Tricritical Behavior in Dynamical Phase Transitions

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We identify a new scenario for dynamical phase transitions associated with time-integrated observables occurring in diffusive systems described by the macroscopic fluctuation theory. It is characterized by the pairwise meeting of first- and second-order bias-induced phase transition curves at two tricritical points. We formulate a simple, general criterion for its appearance and derive an exact Landau theory for the tricritical behavior. The scenario is demonstrated in three examples: the simple symmetric exclusion process biased by an activity-related structural observable; the Katz-Lebowitz-Spohn lattice gas model biased by its current; and in an active lattice gas biased by its entropy production.

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Introduction.—In nonequilibrium statistical mechanics, theoretical results for simple lattice models have guided the understanding of dynamical processes and fluctuations [1–10]. For interacting particle systems, macroscopic fluctuation theory (MFT) [4,11–15] enables the analysis of hydrodynamic scales, exposing behavior independent of microscopic details. Alongside models' typical behavior, MFT predicts rare fluctuations. For example, it identifies the fluctuation mechanism for time-integrated quantities whereby atypical values of the current [16–24] or dynamical activity [25–28] are sustained over long times. These are examples of large deviations, which have also been analyzed numerically [29], and by other theoretical methods [30]. A rich behavior emerges [31–33], including dynamical phase transitions (DPTs), often involving spontaneous symmetry breaking by these (macroscopically atypical) system trajectories [5,18–20,22–29,34].

DPTs are conceptually intriguing, and also provide practical insight, in part because large deviation analyses relate directly to optimal control theory [15,33,35–39]. Here rare events are characterized via extra control forces, added to the system dynamics, to make them become typical. This approach has applications in numerical experiments and for material design [5,33,37,40–44]. In this setting, DPTs signify qualitative changes in the types of control force required.

Several well-studied DPTs occur in the simple symmetric exclusion process (SSEP), with periodic boundary conditions. Its steady states are homogeneous (H), but large deviations toward low activity occur through spatially inhomogeneous (IH) states, while those with large activity exhibit hyperuniformity [26,27]. The transition from H to IH spontaneously breaks translational symmetry and is continuous. In contrast, discontinuous DPTs also arise, in exclusion processes [24] and other models [5].

In this work, we explore a new type of dynamical phase behavior for fluctuations of time-integrated quantities, which manifests as a pair of tricritical points. These live on H-IH phase boundaries and signal a change in character of the H-IH transition, from continuous to discontinuous. We analyze this scenario using MFT, showing it has a universal status—occurring generically when simple criteria are met. We exemplify this with three large-deviation calculations: fluctuations of a structural observable akin to the activity in SSEP, fluctuations of the current in a Katz-Lebowitz-Spohn (KLS) type lattice gas, and fluctuations of the entropy production in an active lattice gas model.

Note that current fluctuations in 1D have been extensively studied [16,18–24], including recent exact solutions via MFT [45–47], for cases where the mobility depends quadratically on density. We show below that tricriticality generically arises when the mobility has an inflection point, absent in those studies, creating a much richer picture for DPTs than previously identified. (The possibility of discontinuous transitions was noted in Ref. [20], but tricritical points have not been explored, to our knowledge.)

Large deviations in SSEP.—We first address fluctuations of time-integrated structural quantities in the SSEP. Consider a one-dimensional periodic lattice with L sites and N particles; each site contains at most one particle, and particles hop to vacant neighbors with rate D_0. To analyze the hydrodynamic scale, let the position of site i be $x = i/L$, and write $\rho(x,t)$ for the hydrodynamic density, with time t measured on the hydrodynamic scale. (The microscopic time is then $\hat{t} = L^2 t$.) Also write $J(x,t)$ for the hydrodynamic current, and denote by $\mathcal{X} = \{\rho(x,t), J(x,t)\}_{x\in[0,1),t\in[0,T]}$ a dynamical trajectory of duration T. Such trajectories respect the continuity equation $\partial_t\rho = -\nabla\cdot J$, so the total density of the system, $\rho_0 = N/L$, is conserved.

Within MFT, the probability of a trajectory is $P(\mathcal{X}) \simeq e^{-LS(\mathcal{X})}$ [2,4,11–15], with action

$$S_T(\mathcal{X}) = \int_0^T dt \int_0^1 dx \frac{|J + D(\rho)\nabla\rho|^2}{2\sigma(\rho)} \tag{1}$$

where $D(\rho) = D_0$ and $\sigma(\rho) = 2D_0\rho(1-\rho)$. Below we retain D and σ as general functions, specializing to SSEP where appropriate. We consider large deviations of time-integrated structural quantities of the form

$$K_T(\mathcal{X}) = L\int_0^T dt \int_0^1 dx\kappa(\rho), \tag{2}$$

with two exemplar choices for $\kappa(\rho)$:

$$\kappa_1(\rho) = \rho(1-\rho), \qquad \kappa_2(\rho) = \rho(1-\rho)^2. \tag{3}$$

For $\kappa = \kappa_1$, K_T measures the dynamical activity [26,27]: it counts the number of possible particle hops (i.e., particles with a vacant neighbor). Meanwhile, κ_2 counts particles with *two* vacant neighbors [48]. Despite their physically similar definitions, these quantities have contrasting large deviation behaviors.

To analyze this, we define the scaled cumulant generating function (CGF) $\Psi(\Lambda) = \lim_{L,T\to\infty}(1/LT)\log\langle e^{\Lambda K_T}\rangle$, where angle brackets indicate a steady-state average. Analogous to a thermodynamic potential, the CGF is a "dynamical free energy" for an ensemble of trajectories biased by the field Λ conjugate to K_T [31–33,37]. For large L, the average is dominated by the most likely trajectory and for large T this is homogeneous in time, so that [48]

$$-\Psi(\Lambda) = \inf_{\rho:\ \int_0^1 dx\rho=\rho_0} \int_0^1 dx[M(\rho)|\nabla\rho|^2 - \Lambda\kappa(\rho)] \tag{4}$$

where $M(\rho) = D(\rho)^2/2\sigma(\rho)$.

An alternative characterization of large deviations involves the rate function $\mathcal{I}$. The probability density for K_T obeys for large L, T

$$\log\mathrm{Prob}[K_T/(LT) \approx k] \simeq -LT\mathcal{I}(k). \tag{5}$$

$\mathcal{I}$ corresponds to a thermodynamic potential dual to Ψ, governing an ensemble of trajectories where K_T is fixed. It can be computed in terms of a dominant path which minimizes the action at constrained K_T:

$$\mathcal{I}(k) = \inf_{\mathcal{X}:\ K_T(\mathcal{X})=kLT} S_T(\mathcal{X})/T. \tag{6}$$

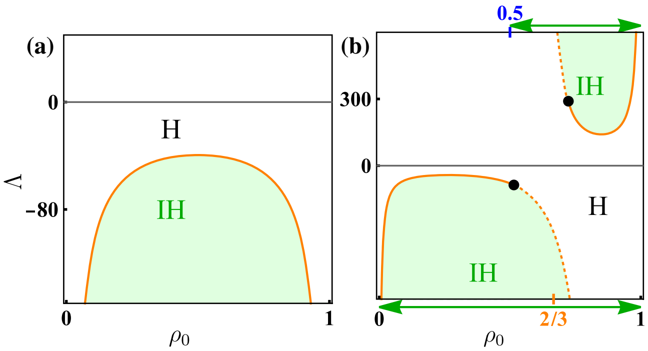

FIG. 1. (a),(b) SSEP dynamical phase diagrams for (a) $\kappa = \kappa_1$; (b) $\kappa = \kappa_2$, showing H and IH states. Arrows demarcate the range of densities ρ_0 for which IH states appear as $|\Lambda_c| \to \infty$. The thick (orange) lines indicate continuous transitions at $\Lambda = \Lambda_{c,2}$ [Eq. (7)], and the dashed continuations indicate discontinuous transitions. These meet at tricritical points (black dots).

As in thermodynamics, enforcing the constraint by Lagrange multiplier shows that $\mathcal{I}$ and Ψ are related by Legendre transform.

Dynamical phase transitions in SSEP.—Figure 1 shows dynamical phase diagrams for large deviations of K_T in ensembles biased via κ_1 and κ_2. Both cases support H-IH phase transitions, but biasing by κ_2 introduces tricritical points, absent for κ_1. To explain this, we first establish a simple condition for discontinuous transitions, related to previous arguments at the microscopic level [5,48,52]. This sufficient condition only involves $\kappa(\rho)$, although discontinuous transitions could also arise for sufficiently elaborate choices of $M(\rho)$ [48].

IH states occur when the minimizer of Eq. (4) has $\rho(x) \neq \rho_0$. The H state is optimal for $\Lambda = 0$, whereas for $\Lambda \to -\infty$ the gradient term is negligible, and we minimize $\int\kappa(\rho)dx$. The outcome depends on the convexity of κ: IH profiles are optimal whenever $\kappa(\rho)$ differs from its lower convex envelope, which is the lower boundary of the convex hull. (This condition is analogous to the double tangent construction for thermodynamic phase separation.) The resulting minimizer has two spatial regions, separated by an interface of width $O(|\Lambda|^{-1/2})$. For both κ_1 and κ_2 these have bulk densities $\rho = 0$, 1.

In such cases, the system is IH for $\Lambda \to -\infty$ but H for $\Lambda = 0$: clearly there must be an intervening DPT where translational symmetry is broken. The same argument applies for $\Lambda \to +\infty$, on replacing κ by $-\kappa$. The arrows in Fig. 1(b) show the regions of IH for large $|\Lambda|$. Only if κ has an inflection point (so that neither of $\pm\kappa$ is convex) do IH states exist for both signs of Λ.

We next establish conditions governing the order of these DPTs. At a *continuous* transition $\rho(x)$ deviates smoothly from ρ_0 as bias is increased. Using Eq. (4) with $\Lambda < 0$, this requires a small perturbation to reduce $\int\kappa(\rho)dx$, implying $\kappa''(\rho_0) < 0$. Conversely, if $\kappa''(\rho_0) > 0$ any transition must be discontinuous. To summarize, for any ρ_0 at which κ

differs from its lower convex envelope then a H-IH transition must occur for some $\Lambda < 0$. If $\kappa''(\rho_0) > 0$, then it *must* be first-order; otherwise it may be continuous or discontinuous. (Analogous results again hold for $\Lambda > 0$, on replacing $\kappa \to -\kappa$.)

Since $\kappa_2''(\rho_0)$ changes sign at $\rho_0 = 2/3$, any H-IH transitions at $\Lambda > 0$ are discontinuous for $\rho_0 < 2/3$, and likewise for $\rho_0 > 2/3$ when $\Lambda < 0$. (In fact, the transitions are discontinuous over broader ranges; see below.) In contrast, $\kappa_1''(\rho_0) < 0$ for all ρ_0: the H-IH transition is always continuous in that case.

To analyze these DPTs quantitatively, we develop a Landau theory [20,24,26,34], valid close to tricriticality. We expand the density as $\rho(x) = \rho_0 + A\cos 2\pi x + B\cos 4\pi x$, where A is a small amplitude and $B = O(A^2)$ [48]. Substituting into Eq. (4) yields

$$-\Psi(\Lambda) \approx -\Lambda\kappa(\rho_0) + \inf_A \left[\frac{\Lambda_{c,2} - \Lambda}{4}\kappa''(\rho_0)A^2 + \beta(\rho_0)A^4\right] \tag{7}$$

where $\approx$ means terms of $O(A^6)$ are omitted; here

$$\Lambda_{c,2} = \frac{8\pi^2 M(\rho_0)}{\kappa''(\rho_0)}, \quad \beta = \frac{\pi^2 M(\rho_0)}{24}[3a(\rho_0) - b^2(\rho_0)], \tag{8}$$

with $a = 2M''/M - \kappa''''/\kappa''$ and $b = 3M'/M - \kappa'''/\kappa''$.

The behavior of the Landau theory [Eq. (7)] is familiar: if $\beta > 0$ there is a continuous transition at $\Lambda_c = \Lambda_{c,2}$ beyond which $A \propto \sqrt{|\Lambda - \Lambda_{c,2}|}$. This happens for the SSEP with $\kappa = \kappa_1$ [26]. From Eq. (8), the sign of $\Lambda_{c,2}$ matches that of κ'', as argued previously.

In contrast, if $\beta < 0$, symmetry breaking can only happen discontinuously, as already noted in Ref. [20]. Points with $\Lambda = \Lambda_{c,2}$ and $\beta(\rho_0) = 0$ are tricritical [53–55]: here the transition changes character from continuous to discontinuous [48]. Note also that wherever $\kappa''(\rho_0) \to 0$, $b^2(\rho_0) \to \infty$. Hence from Eq. (8), β is negative in a range of ρ around any inflection point in κ, such as the one for κ_2 at $\rho_0 = 2/3$ (while generically, as in our examples, staying positive elsewhere). The two tricritical points that limit this range are easily identified since $\Lambda_{c,2}$ and β are explicit functions [48]; see Fig. 1(b).

The full tricritical scenario is illustrated in Fig. 2 and discussed in the Supplemental Material [48]. If $\beta < 0$, and assuming the expansion [Eq. (7)] is stabilized by a term γA^6 with $\gamma > 0$, then precisely at the tricritical point, $\beta = 0$, one finds $A \propto |\Lambda - \Lambda_{c,2}|^{1/4}$. For $\beta < 0$ the transition is discontinuous; it takes place at $\Lambda = \Lambda_{c,1}$ with $|\Lambda_{c,1} - \Lambda_{c,2}| \propto (\rho_0 - \rho_c)^2$. The discontinuity in A grows as $\Delta A|_{\Lambda=\Lambda_{c,1}} \propto |\Lambda_{c,2} - \Lambda_{c,1}|^{1/4}$. These universal, tricritical exponents are exemplified by the theoretical curves in Fig. 2(b) which depend on γ, which we extracted from numerical solutions of Eq. (4) [48].

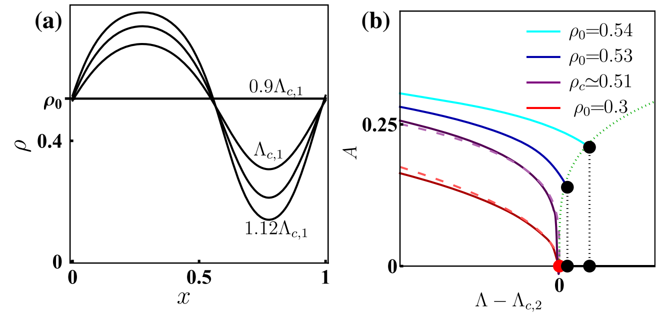

FIG. 2. Tricriticality in SSEP for $\kappa = \kappa_2$. (a) Minimizers of Eq. (4), for $\rho_0 = 0.54$ and $\Lambda = (0.9, 1, 1.11, 1.12) \times \Lambda_{c,1}$, close to the discontinuous transition at $\Lambda_{c,1}$. (b) Amplitude A for various ρ_0, near the tricritical point at $(\rho_0, \Lambda) = (\rho_c, \Lambda_{c,2})$. For $\rho_0 < \rho_c$, a continuous transition occurs at $\Lambda = \Lambda_{c,2}$ where $A \propto |\Lambda_{c,2} - \Lambda|^{1/2}$ (dashed red line) as predicted by Eq. (7). At $\rho_c \simeq 0.515$ the growth follows $A \propto |\Lambda - \Lambda_{c,2}|^{1/4}$ (dashed purple). For $\rho_0 > \rho_c$, a discontinuous transition occurs at $\Lambda = \Lambda_{c,1}$. The discontinuity grows as $\Delta A|_{\Lambda=\Lambda_{c,1}} \propto |\Lambda_{c,2} - \Lambda_{c,1}|^{1/4}$ (dotted green line). Solid lines are numerical solutions of Eq. (4).

Constrained ensemble.—The variational problem [Eq. (4)] is computationally convenient, but additional physical insight is gained via the rate function. Figs. 3(a) and 3(b) show dynamical phase diagrams for the constrained ensemble, indicating the fluctuation mechanism, for different values of K_T, corresponding to optimal paths in Eq. (6). These can be obtained from Ψ by Legendre-Fenchel transform, noting that in the presence of first-order DPTs, such optimal paths are inhomogeneous in time [31,33,48]. The corresponding regions of "timelike phase separation" (analogous to miscibility gaps in thermodynamics [48]) are indicated in Fig. 3(b), further highlighting the presence of discontinuous transitions and tricritical points.

When constructing these phase diagrams, it is important that all homogeneous states are identical in MFT, so the entire H phases in Figs. 1(a) and 1(b) collapse onto the lines $k = \kappa(\rho_0)$ in Figs. 3(a) and 3(b); see also plots in the Supplemental Material [48] showing $\Psi'(\Lambda) = \kappa(\rho_0)$ throughout the H phase. Physically, this reflects that fluctuations of K_T occur by hydrodynamic mechanisms: the slow relaxation of long-wavelength density modes makes their persistent fluctuations much less rare than fluctuations in microscopic structure. However, some values of k are not reached by any hydrodynamic mechanism; in this case the constrained minimization [Eq. (6)] has no solution. Characterization of such fluctuations lies beyond MFT (although some aspects of the inaccessible regime can nonetheless be determined [25,27,28]).

To conclude our study of DPTs in SSEP note that, alongside the emergence of tricritical points, biasing with κ_2 differs from κ_1 in that IH states occur for atypical fluctuations at both high and low κ. At the densities concerned, H states are restricted to a narrow "tightrope"

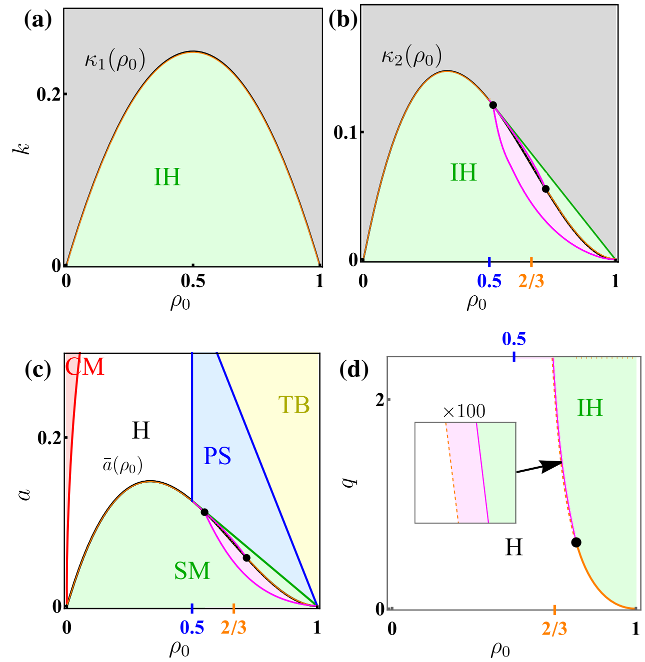

FIG. 3. Dynamical phase diagrams for (a) SSEP conditioned on κ_1, (b) SSEP conditioned on κ_2, (c) active lattice gas conditioned on IEPR, (d) KLS model conditioned on current. Miscibility gaps are denoted by magenta shading [see inset in (d)]. Black dots are tricritical points. Orange tick marks indicate inflection points where $\kappa'' = 0$ [or $\sigma'' = 0$ in (d)]. Blue tick marks indicate the boundaries of regions where $-\kappa$ [or $-\sigma$ in (d)] differs from its lower convex envelope. Gray regions in (a),(b) are not accessible by hydrodynamic fluctuations.

of unbiased dynamics, $k = \kappa(\rho_0)$. [In contrast, for κ_1, IH states arise only for low k fluctuations; states at $k > \kappa(\rho_0)$ remain homogeneous [27].] We emphasize that this phenomenology should be generic in variational problems like Eq. (4), whenever κ has a point of inflection. To illustrate this, we now present two further, very different systems where a similar tricritical scenario arises.

Current fluctuations.—We consider large deviations of the integrated current $Q_T = L\int_0^T dt \int_0^1 dx J(x,t)$ within MFT. For $L, T \to \infty$, the probability that $Q_T \approx qLT$, as a function of q, takes a large deviation form, similar to Eq. (5). Here though, H-IH transitions involve the formation of traveling waves with velocity V, so that $\rho = \rho(x - Vt)$ and $J = J(x - Vt)$ [17–20,23]. The rate function for current then satisfies [48]

$$\mathcal{I}(q) = \inf_{\rho(x),\alpha} \int_0^1 dx[M(\rho)|\nabla\rho|^2 + q^2\kappa_J(\rho;\rho_0,\alpha)], \quad (9)$$

with $\kappa_J = [1 + \alpha(\rho - \rho_0)]^2/[2\sigma(\rho)]$, where $\alpha = V/q$ is a variational parameter. This problem is symmetric in q, so we now restrict to $q \geq 0$.

The minimization problem [Eq. (9)] for $\mathcal{I}(q)$ is similar to the problem in Eq. (4), which previously gave the CGF. Repeating the previous analyses of convexity and the Landau theory yields two analogous results, detailed in the Supplemental Material [48]. First, as $q \to \infty$, a traveling wave state is found whenever $-\sigma(\rho_0)$ differs from its lower convex envelope. Second, the quartic term βA^4 in the corresponding Landau theory has $\beta \to -\infty$ whenever the mobility σ has an inflection point, giving tricritical points ($\beta = 0$).

The mobility σ in this problem plays the same role as κ did in large deviations of K_T for SSEP. This correspondence is further exemplified by a model of Katz-Lebowitz-Spohn type [24,56–58], for a kinetically constrained lattice gas [59]. This is a 1D simple exclusion process where the hop rates depend on the occupancies of neighboring sites as

$$0100 \overset{D_0}{\leftrightarrow} 0010, \quad 1100 \overset{D_0/2}{\leftrightarrow} 1010, \quad 0101 \overset{D_0/2}{\leftrightarrow} 0011.$$

The transition $1101 \leftrightarrow 1011$ is kinetically forbidden [60], but the hydrodynamic behavior still obeys diffusive MFT with $D(\rho) = D_0(1-\rho)$ and $\sigma = 2D_0\rho(1-\rho)^2$ [24].

The resulting phase diagram shows a tricritical point at $q > 0$ [Fig. 3(d)] whose partner lies at negative q (not shown). Since $\sigma(\rho) \propto \kappa_2(\rho)$, this phase diagram resembles the upper half of Fig. 1(b). Its form is robust to variations in hop rates [48].

Active lattice gas.—Our final example considers a $1d$ active lattice gas (ALG) model [61], first introduced to study motility-induced phase separation [62]. It comprises two species of diffusing particles, whose hops are biased in opposite directions, with an additional "tumbling" process where particles change species. Its hydrodynamic behavior can be analyzed within MFT [63,64]; the resulting action $S_{\rm act}$ is analogous to Eq. (1).

We discuss here the emergence of tricritical DPTs in large deviations of the informatic entropy production rate (IEPR), which were previously analyzed in Ref. [65]. Write $\mathcal{X}$ for a hydrodynamic trajectory, and let $\mathcal{X}^R$ be the corresponding time-reversed trajectory. Then the IEPR, $\mathbb{S}_T \equiv [S_{\rm act}(\mathcal{X}) - S_{\rm act}(\mathcal{X}^R)]/T$ [66,67], quantifies time-reversal symmetry breaking at hydrodynamic scales. Its average is $\langle \mathbb{S}_T \rangle = Ls_0\bar{a}(\rho_0)$ where $\bar{a}(\rho) = \rho(1-\rho)^2$ and s_0 is a constant [65]. The IEPR obeys a large deviation principle resembling Eq. (5),

$$\log \mathrm{Prob}[\mathbb{S}_T/(Ls_0) \approx a] \simeq -LT\mathcal{I}(a) \quad (10)$$

where the rate function $\mathcal{I}(a)$ can be characterized variationally, similarly to Eq. (6). The resulting phase diagram, fully derived in Ref. [65], is shown in Fig. 3(c). It is more complex than for the SSEP. As well as the smoothly modulated state (SM) which is analogous to the IH states discussed above, it supports collective motion (CM) and traveling band (TB) states which break the symmetry between species, and a sharply phase-separated (PS) state. Nonetheless, the small-a behavior resembles Fig. 3(b).

In the ALG, the dominant fluctuations involve local particle motions that remain typical for the given local density which becomes nontypical. This results in [65]

$$\mathcal{I}(a) \propto \inf_{\rho(x):\ a=\int_0^1 dx \bar{a}(\rho)} \int dx \mathcal{M}(\rho)|\nabla\rho|^2 \qquad (11)$$

where $\mathcal{M}$ encodes all the cost arising from inhomogeneities of the density [65]. Observing that $\bar{a}(\rho_0) = \kappa_2(\rho_0)$, this variational problem is again similar to Eq. (6) with $\kappa = \kappa_2$. As a result, the behavior in the SM state in Fig. 3(c) is analogous to the inhomogeneous state in Fig. 3(b), including the tricritical points and the timelike phase separation. A central result of this Letter is that the tricritical phenomena unexpectedly encountered in Ref. [65] are *not* specific to the ALG, instead exemplifying a quite general scenario as explored above.

Outlook.—We demonstrated a new class of tricritical behavior that occurs in fluctuations of time-integrated observables when the dynamical action has the general structure of Eq. (4). We gave three examples from the hydrodynamic analysis of large deviations. In all cases, pairs of tricritical points occur on homogeneous-inhomogeneous phase boundaries, separating continuous from discontinuous transitions. Our results significantly enrich the theory of dynamical phase transitions and add to the classes of systems showing tricriticality in nonequilibrium [68–71], for example in fluctuations of instantaneous rather than time-integrated quantities [72].

The discontinuous transitions in Fig. 3 show that even if k is close to its mean value, the large-deviation mechanism may differ strongly from the typical (homogeneous) state: for suitable ρ_0, timelike phase separation can appear once k deviates from $\kappa_2(\rho_0)$, in either direction. Alongside the aforementioned relevance to optimal control and design [5,33,37,40–44], such transitions should be directly realizable in several experimental settings [24]. These include wave transmission in disordered media [73,74] and mesoscopic electronic transport [75,76] where, intriguingly, the relevant mobility can show inflection points [77], as required for tricriticality to emerge.

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