_{|\psi\rangle}(\|\hat{O}|\psi\rangle\|/\||\psi\rangle\|)$. This newly derived bound is applicable to an arbitrary open quantum system, as far as the Born and Markov approximations are validated.

Application of Bound 1 to L-particle systems.—We apply the general bound, Eq. (9), to a system composed of (generally interacting) L identical particles (such as an L-qubit system) subject to N_B heat baths, and we analyze how a heat current scales with L (see Fig. 1). Suppose that the form of the system-environment interaction is given as Eq. (4). In particular, we focus on the case in which the system Hamiltonian is extensive in L, i.e., $\|\hat{H}_S\| = \Theta(L)$. Moreover, we assume that the interaction Hamiltonian $\hat{H}_{\rm int}$ is extensive as well, i.e., $\|\hat{H}_{\rm int}\| = \Theta(L)$, which is a natural assumption for a fair comparison of the heat current with that for a parallel scheme. Hence, we assume $\|\hat{A}_k^{(a)}\| = \Theta(L/c_a)$, which means that the distribution of the interaction energy over each channel k is homogeneous. Furthermore, we assume that $\|\hat{B}_k^{(a)}\| = \Theta(L^0)$, $\Xi_{kl}^{(a)} = \Theta(L^0)$, and $N_B = \Theta(L^0)$ because these quantities are solely determined by the properties of the environment. Consequently, we show that $|J_a(t)|$ and $\sum_{a=1}^{N_B} |J_a(t)|$ scale at most as $\Theta(L^3)$. Remarkably, this scaling exceeds that of the superradiance [42], which is known as a collective emission of photons from L qubits.

In the derivation explained above, we assume that the norm of the system-bath interaction scales linearly with the system size. This assumption is valid for example when a spin ensemble collectively interacts with a cavity where the wavelength of the cavity photon is much larger than the distance between the spins [53,54]. However, when we consider the thermodynamic limit, it might be difficult for the system-coupling strength to scale linearly with the system size [55,56]. Thus, we consider the case in which the system is not large enough to consider the thermodynamic limit but is large enough to observe the collective coupling between the system and environment. For example, the collective effect between a cavity and thousands of superconducting qubits was experimentally observed [54]. Therefore, our method could be realized up to thousands of qubits.

Here, we present an example of an L-qubit system for which the scaling bound $|J_a(t)| \leq \Theta(L^3)$ obtained from Eq. (9) is saturated. We introduce a system Hamiltonian as follows:

$$\hat{H}_S = \omega_q \hat{J}_z, \tag{10}$$

where $\hat{J}_z = \frac{1}{2}\sum_{i=1}^{L} \hat{\sigma}_z^{(i)}$, and $\hat{\sigma}_z^{(i)}$ is the z component of the Pauli operator for the ith qubit with a frequency ω_q. Then, for $\hat{J}_z$ and an operator $\hat{J}^2$ that represents the total angular momentum, we consider a subspace spanned by Dicke states $\{|L/2, M\rangle\}$, which is a simultaneous eigenstate of $\hat{J}^2$ and $\hat{J}_z$, with eigenvalues $(L/2)[(L/2)+1]$ and M, respectively $[M = L/2, (L/2)-1, \ldots, -L/2]$. Throughout this Letter, we omit the label $L/2$ of the Dicke states, i.e., $|M\rangle \equiv |L/2, M\rangle$. Explicitly, the Dicke state $|M\rangle$ is a superposition state of all the computational states having $(L/2)+M$ excited states and $(L/2)-M$ ground states with an equal coefficient for an odd number L.

For an interaction between the L-qubit system and an environment modeled by a single bath (i.e., $N_B = 1$), we consider an "m-body interaction" ($m = 1, 2, \ldots, L$) [44]. In particular, we introduce the following system operator $\hat{A}$ for the interaction Hamiltonian $\hat{H}_{\rm int} = \hat{A} \otimes \hat{B}$:

$$\hat{A} = \frac{gL}{{}_LC_m}\Big[\hat{\sigma}_x^{(1)} \otimes \cdots \otimes \hat{\sigma}_x^{(m)} \otimes \hat{I}^{(m+1)} \otimes \cdots \otimes \hat{I}^{(L)} + (\text{all possible permutations})\Big], \tag{11}$$

where g denotes a constant coupling strength, and $\hat{\sigma}_x^{(i)}$ $[\hat{I}^{(i)}]$ denotes the x component of the Pauli (identity) operator for the ith qubit. Although $\hat{A}$ contains a total of ${}_LC_m = [L!/(L-m)!m!]$ terms, we have $\|\hat{A}\| = \Theta(L)$ owing to the normalization by the prefactor; thus, $\|\hat{H}_{\rm int}\| = \Theta(L)$.

Under the Born and Markov approximations, we derive a Redfield equation for the L-qubit system coupled with a white-noise Markovian bath. Then, for an initial Dicke state $\hat{\rho}_S(0) = |L/2\rangle\langle L/2|$, we obtain an instantaneous heat current $J(0)$ as follows:

$$J(0) = -\gamma_{\rm wn}\omega_q \frac{L^2 m}{{}_LC_m}, \tag{12}$$

where $\gamma_{\rm wn}$ denotes a constant dissipation coefficient for the white-noise environment. For $m = L$, the interaction Hamiltonian induces a direct transition from $|L/2\rangle$ (all-excited state) to $|-L/2\rangle$ (all-ground state), and the absolute value of the heat current scales as $|J(0)| = \Theta(L^3)$. Thus, the universal scaling bound $\Theta(L^3)$ becomes saturated. More generally, for the case of $L - m = \Theta(L^0)$, we can evaluate the scaling as $|J(0)| = \Theta(L^{3+m-L})$ by using an identity ${}_LC_{m-1} = [m/(L-m+1)]_LC_m$. Therefore, in our example, the construction of the interaction for the universal bound $\Theta(L^3)$ to be saturated requires L-body interactions.

Bound 2.—The previous example requires L-body interactions for the scaling bound to be saturated, which might be difficult to realize with the existing technology. Therefore, we consider more feasible cases here. In practical situations, the transfer energy due to an interaction with an environment should be upper-bounded by a threshold value, because the transitions should occur between not-too-distant energy levels of the system Hamiltonian. In particular, we focus on a specific class of the system by setting a constraint that any nondiagonal elements of the noise operators become zero in the system energy basis, if the energy difference exceeds a certain value $\Delta E_k^{(a)}$ for channel k of the ath bath. Mathematically, this reads as follows:

$$|E_i - E_j| > \Delta E_k^{(a)} \Rightarrow \langle i|\hat{A}_k^{(a)}|j\rangle = 0 \quad \forall\ i, j, \tag{13}$$

where we define a spectral decomposition of the system Hamiltonian as $\hat{H}_S = \sum_{i=1}^{N} E_i|i\rangle\langle i|$, and N is the dimension of the Hilbert space of the system. Note that the system is allowed to be degenerate, and a similar assumption was made in Ref. [57]. Under this condition, by using a relation that was also used in Ref. [57], we rigorously derive another new bound on the absolute value of the heat current as follows [44]:

$$|J_a(t)| \le 2\sum_{k,l=1}^{c_a} \Xi_{kl}^{(a)} \Delta E_k^{(a)} \|\hat{A}_k^{(a)}\| \|\hat{A}_l^{(a)}\|. \tag{14}$$

Therefore, if we assume that $\Delta E_k^{(a)} = \Theta(L^0)$ ($\forall\ a, k$), then $|J_a(t)|$ scales at most as $\Theta(L^2)$, and this provides a different scaling bound from that provided by Eq. (9). Moreover, owing to the constraint, we can discuss more realistic cases for the bound to be saturated without many-body interactions.

Bound 2 for superradiance and superabsorption.—We discuss how our results can be applied to noninteracting qubits coupled with a common bath. For L qubits that are coupled with a single bath having a single channel (i.e., $N_B = 1$ and $c_a = 1$), a system Hamiltonian $\hat{H}_{\mathrm{SR}}$ and an interaction Hamiltonian $\hat{H}_{\mathrm{int}}$ are respectively given as follows:

$$\hat{H}_{\mathrm{SR}} = \omega_q \hat{J}_z, \tag{15}$$

$$\hat{H}_{\mathrm{int}} = 2g\hat{J}_x \otimes \hat{B}, \tag{16}$$

where $\hat{J}_x = \frac{1}{2}\sum_{i=1}^{L} \hat{\sigma}_x^{(i)}$, ω_q is a qubit frequency, $\hat{B}$ is a bath operator, and g is a coupling strength between the L-qubit system and the environment. For this system, the condition of Eq. (13) is satisfied for $\Delta E = \omega_q$; Consequently, $\Delta E = \Theta(L^0)$. Then, using Eq. (14), we obtain $|J_a(t)| \le \Theta(L^2)$.

We compare our results with those of previous studies on superradiance [44]. Suppose that L is an odd number. Under a Redfield equation derived for the system of superradiance with a zero temperature bath, we obtain a heat current for an initial state $|1/2\rangle$ as follows:

$$J(0) = -\frac{1}{4}\gamma_0 \omega_q (L+1)^2, \tag{17}$$

where γ_0 denotes a constant dissipation coefficient. This means that superradiance saturates the scaling bound $\Theta(L^2)$ obtained from Eq. (14). (see Fig. 2 for a summary of our results).

Furthermore, we investigate the case of superabsorption, which is interpreted as the reverse process of superradiance [58]. The system Hamiltonian is expressed as follows:

$$\hat{H}_{\mathrm{SA}} = \omega_q \hat{J}_z + \Omega \hat{J}_z^2. \tag{18}$$

A critical component of the superabsorption is the additionally introduced term $\Omega\hat{J}_z^2$. Combined with an engineering of a spectral structure of the environment, the system exhibits an energy absorption process from $|-1/2\rangle$ to $|1/2\rangle$ with a rate of $\Theta(L^2)$. This is superabsorption. However, the superabsorption does not cause the bound in Eq. (14) to be saturated in terms of scaling about L. This is because the interaction induces a transition from $|-L/2\rangle$ to $|-(L/2)+1\rangle$, and the energy difference between these two eigenstates is $\omega_q + (L-1)\Omega$. To use the constraints [Eq. (13)], we must set $\Delta E = \Theta(L)$ for this model. Thus, Eq. (14) provides an upper bound that scales as $\Theta(L^3)$, which cannot be saturated by superabsorption.

Bound on a steady-state heat current.—Although, so far, we have discussed the upper bounds on a heat current for a

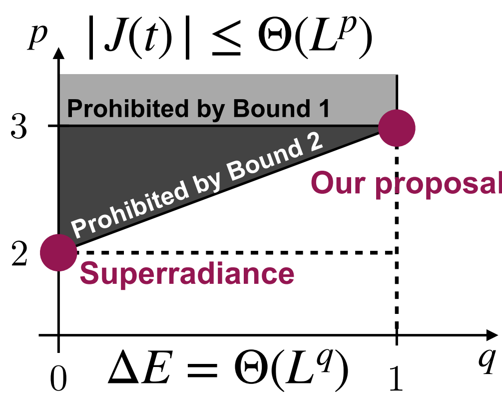

FIG. 2. Scaling bounds on a heat current $J(t)$ flowing into an L-particle open system. The light gray (dark gray) region represents the prohibited scaling by Bound 1 (Bound 2). For simplicity, we consider the case of a single bath and introduce $\Delta E = \max_k \Delta E_k$. Superradiance causes Bound 2 to be saturated, whereas our proposal of using an L-body interaction causes Bound 1 to be saturated. Here, we assume the extensivity of the system Hamiltonian, i.e., $\|\hat{H}_S\| = \Theta(L)$.

given initial state, we can also derive an upper bound on the heat current in a steady state. However, it is difficult to define a steady state for the Redfield equation due to the counter-rotating terms. Thus, we adopt a rotating-wave approximation (RWA) for the Redfield equation, and we obtain the following Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) master equation $[d\hat{\rho}_S(t)/dt] = \mathcal{L}[\hat{\rho}_S(t)]$ [43]:

$$\mathcal{L}[\hat{\rho}] = -i[\hat{H}_S + \hat{H}_{\mathrm{LS}}, \hat{\rho}] + \sum_{a=1}^{N_B} \mathcal{D}_a^{(G)}[\hat{\rho}]. \tag{19}$$

Here, the Lamb shift term $\hat{H}_{\mathrm{LS}}$ satisfies $[\hat{H}_S, \hat{H}_{\mathrm{LS}}] = 0$, and the dissipator $\mathcal{D}_a^{(G)}$ is defined as follows:

$$\mathcal{D}_a^{(G)}[\hat{\rho}] = \sum_{\omega} \sum_{k,l=1}^{c_a} \gamma_{kl,\omega}^{(a)} \left(\hat{A}_{l,\omega}^{(a)} \hat{\rho} \hat{A}_{k,\omega}^{(a)\dagger} - \frac{1}{2} \{ \hat{A}_{k,\omega}^{(a)\dagger} \hat{A}_{l,\omega}^{(a)}, \hat{\rho} \} \right), \tag{20}$$

where $\{\hat{A}, \hat{B}\} = \hat{A}\,\hat{B} + \hat{B}\,\hat{A}$ is an anticommutator. Then, for the GKSL equation, we define a steady state $\hat{\rho}_{\mathrm{ss}}$ for an initial state $\hat{\rho}_S(0)$ as follows:

$$\hat{\rho}_{\mathrm{ss}} = \lim_{t \to \infty} e^{\mathcal{L}t}[\hat{\rho}_S(0)]. \tag{21}$$

First, by using the commutation relationship $[\hat{H}_S, \hat{A}_{l,\omega}^{(a)}] = -\omega \hat{A}_{l,\omega}^{(a)}$ $(\forall\ l, \omega, a)$ that is satisfied for a microscopically derived GKSL equation, we show that $[\hat{H}_S, \hat{\rho}_{\mathrm{ss}}] = 0$ when $[\hat{H}_S, \hat{\rho}_S(0)] = 0$. Second, we show that, if $[\hat{H}_S, \hat{\rho}] = 0$ is satisfied, a heat current $J_a^{(G)}(\hat{\rho}) = \mathrm{Tr}(\hat{H}_S \mathcal{D}_a^{(G)}[\hat{\rho}])$ calculated by the GKSL master equation (after the RWA) is the same as that calculated by the Redfield equation (before the RWA) $J_a(\hat{\rho}) = \mathrm{Tr}(\hat{H}_S \mathcal{D}_a[\hat{\rho}])$. Therefore, we can use the right-hand sides of Eqs. (9) and (14) as the upper bounds on the steady-state heat current $J_{\mathrm{ss},a} = \mathrm{Tr}(\hat{H}_S \mathcal{D}_a^{(G)}[\hat{\rho}_{\mathrm{ss}}])$, which were originally derived for the Redfield equation. Moreover, we find a specific system to show a steady-state heat current of $|J_{\mathrm{ss}}| = \Theta(L^3)$, which saturates Bound 1 in terms of scaling. Consequently, we can construct new quantum-enhanced thermodynamic devices such as a quantum heat engine whose power output P scales as $P = \Theta(L^3)$ while its efficiency is fixed [44].

Conclusion.—In this Letter, we discussed newly derived bounds on a heat current flowing into an open quantum system weakly coupled with an environment. First, we derived a general scaling bound (with the number of particles) on the absolute value of the heat current by assuming the extensivity of a system Hamiltonian and a system-environment interaction Hamiltonian. In particular, we found that the best achievable scaling for the L-particle system is $\Theta(L^3)$ in a limit of large L. However, for the scaling bound to be saturated in our example, an L-body interaction is required, which may be difficult to realize with the existing technology. Then, we derived another bound based on the constraint that any nondiagonal elements of the noise operators become zero with respect to the system energy basis if the energy difference of these two bases exceeds a certain value ΔE. Based on this second bound, we showed that the absolute value of the heat current scales at most as $\Theta(L^2)$ for $\Delta E = \Theta(L^0)$, and this bound is saturated for superradiance. We first revealed the bounds that universally limit how fast a heat current generated by an open L-particle quantum system can scale with the number of particles. Our results are applicable not only to an open quantum system involving an interaction between particles but also to various types of the environment spectrum, both of which are expected to improve the performance of quantum thermodynamic devices such as quantum heat engines, quantum refrigerators, and quantum batteries.

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*s2130043@s.tsukuba.ac.jp
†tokura.yasuhiro.ft@u.tsukuba.ac.jp
‡matsuzaki.yuichiro@aist.go.jp

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