Many-Body Superradiance and Dynamical Mirror Symmetry Breaking in Waveguide QED

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The many-body decay of extended collections of two-level systems remains an open problem. Here, we investigate whether an array of emitters coupled to a one-dimensional bath undergoes Dicke superradiance. This is a process whereby a completely inverted system becomes correlated via dissipation, leading to the release of all the energy in the form of a rapid photon burst. We derive the minimal conditions for the burst to happen as a function of the number of emitters, the chirality of the waveguide, and the single-emitter optical depth, both for ordered and disordered ensembles. Many-body superradiance occurs because the initial fluctuation that triggers the emission is amplified throughout the decay process. In one-dimensional baths, this avalanchelike behavior leads to a spontaneous mirror symmetry breaking, with large shot-to-shot fluctuations in the number of photons emitted to the left and right. Superradiant bursts may thus be a smoking gun for the generation of correlated photon states of exotic quantum statistics.

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The decay rate of a single emitter is dictated by its radiative environment [1–3]. This realization contributed to the development of cavity quantum electrodynamics (QED). Here, highly reflecting mirrors isolate a single optical mode, yielding a localized (or zero-dimensional) reservoir for the emitter, which enhances its decay into the cavity. One-dimensional (1D) baths pertain to "waveguide QED," where an atom is interfaced with a propagating optical mode. Recent years have seen tremendous experimental progress in the field, with platforms including cold atoms coupled to optical nanofibers [4–8], cold atoms [9–11] and quantum dots [12,13] coupled to photonic crystal waveguides, and superconducting qubits coupled to microwave transmission lines [14–16]. Besides altering decay, interfacing several emitters with 1D propagating modes allows engineering of long-range atom-atom interactions [17–20].

The environment also determines the many-body decay of a multiply excited ensemble. A paradigmatic example of many-body decay is Dicke superradiance: a collection of fully inverted emitters phase locks as they decay, emitting a short bright pulse of photons [21,22]. Despite its many-body nature, this problem is solvable in a cavity due to the permutational symmetry that restricts the dynamics to a small subset of states of the (otherwise exponentially large) Hilbert space. In extended systems, atom-atom interactions depend on their positions and many-body decay generates complex dynamics [23–33] that remains to be fully understood.

In free space, superradiance can generate highly directional emission due to the sample geometry [22,34–36]. Symmetry in emission direction can also be broken spontaneously, as atom-atom correlations lead to "memory effects": detecting a photon in one specific direction increases the likelihood for that detector to record subsequent photons. As the direction of emission of the first photon is random, and due to the avalanchelike nature of the process, Dicke superradiance has been predicted to give rise to large shot-to-shot fluctuations in the angular distribution of the far field intensity, both for atomic ensembles [21,37–39] and Bose-Einstein condensates [40]. However, the many optical field modes make the theoretical analysis of this phenomenon challenging.

Here, we investigate the decay of a fully inverted array of emitters into a 1D bath. Leveraging previous work [30] to bypass the exponential growth of the Hilbert space by studying early dynamics, we set constraints on the number of emitters needed to observe superradiance. We investigate chiral and bidirectional waveguides interfaced with ordered and disordered ensembles. In one dimension, due to the confined nature of the optical fields, the buildup of directional correlations translates into spontaneous breaking of mirror symmetry, giving rise to an emergent chirality. We compute the probability distribution of directional emission, which is very broad due to large shot-to-shot fluctuations. The distribution is shown to evolve with time, as Hamiltonian evolution scrambles the correlations imprinted by the dissipative process, thus washing away the memory of the system.

We consider N emitters of resonance frequency ω_0 coupled to a 1D photonic channel, as shown in Fig. 1(a). The waveguide mode mediates interactions between emitters. Tracing out the photonic degrees of freedom under a Born-Markov approximation, the evolution of the emitters' density matrix in the rotating frame is described by the master equation [41,42]

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\mathcal{H}_L + \mathcal{H}_R, \hat{\rho}] + \mathcal{L}_g[\hat{\rho}] + \mathcal{L}_{ng}[\hat{\rho}]. \quad (1)$$

Here, the Hamiltonians $\mathcal{H}_{L(R)}$ allow for distinct coupling to left- and right-propagating waveguide modes (at rates $\Gamma_{L(R)}$ for a single emitter), and read [43]

$$\mathcal{H}_L = -\frac{i\hbar\Gamma_L}{2}\sum_{i<j} e^{ik_{1\mathrm{D}}|z_i - z_j|}\hat{\sigma}^i_{eg}\hat{\sigma}^j_{ge} + \mathrm{H.c.}, \quad (2a)$$

$$\mathcal{H}_R = -\frac{i\hbar\Gamma_R}{2}\sum_{i>j} e^{ik_{1\mathrm{D}}|z_i - z_j|}\hat{\sigma}^i_{eg}\hat{\sigma}^j_{ge} + \mathrm{H.c.}, \quad (2b)$$

where $\hat{\sigma}^i_{ge} = |g_i\rangle\langle e_i|$ is the coherence operator between the ground and excited states of emitter i at position z_i, $k_{1\mathrm{D}}$ is the photon wave vector, and H.c. stands for Hermitian conjugate. The total decay rate of a single emitter into the waveguide is $\Gamma_{1\mathrm{D}} = \Gamma_L + \Gamma_R$. The Lindblad operators $\mathcal{L}_g[\hat{\rho}]$ and $\mathcal{L}_{ng}[\hat{\rho}]$ describe the decay of emitters to guided and nonguided modes, respectively, and read

$$\mathcal{L}_\alpha[\hat{\rho}] = \sum_{i,j=1}^{N}\frac{\Gamma^\alpha_{ij}}{2}(2\hat{\sigma}^j_{ge}\hat{\rho}\hat{\sigma}^i_{eg} - \hat{\rho}\hat{\sigma}^i_{eg}\hat{\sigma}^j_{ge} - \hat{\sigma}^i_{eg}\hat{\sigma}^j_{ge}\hat{\rho}), \quad (3)$$

where $\Gamma^g_{ij} = \Gamma_L e^{ik_{1\mathrm{D}}(z_j - z_i)} + \Gamma_R e^{-ik_{1\mathrm{D}}(z_j - z_i)}$ and $\Gamma^{ng}_{ij} = \Gamma'\delta_{ij}$. We consider that nonguided decay is not collective, either because it represents local parasitic decay or because emitters are far separated and interactions via nonguided modes are negligible.

Emission of photons into the waveguide is correlated due to the shared bath. This is captured by collective jump operators found by diagonalizing the $N \times N$ Hermitian matrix $\mathbb{\Gamma}$ of elements Γ^g_{ij} [35,37]. Photons can only be emitted into the left- or right-propagating modes, and thus $\mathbb{\Gamma}$ has only two nonzero eigenvalues and we can write

$$\mathcal{L}_g[\hat{\rho}] = \sum_{\nu=+,-}\frac{\Gamma_\nu}{2}(2\hat{\mathcal{O}}_\nu\hat{\rho}\hat{\mathcal{O}}^\dagger_\nu - \hat{\rho}\hat{\mathcal{O}}^\dagger_\nu\hat{\mathcal{O}}_\nu - \hat{\mathcal{O}}^\dagger_\nu\hat{\mathcal{O}}_\nu\hat{\rho}), \quad (4)$$

where $\hat{\mathcal{O}}_\nu$ are collective jump operators and Γ_ν are collective decay rates, found as the eigenvectors and eigenvalues of $\mathbb{\Gamma}$, respectively. The $\{+,-\}$ notation indicates that $\hat{\mathcal{O}}_{+(-)}$ generates a photon in a symmetric (antisymmetric) superposition of left- and right-propagating modes.

A fully inverted initial state, $|\psi(t=0)\rangle = |e\rangle^{\otimes N}$, will decay due to vacuum fluctuations, leading to emission of photons into the waveguide at a (normalized) rate

$$R(t) = \frac{1}{N\Gamma_{1\mathrm{D}}}\sum_{\nu=+,-}\Gamma_\nu\langle\hat{\mathcal{O}}^\dagger_\nu\hat{\mathcal{O}}_\nu\rangle. \quad (5)$$

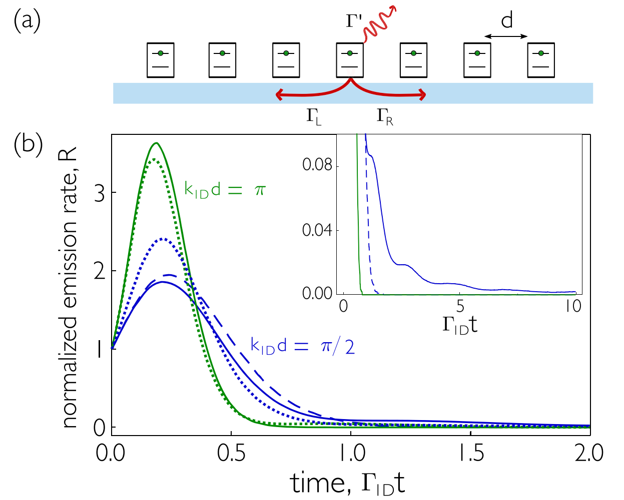

FIG. 1. Many-body superradiance from emitters coupled to a waveguide. (a) Schematic: N emitters of lattice constant d interact via a 1D bath, which supports propagation of photons of wave vector $\pm k_{1\mathrm{D}}$. Single-emitter decay rates into left- and right-propagating modes of the waveguide are $\Gamma_{L/R}$, respectively, and any other parasitic decay is denoted by Γ'. (b) Emission rate into the waveguide for an array of $N = 16$ emitters coupled to a bidirectional (solid lines) and a chiral (dotted) waveguide with $\Gamma_R = 3\Gamma_L$ and $\Gamma' = 0$. Dashed line shows the bidirectional waveguide calculation without Hamiltonian contribution, which is significant at late times (inset).

For large enough N and $\Gamma' = 0$, a superradiant burst occurs for any lattice constant, as shown in Fig. 1(b) for ordered arrays. Calculations are performed using quantum trajectories [44–46]. Maximal superradiance occurs in a bidirectional waveguide (i.e., $\Gamma_R = \Gamma_L$), at the so-called "mirror configuration" ($k_{1\mathrm{D}}d = n\pi$ with $n \in \mathbb{N}$ [51–53]), as this situation corresponds to that studied by Dicke.

Dissipative dynamics are the main driving mechanism for the burst. The coherent (i.e., Hamiltonian) interactions contribute significantly only well beyond the time of maximum emission. At later times the Hamiltonian plays an important role, leading to oscillations in emission as it cycles the atoms between dark and bright states [see inset to Fig. 1(b)].

As we postulated in prior work [30], the minimal condition for a burst is that the first photon enhances the emission of the second. The same insight can be used to derive a condition for a superradiant burst into one particular channel, even if emitters can decay to more than one reservoir (as happens in the presence of finite Γ'). To do so, we adapt the calculation from Ref. [28] of "directional superradiance" and define a second-order correlation function conditioned on measurement only of the waveguide modes

$$\tilde{g}^{(2)}(0) = \frac{\sum_{\nu=\pm}\sum_{\mu=\pm,i}\Gamma_\nu\Gamma_\mu\langle\hat{\mathcal{O}}^\dagger_\mu\hat{\mathcal{O}}^\dagger_\nu\hat{\mathcal{O}}_\nu\hat{\mathcal{O}}_\mu\rangle}{(\sum_{\nu=\pm}\Gamma_\nu\langle\hat{\mathcal{O}}^\dagger_\nu\hat{\mathcal{O}}_\nu\rangle)(\sum_{\mu=\pm,i}\Gamma_\mu\langle\hat{\mathcal{O}}^\dagger_\mu\hat{\mathcal{O}}_\mu\rangle)}. \quad (6)$$

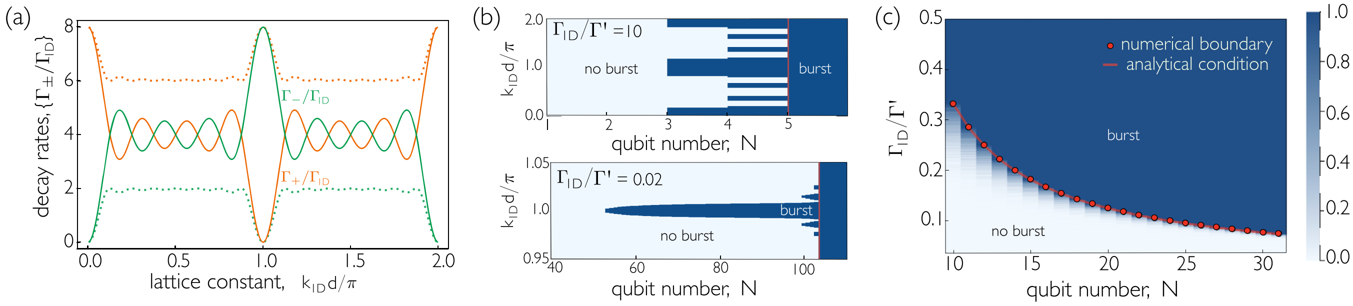

FIG. 2. Predictions of superradiance for ordered and disordered ensembles. (a) Collective decay rates into a bidirectional (solid lines) and a chiral (dotted) waveguide for $N = 8$. Single-emitter decay rates for the chiral waveguide are $\Gamma_R = 3\Gamma_L$. (b) Crossover between burst (dark blue) and no-burst (pale blue) regions in ordered arrays coupled to a bidirectional waveguide. (c) Probability of having a burst in a bidirectional waveguide for spatially disordered ensembles of emitters randomly placed over a section of length $k_{1D}L \gg 2\pi$ [46]. In (b),(c) the red line shows the emitter number that guarantees a burst regardless of separation, as given by Eq. (10).

Here, sums over $\pm$ account for waveguide emission, while sums in i account for local decay. The minimal condition for superradiant emission of photons into the waveguide is found by imposing $\tilde{g}^{(2)}(0) > 1$. Importantly, this condition selects the processes in which the photon emission rate is enhanced into the waveguide only.

For an ensemble of initially inverted emitters this condition becomes [see Supplemental Material [46]]

$$\mathrm{Var}\left(\frac{\{\Gamma_\nu\}}{\Gamma_{1D}}\right) > 1 + \frac{\Gamma'}{\Gamma_{1D}}, \tag{7}$$

where $\mathrm{Var}(\cdot)$ is the variance. This expression is universal; it applies to systems with any number of emitters, in disordered or ordered spatial configurations, and coupled to waveguides with any degree of chirality. A burst occurs if there are only a few dominant decay channels (maximizing the variance), and if collective decay overcomes local loss. As emission is constrained to one dimension, there are at most two bright channels, while $N - 2$ are dark (i.e., of zero decay rate). Therefore, the conditions for a burst are more easily satisfied than for arrays in free space [30,31,54]. Restriction of emission into a 1D bath eliminates most of the competition between different imprinted phase patterns, enabling a more robust phase locking than in free space, where photons can be emitted in all directions.

For ordered arrays of lattice constant d, the two collective decay rates admit the analytical form

$$\Gamma_\pm = \frac{N\Gamma_{1D}}{2} \pm \sqrt{\frac{N^2(\Gamma_L - \Gamma_R)^2}{4} + \Gamma_L\Gamma_R\frac{\sin^2 Nk_{1D}d}{\sin^2 k_{1D}d}}. \tag{8}$$

The two decay rates are generally distinct and finite, as shown in Fig. 2(a), leading to competition between the $\pm$ channels. Different lattice constants give rise to situations ranging from the Dicke model with a single nonzero collective decay rate to maximum competition, where the decay rates are degenerate due to an emergent translational symmetry [46]. For chiral waveguides there is no degeneracy, as any level of chirality breaks translation symmetry. In this case, rates are almost independent of the lattice constant, as interference effects are suppressed.

The minimal burst condition for ordered arrays reads

$$\frac{N(\Gamma_L^2 + \Gamma_R^2)}{\Gamma_{1D}^2} + 2\frac{\Gamma_L\Gamma_R}{N\Gamma_{1D}^2}\frac{\sin^2 Nk_{1D}d}{\sin^2 k_{1D}d} > 2 + \frac{\Gamma'}{\Gamma_{1D}}. \tag{9}$$

Large parasitic decay quenches the superradiant burst for small N, as shown in Fig. 2(b). However (and regardless of the level of independent decay), a burst is always recovered if the number of emitters is increased beyond a certain threshold.

For disordered systems we obtain the minimal burst condition in terms of single-emitter decay rates by placing a lower bound on the trace of $\mathbb{\Gamma}^2$, as the eigenvalues do not admit an analytical form. As demonstrated in the Supplemental Material [46], $\mathrm{Tr}[\mathbb{\Gamma}^2] \geq N^2(\Gamma_R^2 + \Gamma_L^2)$, and the burst is guaranteed to happen for

$$\frac{N(\Gamma_L^2 + \Gamma_R^2)}{\Gamma_{1D}^2} > 2 + \frac{\Gamma'}{\Gamma_{1D}}. \tag{10}$$

Disordered systems saturate this bound [see Fig. 2(c)], while ordered systems may display a burst for lower N due to constructive interference effects [see Eq. (9) and Fig. 2(b)].

Generically, correlations imparted by the jump operators not only produce an accelerated emission of the second photon, but also of subsequent ones. This avalanchelike nature of photon emission implies that an initial fluctuation is amplified throughout the decay process. Shot-to-shot fluctuations in directionality have been predicted in free space [37,40]. In a 1D bath, however, these fluctuations are more striking as there are only two directions, and the fluctuations break mirror symmetry. For instance, if the first photon is measured by a detector to the right, it is very likely that subsequent photons are also detected to the right.

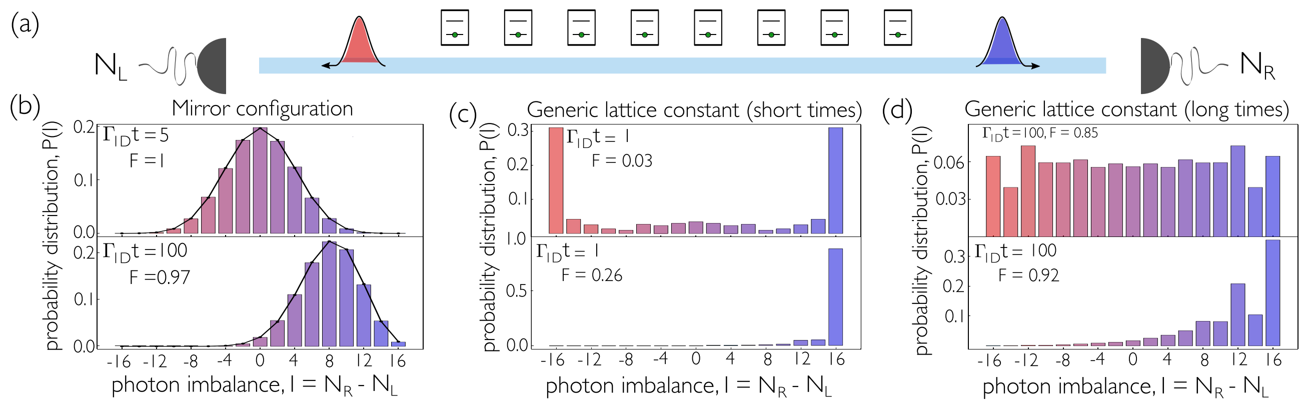

FIG. 3. Emergence of chirality via many-body decay. (a) Superradiant decay leads to directionality that can be investigated by detectors which count the number of right- and left-propagating photons. (b)–(d) Directional photon imbalance for 16 emitters radiating into a symmetric (top) and asymmetric (bottom) waveguide, with $\Gamma' = 0$. For the latter, $\Gamma_R = 3\Gamma_L$. The lattice constant is (b) $k_{1D}d = \pi$ and (c),(d) $k_{1D}d = \pi/\sqrt{3}$. In (c),(d), we investigate short and long times respectively. Only trajectories that reach $|g\rangle^{\otimes N}$ before time t are accounted for, with F denoting the fraction of finished trajectories.

This process gives rise to an emergent chirality even in the case of a bidirectional waveguide. To explore this physics, we unravel $\mathcal{L}_g[\rho]$ in terms of a different pair of operators,

$$\hat{\mathcal{O}}_{L(R)} = \frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{\overset{+}{(-)}ik_{1D}dj}\hat{\sigma}_{ge}^{j} \propto \sqrt{\Gamma_+}\hat{\mathcal{O}}_+ \underset{(-)}{+} i\sqrt{\Gamma_-}\hat{\mathcal{O}}_-, \quad (11)$$

which describe the emission of photons to left- and right-propagating modes with rates $\Gamma_{L(R)}$.

The direction of the first photon is stochastic due to the uncorrelated initial state, with probability depending only on the relative decay rate of each operator, i.e., $p_{L(R)} = \Gamma_{L(R)}/\Gamma_{1D}$. Emergent chirality is already evident at the level of two emissions. If all the photons are emitted into the waveguide (i.e., $\Gamma' = 0$), the detection probabilities for the second photon to be the same as or different to the first one are

$$\tilde{g}^{(2)}_{LL}(0) = 2 - \frac{2}{N}, \quad (12a)$$

$$\tilde{g}^{(2)}_{LR}(0) = 1 - \frac{2}{N} + \frac{1}{N^2}\frac{\sin^2 Nk_{1D}d}{\sin^2 k_{1D}d}. \quad (12b)$$

where $\tilde{g}^{(2)}_{\alpha\beta}(0) = \langle\hat{\mathcal{O}}^\dagger_\alpha\hat{\mathcal{O}}^\dagger_\beta\hat{\mathcal{O}}_\beta\hat{\mathcal{O}}_\alpha\rangle/(\langle\hat{\mathcal{O}}^\dagger_\alpha\hat{\mathcal{O}}_\alpha\rangle\langle\hat{\mathcal{O}}^\dagger_\beta\hat{\mathcal{O}}_\beta\rangle)$.

For large N, the second photon is twice as likely to follow the direction of the first. Subsequent jumps further enhance the chirality [46] (except in the mirror configuration where $\hat{\mathcal{O}}_{L(R)}$ are identical). One can attribute this enhanced chirality to the correlations produced by atom-atom interactions or to the photon detection (far-field measurement is unable to distinguish which atom emitted the photon thus preparing a superposition state [33,39,55]).

For bidirectional waveguides, this emergent chirality is akin to a process of spontaneous symmetry breaking, where mirror symmetry is broken dynamically. A large superradiant burst implies that, for a single realization, most photons are emitted in one direction. Of course, the symmetry is recovered when averaged over realizations, as the first photon is randomly emitted into either direction.

We characterize this behavior by counting the photons emitted in both directions and computing the "photon imbalance," as shown in Fig. 3(a). For a single quantum trajectory—in which atoms evolve from $|e\rangle^{\otimes N}$ to $|g\rangle^{\otimes N}$ via coherent evolution with a non-Hermitian Hamiltonian and decay by the action of jump operators—we define the photon imbalance as the difference in the number of times that the two jump operators act. This corresponds to counting the final number of photons emitted to the right (N_R) and left (N_L) with imbalance $I = N_R - N_L$. The set of possible photon imbalances I has a probability distribution, $\mathcal{P}(I)$. The imbalance distribution depends on the lattice constant and the degree of single-emitter chirality of the waveguide.

In the mirror configuration, as the left and right operators are identical, the normalized probability of emitting a photon in each direction reduces to approximately $p_{L(R)}$ at any stage of the decay. Hence, the photon imbalance roughly follows a binomial distribution, as shown in Fig. 3(b). For the symmetric case, the peak at $I = 0$ reflects the large number of possible emission records that lead to zero imbalance. Minor discrepancies from the binomial originate from the action of the Hamiltonian in between jumps (only for a chiral waveguide) and noise from the finite number of trajectories.

Away from the mirror configuration, the repeated action of a jump operator enhances its probability of acting again, thus amplifying the initial fluctuation and either breaking mirror symmetry (for a bidirectional waveguide) or collectively enhancing chirality (for a chiral one). Repeated action leads to emission that finishes at early times because it produces strongly enhanced photon emission, so the time between jumps is small and Hamiltonian evolution is negligible. This is shown in Fig. 3(c). For a bidirectional waveguide, almost all photons are emitted in one direction. A mildly chiral waveguide becomes almost perfectly chiral.

Hamiltonian evolution becomes relevant for the imbalance statistics at later times [56] by scrambling the states and reducing enhancement (yet still giving rise to a distribution with a very large variance). For a bidirectional waveguide, the competition between enhancement and large number of pathways that yield $I = 0$ produce an almost flat imbalance distribution, as shown in Fig. 3(d). For the chiral waveguide, the chirality enhancement is reduced. Nonetheless, the probability of detecting all photons in a single direction is much greater than the probability predicted by the binomial distribution for independent emission. This resembles a recent prediction for multilevel atoms in a cavity, where there is a higher probability of large imbalances between ground state populations compared to single-atom predictions [57].

In conclusion, we have established a condition for enhanced emission into a preferential channel when emitters decay to multiple reservoirs. We have found the minimal conditions for the emission of a superradiant burst into a 1D bath and determined that the burst should be observable in different experimental setups, such as superconducting qubits coupled to transmission lines and atoms coupled to nanofibers. Many-body superradiance gives rise to an emergent chirality in the system, with large amounts of photons being emitted in one direction. As shown in the Supplemental Material [46], large photon imbalances disappear with increased Γ'. Nevertheless, pronounced imbalances should be observable in state-of-the-art experimental setups with superconducting qubits, where $\Gamma' \simeq 0.01\Gamma_{1D}$ [15], and quantum dots, where $\Gamma' \simeq 0.1\Gamma_{1D}$ [13].

Giant atoms (emitters coupled to the waveguide at multiple points [58–63]) also exhibit superradiance when the parasitic decay is smaller than the individual decay into the waveguide [46]. The interference of each emitter with itself modifies the individual decay rate, and certain configurations result in a decoherence-free system with non-zero coherent interactions [59]. Atoms near these configurations can exhibit monotonic decay. Interestingly, the additional tunability of the coherent interactions may be a resource to compensate the scrambling produced by the "bare" Hamiltonian, thus ensuring a more dissipative dynamics and a larger burst.

An interesting avenue for future research is to investigate the quantum state of photons produced via many-body decay. The mirror configuration produces multiphoton states with similar metrological properties to Fock states [64–68]. However, in this configuration, photons need to be recombined into a single pulse, as they are emitted in both directions. This issue should be partially overcome at different lattice constants or in chiral waveguides. However, in these cases, dynamics may populate dark states, which are prevalent at low excitation densities [7,20,69,70], trapping the last few photons in the pulse. Moreover, the direction of emission is initially random, though this may be overcome by stimulated emission [71,72]. Other promising lines of inquiry include the possibility of using measurement and feedback control on the output light to access entangled dark states and the investigation of non-Markovian effects in many-body decay [61,73–75].

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