Non-Fermi Liquids from Kinetic Constraints in Tilted Optical Lattices

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We study Fermi-Hubbard models with kinetically constrained dynamics that conserves both total particle number and total center of mass, a situation that arises when interacting fermions are placed in strongly tilted optical lattices. Through a combination of analytics and numerics, we show how the kinetic constraints stabilize an exotic non-Fermi liquid phase described by fermions coupled to a gapless bosonic field, which in several respects mimics a dynamical gauge field. This offers a novel route towards the study of non-Fermi liquid phases in the precision environments afforded by ultracold atom platforms.

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Introduction.—A major ongoing program in quantum many body physics is the characterization of phases of matter in which the quasiparticle paradigm breaks down. The most striking examples where this occurs are non-Fermi liquids (NFLs), believed to describe the observed strange metal behavior in a number of quantum materials. The low energy excitations in NFLs typically admit no quasiparticlelike description, with their ground states instead being more aptly thought of as a strongly interacting quantum soup. Our understanding of such states of matter, as well as the conditions in which they may be expected to occur, is very much in its infancy. To this end, it is extremely valuable to have examples of simple microscopic models in which NFLs can be shown to arise, especially so when these models are amenable to experimental realization.

In this Letter we propose just such a model, by demonstrating the emergence of a NFL in a kinetically constrained 2D Fermi-Hubbard model. This model is interesting in its own right, but our interest derives mainly from the fact that it finds a natural realization in strongly tilted optical lattices, a setup which has received recent experimental attention as a platform for studying ergodicity breaking and anomalous diffusion [1–3]. The key physics afforded by the strong tilt is that it provides a way of obtaining dynamics that conserves both total particle number *and* total dipole moment (for us "dipole moment" is synonymous with "center of mass"), with the latter conserved over a prethermal timescale which as we will see can be made extremely long.

In different settings, the kinetic constraints provided by dipole conservation are well known to arrest thermalization and produce a variety of interesting dynamical phenomena [4–8]. More recently, it has been realized that dipole conservation also has profound consequences for the nature of quantum ground states [9–15] and the patterns of symmetry breaking that occur therein [16,17].

Here, we show that when these constraints arise in the context of Fermi-Hubbard models, they produce an exotic NFL state in an experimentally accessible region of parameter space. The low energy theory of this NFL is closely analogous to a famous model in condensed matter physics, namely, that of a Fermi surface coupled to a dynamical U(1) gauge field [18–22]. We leverage this analogy to derive a number of striking features of the NFL state, chief among these being the absence of quasiparticles despite the presence of a sharp Fermi surface, and a vanishing conductivity despite the presence of a nonzero compressibility.

Fermions in strongly tilted optical lattices.—We begin by considering a model of spinless fermions on a tilted 2d square optical lattice, interacting through repulsive nearest-neighbor interactions (see Fig. 1). Spinful fermions are similar, and will be briefly discussed later. Writing the fermion annihilation operators as $f_{\mathbf{r}}$ and letting $n_{\mathbf{r}} \equiv f^\dagger_{\mathbf{r}} f_{\mathbf{r}}$, we consider the microscopic Hamiltonian $H = H_{\rm FH} + H_\Delta$, with the lattice tilt captured by $H_\Delta = \sum_{\mathbf{r},\mathbf{a}} \Delta_a r^a n_{\mathbf{r}}$, and the Fermi-Hubbard part given by

$$H_{\rm FH} = \sum_{\mathbf{r},\mathbf{a}} [-t_a (f^\dagger_{\mathbf{r}} f_{\mathbf{r}+\mathbf{a}} + \text{H.c.}) + V_{0,a} n_{\mathbf{r}} n_{\mathbf{r}+\mathbf{a}}], \quad (1)$$

where $\mathbf{a} = \hat{x}, \hat{y}$ label the unit vectors of the square lattice. The bare nearest-neighbor repulsion $V_{0,a}$ can be engineered by employing atoms with strong dipolar interactions [23,24] or by using Rydberg dressing [25–27]. To simplify the notation we will let both t_a/Δ_a and $V_0 \equiv V_{0,a}$ be independent of a, with $\Delta_x/\Delta_y = t_x/t_y$ unconstrained.

We will be interested in the large-tilt regime t_a/Δ_a, $V_0/\Delta_a \ll 1$, with t_a/V_0 arbitrary. Here it is helpful to pass to a rotating frame which eliminates H_Δ via the time-dependent gauge transformation e^{itH_Δ}. In this frame, the Hamiltonian is

$$H_{\rm rot}(t) = \sum_{\mathbf{r},\mathbf{a}} [-t_a (e^{-i\Delta_a t} f^\dagger_{\mathbf{r}} f_{\mathbf{r}+\mathbf{a}} + \text{H.c.}) + V_0 n_{\mathbf{r}} n_{\mathbf{r}+\mathbf{a}}]. \quad (2)$$

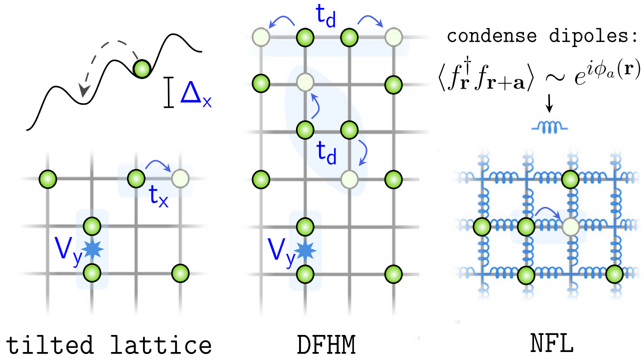

FIG. 1. *Tilted lattice*: We consider an extended Hubbard model in a tilted optical lattice, with single particle hoppings $t_{x,y}$, nearest neighbor repulsions $V_{x,y}$, and tilts along both directions with strengths $\Delta_{x,y}$. *DFHM*: In the large tilt limit the system is described by a dipole-conserving Fermi-Hubbard model, whose dynamics is such that only dipolar bound states—rather than individual fermions—are allowed to move. *NFL:* As the dipole hopping strength t_d increases the dipoles condense, with $f_{\mathbf{r}}^\dagger f_{\mathbf{r}+\mathbf{a}} \sim e^{i\phi_a(\mathbf{r})}$ developing an expectation value, and ϕ_a effectively playing the role of the spatial components of a dynamical U(1) gauge field. The condensate liberates the motion of individual fermions, which form a Fermi surface. As described in the main text, in this regime fluctuations in ϕ_a turn the system into a non-Fermi liquid.

We then perform a standard high-frequency expansion [2,28–32] to perturbatively remove the quickly oscillating phases in the first term. The time-independent part of the resulting expansion conserves the total dipole moments $\sum_{\mathbf{r}} r^a n_{\mathbf{r}}$ because dipoles—being charge neutral objects—can hop freely without picking up any $e^{-i\Delta_a t}$ phases. The result of this expansion [33] is the static Hamiltonian

$$H_{\rm DFH} = -\sum_{\mathbf{r},\mathbf{a}} t_d[d_{\mathbf{r}}^{a\dagger}(d^a_{\mathbf{r}+2\mathbf{a}} + d^a_{\mathbf{r}+\mathbf{a}+\bar{\mathbf{a}}} + d^a_{\mathbf{r}+\mathbf{a}-\bar{\mathbf{a}}} + \mathrm{H.c.})] + \sum_{\mathbf{r},\mathbf{a}} n_{\mathbf{r}}[V n_{\mathbf{r}+\mathbf{a}} + V' n_{\mathbf{r}+2\mathbf{a}} + V''(n_{\mathbf{r}+\mathbf{a}+\bar{\mathbf{a}}} + n_{\mathbf{r}+\mathbf{a}-\bar{\mathbf{a}}})], \quad (3)$$

where we have defined the dipole operators $d^a_{\mathbf{r}} \equiv f^\dagger_{\mathbf{r}} f_{\mathbf{r}+\mathbf{a}}$ and let $\bar{a}$ be the spatial coordinate opposite to a. As expected, $H_{\rm DFH}$ is invariant under the dipole symmetry $f_{\mathbf{r}} \mapsto f_{\mathbf{r}} e^{i\boldsymbol{\alpha}\cdot\mathbf{r}}$, $d^a_{\mathbf{r}} \mapsto d^a_{\mathbf{r}} e^{i\alpha^a}$ for any vector $\boldsymbol{\alpha}$. The coupling constants in $H_{\rm DFH}$ are given by

$$t_d = V_0 \mathsf{t}^2, \quad V = V_0(1-6\mathsf{t}^2), \quad V' = 2t_d, \quad V'' = 4t_d, \quad (4)$$

with the dimensionless hopping strength $\mathsf{t} \equiv t_a/\Delta_a$. We will refer to the model (3) as the dipolar Fermi-Hubbard model (DFHM).

As a time-independent theory, the DFHM only captures the system's dynamics over a (long) prethermal timescale. For (yet longer) times the fermions can exchange energy between $H_{\rm FH}$ and H_Δ, and a system initially prepared in the ground state of $H_{\rm DFH}$ will begin to heat up. We will see later that this is actually not an issue, as the relevant timescale can (in principle) be made arbitrarily long. Before explaining this, however, we first turn our attention to understanding the low-energy physics of $H_{\rm DFH}$.

Theory of the dipolar Fermi-Hubbard model.—In the DFHM, dipole conservation fixes the center of mass of the fermions, which cannot change under time evolution. This precludes a net flow of particles in any many-body ground state, implying a particle number conductivity which is strictly zero at all frequencies and guaranteeing that $H_{\rm DFH}$ always describes an insulating state [9]. In clean systems, a vanishing conductivity almost always comes hand-in-hand with a vanishing compressibility $dn/d\mu = 0$. We will, however, see that for a wide range of t the natural ground state of the DFHM is in fact *compressible*. In this regime the system has a sharp Fermi surface but lacks well-defined Landau quasiparticles, and is therefore an example of a NFL.

To understand the claims in the previous paragraph, we start by considering the limit of small t. Here the repulsive interactions dominate, and various crystalline states may form in a manner dependent on the fermion density. As t is increased, the system can lower its energy by letting dipolar bound states delocalize across the system, by virtue of the dipole hoppings on the first line of (3). For large enough t the dipoles will lower their kinetic energy by condensing, producing a phase where $D^a \equiv \langle d^a\rangle \neq 0$ and spontaneously breaking the dipole symmetry. When applied to the Hamiltonian (3) at half-filling, a mean-field treatment [33] predicts a condensation transition $\mathsf{t} = 1/4$, a value small enough that the perturbative analysis leading to $H_{\rm DFH}$ should remain qualitatively correct.

As also happens in the bosonic version of this model [9–11], the dipole condensate liberates the motion of single fermions: since $d^a_{\mathbf{r}}$ displaces a fermion along $\mathbf{a}$, the expectation value $D^a \neq 0$ gives fermions a nonzero kinetic energy in the $\mathbf{a}$ direction (intuitively, single fermions may now move by "absorbing" dipoles from the condensate). Indeed, assuming a rotation-invariant condensate and writing $D^a \simeq De^{i\phi_a}$ with D constant (allowed as amplitude fluctuations are gapped in the condensate), the first line of (3) becomes the *single-fermion* hopping term

$$H_{\rm hop} = -t_d D \sum_{\mathbf{r},\mathbf{a}} (f^\dagger_{\mathbf{r}} e^{i\phi_a(\mathbf{r})} f_{\mathbf{r}+\mathbf{a}} + \mathrm{H.c.}). \quad (5)$$

If we freeze out the dynamics of ϕ_a, $H_{\rm hop}$ will lead the fermions to form a Fermi surface, with an area set by their density as per Luttinger's theorem. The important question is then to ask what happens when one accounts for the dynamics of the ϕ_a fields. As soon as we introduce these dynamics, the system loses its ability to respond to uniform electric fields, and is rendered insulating. Indeed, turning on a background vector potential A_a in $H_{\rm hop}$ simply amounts to replacing ϕ_a by $\phi_a + A_a$. We can then completely eliminate the coupling of the fermions to A_a through a shift of ϕ_a (see also [34]). Since ϕ_a is the Goldstone mode for the broken dipole symmetry, all other terms in the effective Hamiltonian can only involve gradients of ϕ_a, and thus after the shift, the Hamiltonian can only depend on gradients of A_a. This then leads to a particle conductivity $\sigma(\omega, \mathbf{q})$ that vanishes for all ω as $\mathbf{q} \to 0$.

To deepen our understanding of this phase, we pass to a field theory description by writing $f_\mathbf{r} \simeq \int d\theta e^{i\mathbf{K}_F(\theta)\cdot\mathbf{r}}\psi_\theta(\mathbf{r})$, where $\mathbf{K}_F(\theta)$ is the Fermi momentum at an angle θ on the Fermi surface. Standard arguments then lead to the imaginary-time Lagrangian

$$\mathcal{L}_{\rm DFH} = \int d\theta \psi_\theta^\dagger \Big(\partial_\tau - i\mathbf{v}_\theta \cdot \nabla + \frac{\varkappa}{2}\nabla_\parallel^2 + \sum_a g_a(\theta)\phi_a\Big)\psi_\theta + \sum_a \kappa_D (\partial_\tau \phi_a)^2 + \sum_{a,b} K_{a,b}(\nabla_a \phi_b)^2. \quad (6)$$

In writing the above we have approximated the dispersion of ψ_θ to include only the leading terms in the momentum deviation from $\mathbf{K}_F(\theta)$, written the Fermi velocity as $\mathbf{v}_\theta$, let $\varkappa$ denote the Fermi surface curvature, and taken $\nabla_\parallel$ as the derivative along the Fermi surface.

In the important "Yukawa" term $g_a(\theta)\phi_a\psi_\theta^\dagger\psi_\theta$, the coupling function $g_a(\theta)$ is strongly constrained by dipole symmetry, which sends $\psi_\theta \to e^{i\alpha\cdot\mathbf{r}}\psi_\theta$, $\phi_a \to \phi_a + \alpha_a$ for any constant vector α_a. The requirement that (6) be invariant then gives the constraint

$$g_a(\theta) = -v_a(\theta). \quad (7)$$

This implies that ϕ_a couples to the fermions in *exactly* the same way as the spatial part of a U(1) gauge field. This draws a connection between the DFHM and a Fermi surface coupled to a dynamical U(1) gauge field, a system with a long history in condensed matter physics. In both models the modes that couple to the fermions are vector fields that are guaranteed to be gapless—by gauge invariance in the gauge field case, and by their origin as Goldstone modes in the DFHM. Crucially, the coupling between ϕ_a and the fermions is not "soft," remaining nonzero even at zero momentum (soft couplings are irrelevant under RG, and fail to induce NFL behavior). In line with the general framework of Ref. [35], this is made possible by the fact that the dipole charge and the total momentum P^b satisfy $\langle[\sum_\mathbf{r} r^a n_\mathbf{r}, P^b]\rangle = i\delta^{a,b}\sum_\mathbf{r}\langle n_\mathbf{r}\rangle \neq 0$, the nonvanishing of which is necessary to avoid obtaining a soft coupling.

An important difference compared to the Fermi surface + gauge field problem is that in the DFHM, there is no analog of a time component of the gauge field. For fermions coupled to a dynamical gauge field a_μ, the coupling to a_0 renders the theory incompressible: an external probe potential A_0 (the susceptibility to which the compressibility corresponds) evokes no response, as A_0 can be absorbed into a_0. Since there is no analog of a_0 in the DFHM this argument does not apply, and indeed it is well known that a Fermi surface coupled to a gapless boson is generically compressible [21,36–39]. Remarkably, we thus manage to obtain a system with both vanishing conductivity *and* nonzero compressibility (as also occurs in the "Bose-Einstein insulator" phase of the dipole-conserving Bose-Hubbard model [9]).

To demonstrate the NFL nature of $\mathcal{L}_{\rm DFH}$, we note that it is essentially the same as the action that arises at the "Hertz-Millis" theory [40,41] of the quantum critical point associated with the onset of loop current order in a metal [34] [but with the crucial restriction (7) coming from dipole conservation]. Like in that case, fluctuations of ϕ_a turn the system into an NFL. Indeed, standard calculations show that at the Fermi surface, the fermion self energy has the form $\Sigma_f(\mathbf{K}, i\omega) = i\,{\rm sgn}(\omega)|\omega|^\delta$ with the exponent $\delta < 1$ (a variety of theoretical approximations all converge on $\delta = 2/3$ [19,21,36,37,42,43], which is also the exponent of the low-temperature specific heat, $C \sim T^\delta$). This shows that there are no sharply defined quasiparticles in this model, despite the existence of a sharply defined Fermi surface. We also note that this model has no weak-coupling pairing instability, due to the strong repulsive interaction between fermions on antipodal patches mediated by ϕ_a [44].

Numerics.—We now provide a first step towards testing the above theoretical predictions by performing DMRG on small cylinders with the DFHM Hamiltonian (3). We focus on the case of half-filling so as to compare with predictions from mean field, which predicts a dipole condensation transition at $\mathsf{t} = 1/4$. Figure 2 shows the DMRG results for

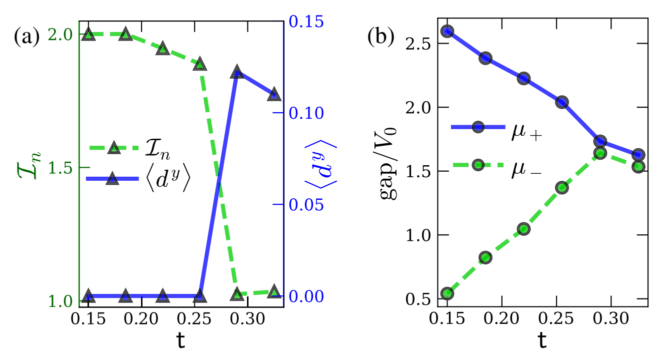

FIG. 2. DMRG results for the dipolar Fermi-Hubbard Hamiltonian $H_{\rm DFH}$ at half-filling on a cylinder of size $(L_x, L_y) = (20, 6)$ and at bond dimension $\chi = 400$. (a) the inverse participation ratio $\mathcal{I}_n$ of the density and the expectation value $\langle d_\mathbf{r}^y\rangle = \langle f_\mathbf{r}^\dagger f_{\mathbf{r}+\mathbf{y}}\rangle$ averaged over lattice sites. As judged by $\mathcal{I}_n$ charge order occurs at small t but melts at $\mathsf{t}_* \approx 0.275$; the plot of $\langle d^y\rangle$ shows this is also where dipole condensation occurs. (b) The energy cost to add or remove a fermion, $\mu_\pm \equiv \pm E(N \pm 1) \mp E(N)$, with $E(N)$ the ground state energy in the sector with total charge N. The charge gap $\mu_+ - \mu_-$ closes at the same location where dipole condensation occurs, suggesting the onset of the NFL.

a cylinder of modest size $(L_x, L_y) = (20, 6)$. For a range of t near $1/4$ we compute the expectation values of the dipole operators and the inverse participation ratio $\mathcal{I}_n \equiv L_x L_y \langle \sum_i n_i^2 \rangle / (\sum_i \langle n_i \rangle)^2$ (Fig. 2 left; $\langle d^x \rangle$ is similar to $\langle d^y \rangle$ but smaller in magnitude, presumably due to finite-size effects). We find $\mathcal{I}_n \approx 2$, $\langle d^y \rangle = 0$ for $\mathsf{t} \leq \mathsf{t}_*$ (as expected from a charge-ordered state) and $\mathcal{I}_n \approx 1$, $\langle d^y \rangle > 0$ for $\mathsf{t} > \mathsf{t}_*$ (as expected from a dipole condensate), where the critical value $\mathsf{t}_* \approx 0.275$ is respectably close to the mean-field estimate.

To investigate the state at $\mathsf{t} > \mathsf{t}_*$ we compute the chemical potentials $\mu_\pm \equiv \pm E(N \pm 1) \mp E(N)$, where $E(N)$ is the ground state energy in the symmetry sector with total charge N. The gap to charged excitations is given by $\mu_+ - \mu_-$, which is seen to approximately close at t_* (Fig. 2, right). This suggests that at t_* the system undergoes a (presumably first-order) transition into the NFL described by (6). While this is all in accordance with our theoretical analysis, these numerics do not answer questions about the doping dependence of t_*, or reveal the nature of the correlations present in the NFL phase. A proper treatment of these questions is left to future work.

Experimental considerations.—The most obvious experimental signature of the NFL state is the simultaneous presence of both a nonzero compressibility and a vanishing particle conductivity, both of which can be directly measured from density snapshots taken in quantum gas microscopes [45,46]. The dipole condensate can be directly detected through correlation functions of the dipole operators $f_{\mathbf{r}}^\dagger f_{\mathbf{r}+\mathbf{a}}$, which can be measured using superlattice potentials as detailed in Refs. [47,48]; these correlation functions are long-ranged in the NFL but short ranged in the charge-ordered state. The Fermi surface itself can be detected in principle by looking for Friedel oscillations in the density-density correlation function [49], which are even stronger [37] than in a conventional Fermi liquid.

We now discuss issues relating to the experimental preparation of the NFL state. In one possible protocol, the system is prepared in a uniform density product state at zero single particle hopping $t_a = 0$ and zero tilt $\Delta_a = 0$. Δ_a is then diabatically switched on to a value much larger than the Hubbard interaction and the dimensionless hopping strength $\mathsf{t} \equiv t_a/\Delta_a$ is slowly increased, with the goal of reaching the NFL regime while keeping the dipole-conserving system at an effective temperature $T \lesssim T_F$, with $T_F \sim t_d = V_0 \mathsf{t}^2$ the Fermi temperature.

At this point in the discussion, the prethermal nature of H_{DFH} becomes important. In going from (2) to (3) we only kept the time-independent part of the effective Hamiltonian; a more complete analysis shows that in fact $H = H_{\mathrm{DFH}} + \mathcal{V}(t)$, with the most important part of $\mathcal{V}(t)$ being $\mathcal{V}(t) = V_0 \mathsf{t}^2 \sum_{\mathbf{r}; s=\pm 1} e^{i(\Delta_x + s\Delta_y)t} \mathcal{O}_{\mathbf{r}}^s + \mathrm{H.c.}$, where $\mathcal{O}_{\mathbf{r}}^s$ is a rather complicated four-fermion interaction with a net dipole moment of 1 (s) in the x (y) direction [33]. $\mathcal{V}(t)$ causes a system initially prepared in the ground state of H_{DFH} to heat up. Furthermore, if $|\Delta_x| = |\Delta_y|$, $\mathcal{V}(t)$ contains *time-independent* terms which break one linear combination of the two components of the dipole moment symmetry (if $|\Delta_x|/|\Delta_y| = p/q$ is rational, such terms will arise at qth order in perturbation theory). Breaking the symmetry in this way will generically yield a nonzero conductivity along one spatial direction and produce a crossover to an anisotropic phase that preempts the NFL at large scales. Fortunately, we now argue that these problems are not as severe as they might appear.

The issue of $\mathcal{V}(t)$ containing time-independent dipole-violating terms can be circumvented simply by taking $|\Delta_x|/|\Delta_y|$ to be irrational [5]. However, even when $|\Delta_x| = |\Delta_y|$, the time-independent part of $\mathcal{V}(t)$ is highly irrelevant and only produces a violation of (7) through 2-loop diagrams that are suppressed by further powers of t. In practice, these symmetry-breaking terms may thus only lead to a crossover out of the NFL at length scales larger than experimentally relevant system sizes.

To assess the effects of heating, we estimate the heating rate r of a state initially prepared in the ground state of H_{DFH} and then time evolved with $H_{\mathrm{DFH}} + \mathcal{V}(t)$. r can be bounded using the theory of Floquet prethermalization [50–53] as

$$r < C' V_0^2 e^{-C|\Delta_x|/J}, \tag{8}$$

where $C, C' > 0$ are dimensionless constants depending on $|\Delta_x|/|\Delta_y|$, and where J is an energy scale determined by the maximum amount of energy locally absorbable by H_{DFH} (if $|\Delta_x|/|\Delta_y|$ is irrational, $|\Delta_x|/J$ in the exponent is replaced by $\sqrt{|\Delta_x|/J}$ [53]).

From the couplings given in (4), we see that *all* of the terms in H_{DFH} are proportional to V_0, meaning that $J = C'' V_0$ for another dimensionless constant C''. Crucially though, the parameter t which tunes between the different phases of H_{DFH} is *independent* of V_0. This implies that by decreasing V_0 and keeping t fixed we can make $|\Delta_x|/J$ arbitrarily large—and hence r arbitrarily small—all while remaining at a *fixed* point in the phase diagram. This parametric suppression of r means that the issue of prethermal heating can in principle be sidestepped simply by working at weak bare interactions.

Discussion.—We have demonstrated the emergence of a rather exotic non-Fermi liquid (NFL) from a simple dipole-conserving Fermi-Hubbard model. This model has a natural realization in strongly tilted optical lattices, and in the NFL regime is described by fermions coupled to an emergent bosonic mode which plays the role of a spatial gauge field. This provides an ultracold-atoms path towards the study of strongly interacting fermions and gauge fields in a manner rather distinct from approaches that build in gauge fields at a more microscopic level [54–56].

As always with ultracold atoms, the experimental crux is likely to be whether or not one can access the temperatures

$T \lesssim T_F$ required to probe the physics of the NFL ground state. With this in mind it is natural to wonder if the kinetic constraints imposed by dipole conservation lead to any interesting *dynamical* signatures of the NFL regime, which could be more readily identified in experiments unable to perform a sufficiently adiabatic parameter sweep.

While our focus so far has been on systems of spinless fermions, similar physics is also realizable in spinful models. A natural place to look is the tilted Fermi-Hubbard model

$$H = \sum_{\mathbf{r},\mathbf{a},\sigma}[-t_a(f^{\dagger}_{\mathbf{r},\sigma}f_{\mathbf{r}+\mathbf{a},\sigma} + \mathrm{H.c.}) + \Delta_a r^a n_{\mathbf{r},\sigma}] + V_0 \sum_{\mathbf{r}} n_{\mathbf{r},\uparrow} n_{\mathbf{r},\downarrow}. \tag{9}$$

In the large Δ_a limit, this Hamiltonian yields a dipole-conserving model with nearest-neighbor interactions and a Heisenberg exchange proportional to V_0. At half filling and with *attractive* bare interactions, mean-field calculations predict a transition at $t_a/\Delta_a \gtrsim 0.26$, where the dipole operators $f^{\dagger}_{\mathbf{r},\sigma}f_{\mathbf{r}+\mathbf{a},\sigma}$ condense and subsequently produce an NFL. We leave a more thorough investigation of this physics to future work.

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Note added.—Recently, we learned of a related work to appear by A. Anakru and Z. Bi.

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