Wang *et al.* Reply: In Ref. [1], we proposed that the apical oxygen vacancies act as anisotropic scattering impurities. Within the Born approximation, they lead to a quasiparticle scattering rate $\Gamma_d \cos^2(2\theta)$, that is maximal (zero) in the antinodal (nodal) direction. Together with an isotropic scattering rate Γ_s that is common in real samples [2] and $\Gamma_s \ll \Gamma_d$, the theory explains nicely the puzzling experimental results in the overdoped $La_{2-x}Sr_xCuO_4$ thin films: (1) The superfluid density $\rho_s(T)$ drops linearly with increasing temperature T, and at the same time $\rho_s(0)$ scales quasilinearly with the transition temperature T_c [3]; (2) the optical conductivity extrapolated to zero frequency appears to be lower than the transport conductivity [4].

In a recent comment [5] to Ref. [1], the importance of the nature of the impurity scattering is reemphasized, and some issues are raised regarding to the approach in Ref. [1], which we evaluate term by term below.

(i) Reference [5] points out that the oxygen vacancy leads to a change of local electrostatic potentials and may act as potential scattering center. We agree with this point, and in fact this provides a possible origin of Γ_s in our assumption, which we did not specify explicitly. However, this does not rule out the importance of the anisotropic scattering we proposed. According to Ref. [5], the change of the electrostatic potential is about 90 meV and 3 meV on nearest and next nearest coppers below the oxygen vacancy. This energy scale is minute in comparison to the local Hubbard interaction on copper (about 10.5 eV) and also to the charge transfer gap (about 4 eV) related to the in-plane oxygen [6]. As a result, this can barely change the effective parameters for the Zhang-Rice singlet [7], the effective degrees of freedom in doped cuprates. In contrast, the hopping between the p_z orbital (of the apical oxygen) and the p_x/p_y orbitals (of in-plane oxygens) should be of the same order of magnitude to that between the nearest p_x and p_y orbitals (about 0.65 eV). As such, the leading effect of the apical oxygen vacancy is the depletion of the virtual hopping of the Zhang-Rice singlet via the p_z orbital. It is this effect that leads to the anisotropic scattering we discussed in Ref. [1]. Note that this effect is the difference between the cases in the absence and presence of the apical oxygen, and the Zhang-Rice physics is not easily captured in the *ab initio* calculations.

In addition, Ref. [5] also proposed the scattering due to the Sr atoms, which is momentum dependent but is also weak.

(ii) Reference [5] stated that we did not consider the change of Fermi surface topology as the van Hove point is passed. In fact, we emphasized that we are concerned with the overdoped regime, where the Fermi surface topology no longer changes. In addition, the linear drop of $\rho_s(T)$ relies on the nodal structure of the Γ_d scattering only, independently of the concrete Fermi surface topology.

(iii) Reference [5] stated that we did not consider self-energy correction in the superconducting state. In fact, our concrete self-consistent calculations [8] show that in the dirty limit [to be consistent with the scaling between $\rho_s(0)$ and T_c in the overdoped regime], the self-energy is diagonal, purely imaginary, hardly changes in frequency, and is essentially given by the bare scattering rate. As such, the self-consistency leads to quantitative but not qualitative changes in the dirty limit considered in Ref. [1].

(iv) Reference [5] stated that we did not consider the vertex corrections. In fact, in the Born limit, the dressed vertex reads $\Lambda_k = v_k + n_{\rm imp} \sum_{k'} |V_{kk'}|^2 G^2(i\omega_n, k')\Lambda_{k'}/N$, where k is momentum, v_k is the bare vertex (or group velocity), $n_{\rm imp}$ is the impurity concentration, N is the number of lattice sites, G is the Matsubara Green's function, and $V_{kk'}$ is the scattering matrix (assumed to arise from the same type of scattering centers). As an example, consider $V_{kk'} = V_s + V_d f_k f_{k'}$, where $f_k = \cos k_x - \cos k_y$ is the d-wave form factor. Since $\Lambda_{k'}$ is odd in k', while $V_{kk'}$ and $G(i\omega_n, k')$ are even in k', the vertex correction vanishes by symmetry. If the s-wave and d-wave scattering centers are from different types of scattering centers, the vertex correction adds up, but the symmetry again rules out the vertex correction up to the linear order in both types of impurity concentrations.

To conclude, Ref. [5] proposed possible potential scattering effect of oxygen vacancies, which is included implicitly as Γ_s in Ref. [1]. The other concerns do not change our results qualitatively. The essential point is the quasinodal structure in the total scattering rate. This may follow from different scattering mechanisms [1,9], and it is likely that the difference does not arise at the level of final results.

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