## Test of Light-Lepton Universality in the Rates of Inclusive Semileptonic $B$-Meson Decays at Belle II

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#### Abstract

We present the first measurement of the ratio of branching fractions of inclusive semileptonic $B$-meson decays, $R\left(X_{e / \mu}\right)=\mathcal{B}(B \rightarrow X e \nu) / \mathcal{B}(B \rightarrow X \mu \nu)$, a precision test of electron-muon universality, using data corresponding to $189 \mathrm{fb}^{-1}$ from electron-positron collisions collected with the Belle II detector. In events where the partner $B$ meson is fully reconstructed, we use fits to the lepton momentum spectra above $1.3 \mathrm{GeV} / c$ to obtain $R\left(X_{e / \mu}\right)=1.007 \pm 0.009$ (stat) $\pm 0.019$ (syst), which is the most precise leptonuniversality test of its kind and agrees with the standard-model expectation.


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In the standard model, all charged leptons share the same electroweak coupling, a symmetry known as lepton universality. Semileptonic $B$-meson decays involving the quark transition $b \rightarrow c(e, \mu, \tau) \nu$ provide excellent sensitivity to potential lepton-universality-violating (LUV) physics. Persistent evidence for LUV in the rate of semileptonic decays to $\tau$ leptons relative to the light leptons, $\ell \in(e, \mu)$, has been found in the combination of results from the $B A B A R$, Belle, and LHCb experiments [1-8]. Previous direct searches for LUV between the light leptons in semileptonic $B$-meson decays have measured the branch-ing-fraction ratio in a single exclusive charmed hadron decay mode [9] or the shapes of kinematic distributions of all decays to charmed hadrons [10].

We present here the first measurement of the inclusive branching-fraction ratio $R\left(X_{e / \mu}\right)=\mathcal{B}(B \rightarrow X e \nu) /$ $\mathcal{B}(B \rightarrow X \mu \nu)$, the most precise test of $e-\mu$ universality in semileptonic $B$-meson decays to date. We indicate with $X$ the generic hadronic final state of the semileptonic decay of any flavor of $B$ meson originating from $b \rightarrow c \ell \nu$ or, rarely, $b \rightarrow u \ell \nu$ quark transitions. We use a Belle II collision dataset collected between 2019 and 2021 at a center-of-mass (c.m.) energy of $\sqrt{s}=10.58 \mathrm{GeV}$, corresponding to the mass of the $\Upsilon(4 S)$ resonance, which almost exclusively decays into a pair of $B$ mesons of opposite flavor $\left(B^{0} \bar{B}^{0}\right.$, $B^{+} B^{-}$). The integrated luminosity of the dataset is $189 \mathrm{fb}^{-1}$, equivalent to approximately $198 \times 10^{6} B \bar{B}$ pairs. We use an additional off-resonance collision dataset, collected at an energy 60 MeV below the $\Upsilon(4 S)$ resonance and corresponding to an integrated luminosity of $18 \mathrm{fb}^{-1}$, to determine expected backgrounds from continuum processes $e^{+} e^{-} \rightarrow q \bar{q}$, where $q$ indicates $u, d, s$, or $c$ quarks. We reconstruct $B$ mesons decaying fully hadronically (the partner $B$ ) and associate remaining particles with their accompanying $B$ meson (the signal $B$ ). We identify leptons from among these remaining particles and extract the signal yield from a fit to the distribution of $p_{\ell}^{B}$, the lepton momentum in the rest frame of the signal $B$ meson.

[^0]The Belle II detector [11,12] operates at the SuperKEKB asymmetric-energy electron-positron collider [13] at KEK. The detector consists of several nested subsystems arranged in a closed cylinder around the interaction region and nearly coaxial with the beams. The cylindrical portion is referred to as the barrel, which is closed by the forward and backward end caps. The innermost subsystem is the vertex detector, composed of two layers of silicon pixels and four outer layers of silicon strip detectors. During data collection for this analysis the outermost pixel layer only covered $15 \%$ of the azimuthal angle. Charged-particle trajectories (tracks) are reconstructed by a small-cell drift chamber (CDC) filled with a $\mathrm{He}(50 \%)$ and $\mathrm{C}_{2} \mathrm{H}_{6}(50 \%)$ gas mixture, which also provides a measurement of ionization energy loss for particle identification. A Cherenkov-light imaging and time-of-propagation detector provides charged pion and kaon identification in the barrel region, while in the forward end cap a proximity-focusing, ring-imaging Cherenkov detector with an aerogel radiator is used. An electromagnetic calorimeter (ECL) consisting of $\mathrm{Cs}(\mathrm{I})$ crystals provides photon and electron identification in the barrel and both end caps. All of the above subsystems are embedded in a uniform 1.5 T magnetic field that is nearly aligned with the electron beam and is generated by a superconducting solenoid situated outside the ECL. The outermost subsystem, the $K_{L}^{0}$ and muon identification detector, consists of scintillator strips in the end caps and the inner part of the barrel, and resistive plate chambers in the outer barrel, interleaved with iron plates that serve as a magnetic flux return yoke.

We use Monte Carlo simulation to produce signal and background models, and to calculate reconstruction efficiencies and detector acceptance. The processes are simulated with the EvtGen [14], PYthia [15], and ккмс [16] software packages. Final-state radiation of photons from stable charged particles is simulated using the pнотоs software package [17]. Detector simulation is performed with the GEANT4 [18] software package. Simulated beam-induced backgrounds are added to the events [19]. Events are subsequently reconstructed and analyzed in the same fashion as the collision data with the Belle II analysis software framework, basf2 [20,21]. The simulated $e^{+} e^{-} \rightarrow \Upsilon(4 S) \rightarrow B \bar{B}$ samples contain known semileptonic and hadronic $B$ decays. The signal model includes the following known exclusive decays (charge
conjugation is implied throughout): $B \rightarrow D \ell \nu, B \rightarrow D^{*} \ell \nu$, and $B \rightarrow D^{* *} \ell \nu$, where $D^{* *}$ collectively indicates the excited charmed states $D_{0}^{*}, D_{1}^{\prime}, D_{1}$, and $D_{2}^{*}$, whose masses and widths are taken from Ref. [22]. The $B \rightarrow D^{(*)} \ell \nu$ decays are modeled with the Boyd-Grinstein-Lebed [23-25] form-factor parametrization. The modeling of $B \rightarrow D^{* *} \ell \nu$ decays is based on the Bernlochner-LigetiRobinson (BLR) model [26,27].

Semileptonic $B$ decays into the nonresonant final states $B \rightarrow D^{(*)} \pi \pi \ell \nu$ and $B \rightarrow D^{(*)} \eta \ell \nu$ are used to model the difference between the sum of individual branching fractions of exclusive decays, $B \rightarrow D^{(*)} \ell \nu$ and $B \rightarrow D^{* *} \ell \nu$, and the measured total semileptonic $B$ decay width [22]. These "gap modes" are included in dedicated simulated samples that use intermediate, broad $D^{* *}$ resonances and are modeled with BLR. Resonant and nonresonant Cabibbosuppressed semileptonic $B$ decays including $b \rightarrow u \ell \nu$ quark transitions are simulated simultaneously in a hybrid model [28] that is updated according to Ref. [29].

We use the Full Event Interpretation (FEI) algorithm [30] to reconstruct the signal $B$ meson's partner in a fully hadronic decay mode, labeled $B_{\mathrm{tag}}$. This separates the finalstate particles from the two $B$ mesons as well as increases the signal purity. We use three variables to maximize the purity of the $B_{\text {tag }}$ selection: The beam-constrained (bc) mass $M_{\mathrm{bc}}=\sqrt{(\sqrt{s} / 2)^{2}-\left|\vec{p}_{B}^{*}\right|^{2}}$, the energy difference $\Delta E=E_{B}^{*}-\sqrt{s} / 2$, and a confidence score produced by the FEI to classify $B$ mesons, $\mathcal{P}_{\text {FEI }}$, which has a value between zero (indicating low confidence in the tag reconstruction) and one (high confidence). The quantities $\vec{p}_{B}^{*}$ and $E_{B}^{*}$ denote the $B_{\text {tag }}$ momentum vector and energy in the c.m. frame, respectively. We select $B_{\mathrm{tag}}$ candidates with $M_{\mathrm{bc}} \in[5.2725,5.2850] \mathrm{GeV} / c^{2}, \Delta E \in[-0.15,0.10] \mathrm{GeV}$, and $\mathcal{P}_{\text {FEI }}>0.1$. If multiple $B_{\text {tag }}$ candidates pass these selections in an event, we choose the one with the highest value of $\mathcal{P}_{\text {FEI }}$. Approximately $82 \%$ of the selected tag candidates are correctly reconstructed, accounting for roughly $0.1 \%$ of all $\Upsilon(4 S)$ events [30].

We reconstruct signal-lepton candidates with $p_{\ell}^{B}>$ $1.3 \mathrm{GeV} / c$ from the remaining tracks after the $B_{\text {tag }}$ reconstruction. We require that the lepton charge corresponds to the charge of a primary lepton from the semileptonic decay of a signal $B$ meson that has the opposite flavor to the $B_{\text {tag }}$ candidate. The lepton track candidates are extrapolated to the point of closest approach to the measured interaction point, which is required to be within 1 cm in the radial direction and within 3 cm along the beam axis, and are required to point within the CDC angular acceptance.

Muon candidates are required to have transverse momentum $p_{T}>0.4 \mathrm{GeV} / c$ and are identified by means of a discriminator defined as the ratio $\mathcal{L}_{\mu} /\left(\mathcal{L}_{e}+\mathcal{L}_{\mu}+\mathcal{L}_{\pi}+\right.$ $\mathcal{L}_{K}+\mathcal{L}_{p}+\mathcal{L}_{d}$ ), where the identification likelihood $\mathcal{L}_{i}$ for each charged-particle hypothesis $i$ combines
particle-identification information from all subdetectors that provide it. The resulting efficiency is measured from dedicated control channels to be on average $90 \%$ for $p>1 \mathrm{GeV} / c$, corresponding to an average muon misidentification probability for pions and kaons of $3 \%$.

Electron candidates are required to have $p_{T}>0.3 \mathrm{GeV} / c$. We correct their four-momenta to recover bremsstrahlung radiation by adding energy depositions in the ECL (clusters) that are not matched to any track and that are found within a cone centered on the electron direction. The opening angle of this cone depends on the momentum magnitude and is optimized using simulation. We validate the bremsstrahlung correction using an inclusive sample of $J / \psi \rightarrow e^{+} e^{-}$candidates in experimental data. Electron candidates are identified by means of a multiclass boosted-decision-tree classifier that exploits several ECL-cluster observables in combination with particle-identification likelihoods from the other Belle II subsystems [31] defined analogously to the muon likelihood. The classifier thresholds are tuned in a three-dimensional grid of lab-frame momentum ( $\left.p^{\text {lab }}\right)$, polar angle $\left(\theta^{\text {lab }}\right)$, and charge $(q)$ intervals to achieve a uniform $80 \%$ identification efficiency. The misidentification probability for pions (kaons) with $p>1 \mathrm{GeV} / c$ in the barrel is on average $0.01(<0.001) \%$. If two or more signal-lepton candidates from the same event pass the above selections, we select the lepton with the highest identification likelihood. We obtain correction weights, typically near 1.0 , and uncertainties, for lepton-identification efficiencies and hadronmisidentification probabilities from auxiliary measurements in discrete intervals of ( $p^{\text {lab }}, \theta^{\text {lab }}, q$ ) using dedicated data samples. We calibrate the lepton-identification efficiencies using $J / \psi \rightarrow \ell^{+} \ell^{-}, \quad e^{+} e^{-} \rightarrow \ell^{+} \ell^{-}(\gamma)$, and $e^{+} e^{-} \rightarrow$ $\left(e^{+} e^{-}\right) \ell^{+} \ell^{-}$events. For charged kaons, we calibrate misidentification probabilities using $D^{*+} \rightarrow D^{0}\left(\rightarrow K^{-} \pi^{+}\right) \pi^{+}$ events; for charged pions, we use $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$and $e^{+} e^{-} \rightarrow$ $\tau^{ \pm}$(1-prong) $\tau^{\mp}$ (3-prong) events, where in the latter case one $\tau$ lepton is reconstructed in its decay modes with three charged hadrons.

All remaining ECL clusters not associated with a track that pass the following quality criteria are then combined to form the $X$ system. ECL clusters are required to be more than 30 cm away from the nearest extrapolated track and to have energies greater than $0.04,0.055$, and 0.09 GeV in the forward, barrel, and backward regions of the ECL, respectively. Tracks are required to be consistent with originating from the interaction point (within 2 cm in the radial direction and 4 cm along the beam axis), be in the CDC polar-angle acceptance, and have at least one measurement point in the CDC. Mass hypotheses are assigned to each charged particle by checking particle-identification criteria in a specific sequence (electron, muon, kaon, proton) and assigning the hypothesis of the first satisfied criterion. Remaining charged particles are considered to be pions.

We suppress continuum background with a boosted decision tree trained on simulated data that exploits 21 event-topology variables built from particle candidates that pass the same selection criteria as those used for the $X$ system reconstruction. These variables quantify the spatial distribution of momentum and energy in the events in order to discriminate between $B \bar{B}$ events, which are largely isotropic, and continuum events, which tend to have a back-to-back structure [32]. We select events identified as $B \bar{B}$-like, which rejects $55 \%$ of the continuum background while retaining $97 \%$ of the $B \bar{B}$ candidates.

We describe the remaining continuum background using off-resonance data with the yield scaled by the squared ratio of off- to on-resonance c.m. collision energies $c_{\text {off-res }}=$ $\left(\sqrt{\text { off-res }} / \sqrt{s}_{\Upsilon(4 S)}\right)^{2}=0.989$ to account for a factor of $1 / s$ in the $e^{+} e^{-} \rightarrow q \bar{q}$ cross section. The energy and momentum of particles in this dataset are also scaled by $1 / \sqrt{c_{\text {off-res }}}=1.006$ to account for the reduced c.m. energy available with respect to collisions at the $\Upsilon(4 S)$ resonance.

We extract the signal yields $N_{\ell}^{\text {meas }}$ with simultaneous binned maximum-likelihood template fits to the $p_{e}^{B}$ and $p_{\mu}^{B}$ spectra in the range $p_{\ell}^{B} \in[1.3,2.3] \mathrm{GeV} / c$ subdivided into ten equal intervals (bins), where the last bin includes any overflow events. The lower limit on $p_{\ell}^{B}$ is chosen to reduce backgrounds, and to suppress $B \rightarrow X \tau \nu$ decays to a negligible level.

We define three components for each lepton flavor, which we fit simultaneously. The signal component, $B \rightarrow X \ell \nu$, has an unconstrained yield. The continuum component has a Gaussian constraint on its yield derived from off-resonance data. The background component mostly contains events with hadrons misidentified as leptons (fakes) and correctly reconstructed lepton candidates originating mainly from decays of charmed hadrons (secondaries). The yield of this component has a Gaussian constraint derived from a fit to data in a samecharge control channel containing events with two $B$ mesons reconstructed with the same flavor and therefore enriched with fakes and secondaries, but also including $B \rightarrow X \ell \nu$ from neutral $B$-meson oscillations. We perform this control-channel fit with electrons and muons simultaneously and with unconstrained background and $B \rightarrow$ $X \ell \nu$ yields. We further verify that it is robust against arbitrary variations of the predicted yields of any of the components.

The statistical and systematic uncertainties are incorporated in the likelihood definition via nuisance parameters, one for each $p_{\ell}^{B}$ bin for each component. Constraints on the nuisance parameters are encoded in a global covariance matrix for bins and components, constructed by summing the covariance matrices of all individual uncertainty sources.

The uncertainties associated with the lepton-identification-efficiency and hadron-misidentification weights are provided by auxiliary measurements, as
previously described. They are propagated to $R\left(X_{e / \mu}\right)$ uncertainties under the following assumptions for leptons (or hadron fakes) of a given type: uncertainties within the same ( $p^{\text {lab }}, \theta^{\text {lab }}, q$ ) bin are fully correlated for events from different components; statistical (systematic) uncertainties are fully uncorrelated (correlated) for events in different bins and components.

We obtain event weights from branching-fraction uncertainties by performing Gaussian variations of each using central values and widths from the best experimental determinations and their uncertainties. For $B \rightarrow X_{u} \ell \nu$ and $B \rightarrow D^{(*)} \ell \nu$, we use the latest values [22,33], combining the results of neutral and charged $B$ mesons under the assumption of isospin symmetry in the latter case. For the remaining $b \rightarrow c \ell \nu$ decays, not all possible final states have been measured to date. We estimate their unknown branching fractions by extrapolating from existing measurements to the unobserved $D^{* *}$ final-state decays, again assuming isospin symmetry. Among the nonresonant gap modes, only the decay $B \rightarrow D^{(*)} \pi^{+} \pi^{-} \ell \bar{\nu}_{\ell}$ is measured [34]. This result is extrapolated to the other charge configurations to estimate their total branching fractions. The remaining gap modes, $B \rightarrow D^{(*)} \eta \ell \nu$, are assigned a $100 \%$ branching fraction uncertainty. The fit to data reduces the uncertainties on the gap-mode branching fractions by exploiting the differences in shape between these modes and the remaining signal. Form-factor parameters are varied within their uncertainties (including correlations) using the HAMMER software package [35]. Uncertainties in the number of selected signal events from uncertainties in branching fractions and form-factor parameters are assumed to be fully correlated between the electron and muon channels.

The ratios of the $B_{\mathrm{tag}}$ reconstruction efficiencies in data and simulation for each used $B$ hadronic decay mode are all compatible between the electron and muon channels within their statistical uncertainties. Therefore, we conclude they fully cancel out in the $R\left(X_{e / \mu}\right)$ ratio, and assign no further systematic uncertainty.

After all selections and corrections, we determine total signal efficiencies by extracting the selected signal yields $N_{\ell}^{\text {sel }}$ from fits to the simulated spectra and dividing by the number of generated events in the full phase space $N_{\ell}^{\mathrm{gen}}$. The electron efficiency is $(1.77 \pm 0.04) \times 10^{-3}$, and the muon efficiency is $(2.14 \pm 0.06) \times 10^{-3}$. The correlation between the two efficiencies due to shared systematic uncertainties is 0.76 .

We fit the experimental $p_{\ell}^{B}$ spectra in the same-charge control and opposite-charge signal samples, as shown in Fig. 1. We measure $N_{e}^{\text {meas }}=50960 \pm 290$ and $N_{\mu}^{\text {meas }}=$ $61300 \pm 400$ signal events in the electron and muon modes, respectively, with a correlation of 0.027 . The global $\chi^{2}$ value after the fit is 21.5 with 18 degrees of freedom, corresponding to a $p$ value of 0.25 .


FIG. 1. Same-charge control channel (left) and opposite-charge signal (right) spectra of the lepton momentum in the $B_{\text {sig }}$ rest frame $p_{\ell}^{B}$ with the fit results overlaid. The background component mostly contains events with fake or secondary leptons. The last bin contains overflow events. The hatched area shows the total statistical plus systematic uncertainty, added in quadrature in each bin.

From the yields, we calculate $R\left(X_{e / \mu}\right)$ using

$$
\begin{equation*}
R\left(X_{e / \mu}\right)=\frac{N_{e}^{\text {meas }}}{N_{\mu}^{\text {meas }}} \times \frac{N_{\mu}^{\text {sel }}}{N_{e}^{\text {sel }}} \times \frac{N_{e}^{\text {gen }}}{N_{\mu}^{\text {gen }}} \tag{1}
\end{equation*}
$$

We estimate the size of each systematic uncertainty by first fitting the simulated spectrum with only statistical fluctuations allowed. We then enable fluctuations from one systematic source and take the quadrature difference of the two to be the uncertainty from that source. We further validate these uncertainties by generating a large number of test datasets obtained by modifying the simulated dataset, each corresponding to a specific systematic variation, and observing the resulting variation in the extracted value of $R\left(X_{e / \mu}\right)$. The resulting uncertainties are summarized in Table I. The largest uncertainty, of $1.9 \%$, is associated with the lepton-identification efficiencies and misidentification probabilities. In the $R\left(X_{e / \mu}\right)$ ratio, branching-fraction and form-factor uncertainties largely cancel, with residual uncertainties arising from coupling between signal and background template shapes. Uncertainties associated with track finding efficiencies are negligible.

TABLE I. Statistical and systematic uncertainties on the value of $R\left(X_{e / \mu}\right)$ from the most significant sources.

| Source | Uncertainty [\%] |
| :--- | :---: |
| Sample size | 0.9 |
| Lepton identification | 1.9 |
| $X \ell \nu$ branching fractions | 0.2 |
| $X_{c} \ell \nu$ form factors | 0.1 |
| Total | 2.1 |

We find an $R\left(X_{e / \mu}\right)$ value of

$$
\begin{equation*}
R\left(X_{e / \mu}\right)=1.007 \pm 0.009(\text { stat }) \pm 0.019(\text { syst }) \tag{2}
\end{equation*}
$$

which agrees with a previous measurement from Belle in exclusive $B \rightarrow D^{*} \ell \nu$ decays [9]. In order to reduce model dependence, we also provide a fiducial measurement by recalculating $N_{\ell}^{\text {gen }}$ of Eq. (1) in the restricted phase space defined by selecting events with a generated $B$-frame lepton momentum above $1.3 \mathrm{GeV} / c$, leading to an overall scaling of $R\left(X_{e / \mu}\right)$ by 0.998 . The result is

$$
\begin{align*}
R\left(X_{e / \mu} \mid p_{\ell}^{B}>1.3 \mathrm{GeV} / c\right)= & 1.005 \pm 0.009(\text { stat }) \\
& \pm 0.019(\mathrm{syst}) . \tag{3}
\end{align*}
$$

In order to test the dependence of the result on the chosen lower threshold on $p_{\ell}^{B}$, we measure $R\left(X_{e / \mu}\right)$ while changing the nominal value of $1.3 \mathrm{GeV} / c$ to $1.1,1.2$, and $1.4 \mathrm{GeV} / c$. The values are mutually consistent with a $p$ value of 0.27 , taking into account the correlations between uncertainties of the four measurements. Similarly, the result is consistent between subsets of the full dataset when split by lepton charge, tag flavor, and by data-taking period. We find that the bremsstrahlung recovery procedure has negligible impact on the result. Furthermore, we check the impact on $R\left(X_{e / \mu}\right)$ of the modeling of charmed $D$ meson decays by varying the branching ratio of each decay $D \rightarrow K+$ anything within its uncertainty as provided in Ref. [22] while fixing the total event normalization. The effect is negligible. No evidence for a significant bias associated with the selection of a single candidate in the case of multiple candidates as described in Ref. [36] is observed.

Our result is the most precise branching fraction-based test of electron-muon universality in semileptonic $B$ decays. The measurement in the full phase space, Eq. (2), is consistent with the standard-model prediction of $1.006 \pm 0.001$ [37].

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[1] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. Lett. 109, 101802 (2012).
[2] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 88, 072012 (2013).
[3] M. Huschle et al. (Belle Collaboration), Phys. Rev. D 92, 072014 (2015).
[4] G. Caria et al. (Belle Collaboration), Phys. Rev. Lett. 124, 161803 (2020).
[5] S. Hirose et al. (Belle Collaboration), Phys. Rev. Lett. 118, 211801 (2017).
[6] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 115, 111803 (2015); 115, 159901(E) (2015).
[7] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 120, 171802 (2018).
[8] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 97, 072013 (2018).
[9] E. Waheed et al. (Belle Collaboration), Phys. Rev. D 100, 052007 (2019).
[10] R. van Tonder et al. (Belle Collaboration), Phys. Rev. D 104, 112011 (2021).
[11] T. Abe et al. (Belle II Collaboration), arXiv:1011.0352.
[12] W. Altmannshofer et al., Prog. Theor. Exp. Phys. 2019, 123C01 (2019).
[13] K. Akai, K. Furukawa, and H. Koiso (SuperKEKB accelerator team), Nucl. Instrum. Methods Phys. Res., Sect. A 907, 188 (2018).
[14] D. J. Lange, Nucl. Instrum. Methods Phys. Res., Sect. A 462, 152 (2001).
[15] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P.Z. Skands, Comput. Phys. Commun. 191, 159 (2015).
[16] S. Jadach, B. F. L. Ward, and Z. Wa̧s, Comput. Phys. Commun. 130, 260 (2000).
[17] E. Barberio, B. van Eijk, and Z. Wąs, Comput. Phys. Commun. 66, 115 (1991).
[18] S. Agostinelli et al. (GEANT4 Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 506, 250 (2003).
[19] A. Natochii et al., arXiv:2203.05731.
[20] T. Kuhr, C. Pulvermacher, M. Ritter, T. Hauth, and N. Braun (Belle II Framework Software Group), Comput. Software Big Sci. 3, 1 (2019).
[21] (Belle II Collaboration), 10.5281/zenodo.5574115.
[22] R. L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022).
[23] C. G. Boyd, B. Grinstein, and R. F. Lebed, Phys. Rev. Lett. 74, 4603 (1995).
[24] R. Glattauer et al. (Belle Collaboration), Phys. Rev. D 93, 032006 (2016).
[25] D. Ferlewicz, P. Urquijo, and E. Waheed, Phys. Rev. D 103, 073005 (2021).
[26] F. U. Bernlochner, Z. Ligeti, and D. J. Robinson, Phys. Rev. D 97, 075011 (2018).
[27] F. U. Bernlochner and Z. Ligeti, Phys. Rev. D 95, 014022 (2017).
[28] C. Ramirez, J. F. Donoghue, and G. Burdman, Phys. Rev. D 41, 1496 (1990).
[29] C. Patrignani et al. (Particle Data Group), Chin. Phys. C 40, 100001 (2016).
[30] T. Keck et al., Comput. Software Big Sci. 3, 6 (2019).
[31] M. Milesi, J. Tan, and P. Urquijo, EPJ Web Conf. 245, 06023 (2020).
[32] A. J. Bevan et al., Eur. Phys. J. C 74, 3026 (2014).
[33] Y. Amhis et al. (Heavy Flavor Averaging Group (HFLAV)), Eur. Phys. J. C 81, 226 (2021).
[34] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. Lett. 116, 041801 (2016).
[35] F. U. Bernlochner, S. Duell, Z. Ligeti, M. Papucci, and D. J. Robinson, Eur. Phys. J. C 80, 883 (2020).
[36] P. Koppenburg, arXiv:1703.01128.
[37] M. Rahimi and K. K. Vos, J. High Energy Phys. 11 (2022) 007.


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