Editors' Suggestion

Measuring Topological Entanglement Entropy Using Maxwell Relations

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(Received 31 December 2022; accepted 8 June 2023; published 7 July 2023)

Topological entanglement entropy (TEE) is a key diagnostic of topological order, allowing one to detect the presence of Abelian or non-Abelian anyons. However, there are currently no experimentally feasible protocols to measure TEE in condensed matter systems. Here, we propose a scheme to measure the TEE of chiral topological phases, carrying protected edge states, based on a nontrivial connection with the thermodynamic entropy change occurring in a quantum point contact (QPC) as it pinches off the topological liquid into two. We show how this entropy change can be extracted using Maxwell relations from charge detection of a nearby quantum dot. We demonstrate this explicitly for the Abelian Laughlin states, using an exact solution of the sine-Gordon model describing the universal crossover in the QPC. Our approach might open a new thermodynamic detection scheme of topological states also with non-Abelian statistics.

DOI: 10.1103/PhysRevLett.131.016601

Introduction.—Entanglement describes nonlocal correlations between quantum objects and is essential in understanding quantum many-body systems. It is also at the heart of quantum computation and information sciences and plays a pivotal role in models such as topological quantum computation [1] and measurement-based quantum computation [2]. Topological entanglement entropy (TEE) [3–5] is the ultimate diagnostic of topological order as defined by fractional quasiparticles carrying anyonic statistics. In a $(2+1)$-dimensional topological phase where the system size is larger than the correlation length, the entanglement entropy in the ground state $|\Psi\rangle$ between subsystems A and B, $S_{\mathrm{EE}} = -\mathrm{Tr}[\rho_A \log \rho_A]$ where $\rho_A = \mathrm{Tr}_B[|\Psi\rangle\langle\Psi|]$, takes the general form [3,4]

$$S_{\mathrm{EE}} = \alpha L - \gamma + \cdots. \tag{1}$$

Here L is the length of the entanglement cut, α is a nonuniversal constant describing short range entanglement, and the second subleading term is the TEE. The TEE $\gamma = \log \mathcal{D}$ is uniquely related to the total quantum dimension of the topological phase, $\mathcal{D} = \sqrt{\sum_a d_a^2}$, with d_a being the quantum dimensions of each individual anyon type labeled by a. In the presence of N anyons of type a, the gapped topological liquid has a degeneracy that scales as d_a^N, so that $d_a > 1$ ($d_a = 1$) refers to a non-Abelian (Abelian) anyon. For instance, there are m Abelian anyons in the fractional quantum Hall (FQH) Laughlin state with filling fraction $\nu = 1/m$, with $d_a = 1$ ($a = 0, 1, \ldots, m-1$) and $\mathcal{D} = \sqrt{m}$; the Moore-Read state at $\nu = 5/2$ has four Abelian anyons and two non-Abelian anyons having $d_a = \sqrt{2}$, with $\mathcal{D} = 2\sqrt{2}$.

Measuring entanglement entropy in many-body systems is a daunting task since it requires full state tomography. Variants of S_{EE} such as the Rényi entanglement entropy can be accessed in controllable many-body quantum simulators, such as cold atoms or trapped ions using many-copy methods [6–9] or randomized measurement techniques [10–12]. The latter, remarkably, was employed recently to measure the Rényi TEE of Kitaev's toric code [13] prepared on a quantum processor [14]. Unambiguously extracting the subleading TEE term γ requires dividing the system into three subsystems and forming appropriate combinations of the different partitions such as to cancel the leading term [3,4]. This was successfully implemented [14] thanks to the zero correlation length of the toric code eigenstates. However, measuring TEE in condensed matter systems, such as in the realm of the FQH effect in two-dimensional electron systems, remains elusive.

Interestingly, there exists an intimate relation between TEE and a thermodynamic entropy loss associated with a quantum point contact (QPC) [15]. The QPC allows tunneling between two points on the edge (see parameter λ in Fig. 2 below). For an edge carrying fractional quasiparticles, this is a relevant perturbation that introduces an energy scale T_B [16]. For $T \gg T_B$ the system is described by the ultraviolet (UV) fixed point that is unaffected by tunneling. As temperature decreases below T_B, tunneling processes proliferate and the system flows to the infrared (IR) fixed point where the droplet is disconnected into two droplets A and B. Their anyonic charge a can no longer fluctuate since each droplet can not support an overall fractional anyonic charge, leading to an entropy reduction. Using the bulk-edge correspondence, it was shown for a general $(2+1)$-dimensional chiral topological

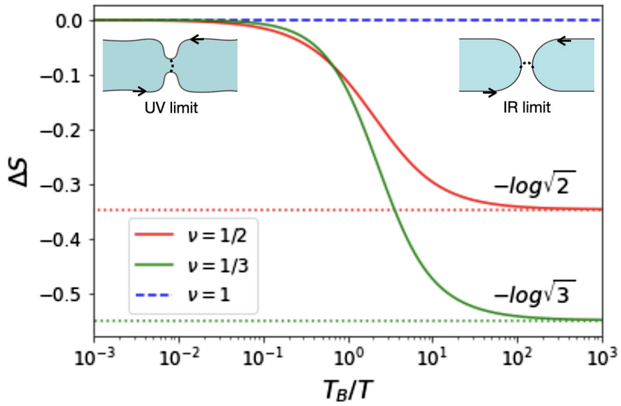

FIG. 1. Exact TBA results for the boundary entropy change in a QPC at $\nu = 1, 1/2, 1/3$ with a net entropy difference of 0, $\log\sqrt{2}$, and $\log\sqrt{3}$, respectively. The results for $\nu = 1/2$ are also obtained using refermionization [25].

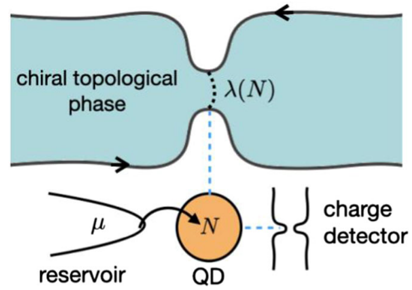

FIG. 2. A QPC in a chiral topological phase facilitates tunneling between the edge states. The tunneling strength λ is controlled by the charge on a nearby QD, $\lambda = \lambda(N)$, that itself depends on the chemical potential μ of a nearby weakly tunnel-coupled reservoir. An adjacent charge detector measures the charge of the QD $\langle N \rangle$. Sufficiently charging the QD drives the QPC between UV to IR limits.

phase that the thermodynamic entropy change of the QPC coincides with a bulk property being precisely the TEE [15],

$$S_{\mathrm{UV}} - S_{\mathrm{IR}} = \log \mathcal{D}. \tag{2}$$

This fundamental relation, derived 16 years ago, suggests that the TEE can "simply" be read off from the temperature dependence of the entropy, as shown in Fig. 1. Indeed, over the past decades there has been a surge of interest in measuring thermodynamic entropy in the bulk of mesoscopic systems [17–21], with specific emphasis on the entropy contribution of non-Abelian anyons [22–24]. However, extracting the TEE fundamentally requires one to measure an entropy change of order unity associated with a change of the topology of the surface. Thus, isolating the $\mathcal{O}(1)$ entropy change in Eq. (2) from bulk contributions, including those of the 1D edge states themselves, remained elusive.

More recently, local measurements of entropy of electronic states were demonstrated in quantum dot (QD) systems via transport in specific systems [41,42] and using a more general framework based on charge detection and Maxwell relations [43–45]. Some promising applications of using this general framework were theoretically identified earlier [22–24,46–51]. In Refs. [43–45] a QPC is used as a charge detector *weakly coupled* to a QD. Then, Maxwell relations are used to infer the entropy change of the dot.

Here, we propose to *strongly* couple a FQH QPC to a trivial QD. Employing Maxwell relations, we show that one can infer the change in entropy of the joint system, including both the QD and, importantly, the QPC, as the latter switches from being fully transmitting to fully reflecting. At fixed temperature, such a measurement allows one to capture the $\mathcal{O}(1)$ topological entropy change in Eq. (2) and thus extract the TEE.

Model.—The chiral edges of a FQH system described by an Abelian Laughlin state with filling factor $\nu = 1/m$ are described by the bosonized Hamiltonian [25,52],

$$H_0 = \frac{1}{4\pi\nu} \int dx (\partial_x \varphi_L)^2 + (\partial_x \varphi_R)^2, \tag{3}$$

where L/R denotes left and right movers. Here we have set the edge velocity to unity. The QPC at $x = 0$ induces tunneling of quasiparticles between the edge states, which is described by [16,53]

$$H_B = \lambda \cos[\varphi_L(x=0) - \varphi_R(x=0)], \tag{4}$$

where λ is the tunneling strength. For $m > 1$, it leads to the energy scale $T_B = C\lambda^{[1/(1-\nu)]}$, across which the crossover from UV to IR limits happen, with C being a nonuniversal constant of appropriate dimensions [16]. The integer quantum Hall case $m = 1$ is also described by the same Hamiltonian, but in this case the tunneling of electrons is marginal in the renormalization group sense and there is no crossover. While for experiments in electronic systems only odd values of m are relevant, corresponding to fermions, we also consider theoretically the case of even m, corresponding to a Laughlin state of bosons.

In general, for $\nu = 1/m$, the model can be mapped into the boundary sine-Gordon model, which is integrable [54]. It can be solved using the thermodynamic Bethe ansatz (TBA) [25,55–57], which allows one to obtain the free energy $F[T, T_B]$, and from it, the boundary entropy $S = -\partial_T F$.

Applying the TBA, we compute the entropy along the full crossover from the UV to IR fixed points (see Supplemental Material [25]), see Fig. 1, which gives $\mathcal{S}_{\mathrm{UV}} - \mathcal{S}_{\mathrm{IR}} = \log\sqrt{m}$. This yields a finite entropy change only in the fractional case, $m > 1$. We included also the case $\nu = 1/2$, which can be solved exactly using refermionization and which corresponds to an effective Majorana fermion [16,25,49,58,59]. Similar methods [60,61] can be applied to extract the entropy of parafermion modes.

Next, we present two schemes to realize the crossover in the QPC between the UV and IR limits and measure the resulting thermodynamic entropy change.

Scheme 1: Side-coupled quantum dot.—We now illustrate the TEE measurement protocol for the $\nu = 1/m$ Laughlin states using a side-coupled quantum dot following the approach of Ref. [50]. As shown in Fig. 2, we attach to the QPC a QD in the Coulomb blockade regime described by a classical energy function $E(N,\mu) = E_c N^2 - \mu N$. Here E_c is the charging energy, N is the number of electrons in the QD, and μ is a local chemical potential of the QD, controlled by a gate voltage. The QD interacts electrostatically with the QPC, as described by a dependence of the tunneling amplitude on the number operator of the QD, $\lambda = \lambda(N)$. Thus, the crossover energy scale T_B is controlled by N, $T_B = T_B(N)$. We now show how, from such a dependence, one can extract $S_{\rm UV} - S_{\rm IR}$.

Under these conditions, the partition function of the combined system is

$$Z_{\rm tot} = \sum_N e^{-\frac{1}{T}[F(T,T_B(N))+E(N,\mu)]}, \tag{5}$$

where $F(T, T_B(N))$ is the free energy corresponding to the Hamiltonian in Eqs. (3) and (4) with $\lambda \to \lambda(N)$. By attaching a charge detector to the QD, using the Maxwell relation $(d\langle N\rangle/dT) = (dS/d\mu)$, one can extract the entropy change produced by a change of μ,

$$\Delta S_{\mu_1\to\mu_2} = \int_{\mu_1}^{\mu_2} \frac{d\langle N\rangle}{dT} d\mu. \tag{6}$$

When $T \ll E_c$, upon increasing μ, there are several quantized charge steps in N, see inset of Fig. 3(a), and the QPC gets closer to pinch-off, corresponding to an increase in T_B. The desired entropy change will occur if, by charging the QD by ΔN electrons, the QPC transitions from the UV limit ($T_B \ll T$) to the IR limit ($T_B \gg T$).

In Fig. 3, we assume an N dependence of $T_B(N)$ such that across a charging of $\Delta N = 3$ electrons one achieves $T_B(N) \ll T \ll T_B(N + \Delta N)$. In Fig. 3(b) we compute $(d\langle N\rangle/dT)$, where $\langle N\rangle$ is extracted from the total free energy $F_{\rm tot} = -T \ln Z_{\rm tot}$ as $\langle N\rangle = -\partial F_{\rm tot}/\partial\mu$. The total entropy change $S_{\mu_1=0\to\mu}$ from a selected μ_1 until a varying μ is shown in Fig. 3(c). We can see that this entropy contains a series of log2 peaks corresponding to charge degeneracies of the QD, riding on top of the slowly varying entropy along the crossover for $\nu \neq 1$. For $T \ll E_c$, these two effects are well separated and we can measure the TEE by taking the difference between the corresponding entropy plateaus where $\langle N\rangle$ weakly fluctuates; see dashed horizontal lines in Fig. 3(c).

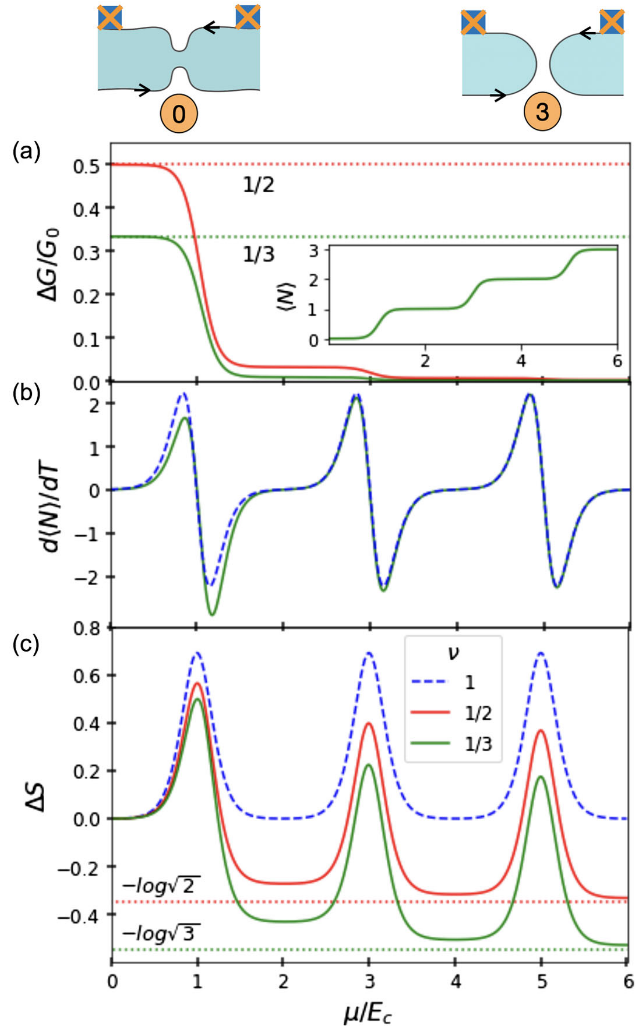

FIG. 3. Illustration of the proposed TEE measurement. (a) The conductance between the Ohmic contacts for filling factor $\nu = 1/2, 1/3$ changes upon charging a nearby QD by varying the chemical potential μ of the QD. The charging curve is shown in the inset. Here $T/E_c = 0.1$, $G_0 = e^2/h$, and $T_B(N)/E_c = e^{N/2} - 1$ [50]. (b) Temperature derivative of the QD charge as a function of μ. For $\nu = 1$ (dashed) the curve is exactly antisymmetric, while for $\nu = 1/3$ (solid) it is not. (c) The resulting entropy change (QPC + QD), obtained by integrating $d\langle N\rangle/dT$ using the Maxwell relation (6), for $\nu = 1$ (dashed blue) and $\nu = 1/2, 1/3$ (solid curves). The peaks are associated with the log2 entropy of the dot at its charge steps. The plateaus of the entropy decrease for $\nu = 1/2, 1/3$ as a function of μ, and converge to the desired TEE once the UV-IR crossover is accomplished.

Several experiments observed the crossover between UV and IR limits of a gate-tuned QPC for several FQH states through the conductance and the shot noise, which crosses from quasiparticle tunneling to electron tunneling [62–65]. Our method outlined in Fig. 2, however, requires one to drive this crossover as a function of the chemical potential of the dot. The conductance for such a crossover is plotted in Fig. 3(a).

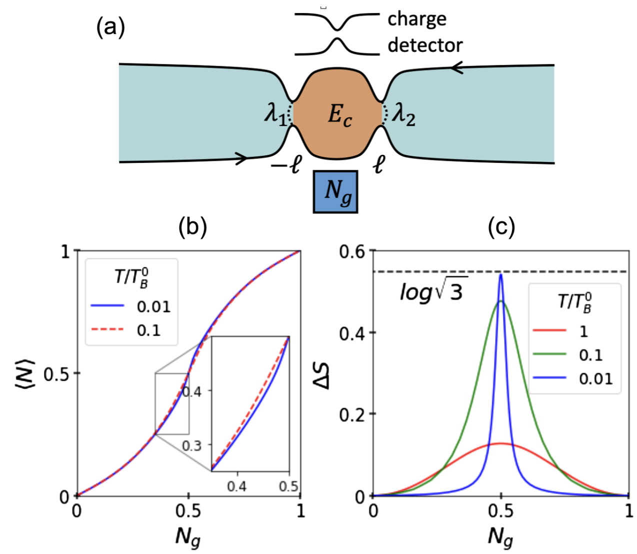

FIG. 4. (a) Two QPCs in series creating a QD with Coulomb interaction E_c within the FQH system. By tuning the gate voltage N_g at fixed $\lambda_{1,2}$, this resonant double barrierlike system yields an effective backscattering strength in Eq. (8) which drops at resonance and vanishes for symmetric barriers. The charge detector measures the charge of the QD. (b) The average occupation of the QD for the case of symmetric barriers at $\nu = 1/3$. Increasing temperature shifts the curve to right or left for N_g above or below 1/2. (c) Entropy change inferred from Maxwell relation for several ratios of T/T_B^0. As $T \to 0$, $N_g = 0$ tends to the IR limit and the entropy difference at $N_g = 1/2$ (which is always the UV limit) tends to the TEE.

In practice, however, a side-coupled QD as in Fig. 2 may have a limited electrostatic affect on the QPC. To enhance the effect, we now discuss a different setup with the QD embedded in the constriction.

Scheme 2: Coulomb blockade in the FQH regime.—One can realize a nearly complete change of transmission by replacing the QPC by a double barrier consisting of two QPCs, see Fig. 4(a). For noninteracting electrons, it is well known [66] that the transmission has an abrupt resonance peak that is maximized for symmetric barriers. Here we demonstrate that this resonance effect also occurs in a double barrier system on a fractional edge.

As shown in Fig. 4(a), we consider two QPCs in series separated by a distance 2ℓ, having backscattering amplitudes $\lambda_{1,2}$, and defining a QD characterized by a charging energy E_c. In this case, the Hamiltonian is

$$H_{\mathrm{QD}} = \lambda_1 \cos[\varphi_L(-\ell) - \varphi_R(-\ell)] + \lambda_2 \cos[\varphi_L(\ell) - \varphi_R(\ell)] + \frac{E_c}{4\pi^2}\left(\sum_{i=L,R}[\varphi_i(-\ell) - \varphi_i(\ell)] - 2\pi N_g\right)^2. \tag{7}$$

In the limit of a large charging energy $E_c\ell \gg 1$, one obtains an effective N_g-dependent boundary condition for the bosonic field [25,67–69]. As a result, the two QPCs behave as one effective QPC with

$$H'_B = \lambda_{\mathrm{eff}} e^{i[\varphi_L(0) - \varphi_R(0)]} + \mathrm{H.c.}, \tag{8}$$

where

$$\lambda_{\mathrm{eff}} \propto \lambda_1 e^{-i\pi N_g} + \lambda_2 e^{i\pi N_g}. \tag{9}$$

This effective model holds for $E_c \gg T_{B1,2} = C\lambda_{1,2}^{1/(1-\nu)}$. We assume a finite reflection at the QPCs such that $T_{B1,2} \gg T$. However, using Eq. (9), we see that, when the barriers are symmetric $\lambda_1 = \lambda_2 \equiv \lambda_0$, we have $\lambda_{\mathrm{eff}} \propto \lambda_0 \cos(\pi N_g)$, or equivalently, $T_B = T_B^0 |\cos(\pi N_g)|^{1/(1-\nu)}$. Hence at $N_g = 1/2$, the effective barrier vanishes and $T_B = 0$.

Thus, the TEE can be extracted as the entropy reduction between the resonance condition $N_g = 1/2$ and the off-resonance limit $N_g = 0$, 1. This entropy difference can be directly measured by attaching a charge detector to the QD and using the Maxwell relations. Equation (6) applies with $\mu \to 2E_c N_g$.

In Figs. 4(b) and 4(c), we plot $\langle N\rangle$ and the extracted entropy. Here $\langle N\rangle = -(1/2E_c)(\partial F/\partial N_g)$ is computed from the TBA free energy $F(T, T_B)$ with $T_B = T_B[\lambda_{\mathrm{eff}}(N_g)]$ carrying the N_g dependence via Eq. (9). Different than the side-coupled QD, by using the resonance effect, the crossover is fully accomplished along the way from a Coulomb peak ($N_g = 1/2$) and a nearby Coulomb valley ($N_g = 0$, 1). Also, different from the side-coupled QD that was only weakly coupled to the lead, and hence led to clear log2 entropy peaks at its charge degeneracy points [Fig. 3(c)], here the strong tunnel-coupled QD has a negligible contribution to the entropy, which scales as $S_{\mathrm{QD}} \sim (T_B^{2-2\nu}/T^{2-2\nu})(T^2/E_c^2)$ [25]. However, in order to explore the full UV-IR crossover, the barriers should be high enough such that $T_B^0 \gg T$, see Fig. 4(c).

Discussion.—In the above schemes, to isolate the subleading TEE term γ in Eq. (1), we assume that when the FQH droplet splits into two, the change in the length of the edge, $L \to L + \Delta L$, leads only to a negligible entropy change as compared to the order $\mathcal{O}(1)$ TEE. We can now justify this assumption. The resulting entropy change of the 1D gapless edge modes with velocity v_F is

$$\Delta S_L \sim \frac{T}{\hbar v_F}\Delta L. \tag{10}$$

Since the TEE part of the entropy change is $O(1)$, we can neglect ΔS_L if $\Delta L \ll \hbar v_F/T$. For $T = 100$ mK and $v_F = 10^6$ m/s, we find that the requirement $\Delta L \ll 10^{-4}$ m is easily satisfied.

Whereas our proposed scheme to extract the TEE relies on bulk-edge correspondence, one could inquire about nonuniversal effects near the edge. For example, if the QPC edge modes interact among themselves or with nearby edge states emerging, e.g., due to edge reconstruction [70], then there will be a correction to the extracted entropy change.

Similar to a Luttinger liquid [71] whose entropy per length ΔL is given by Eq. (10) with v_F being renormalized by interactions, we argue that, at low temperatures, the interaction-induced corrections take the form of Eq. (10), with ΔL being replaced by the effective interaction length near the QPC L_{int}. This scaling can also be obtained perturbatively in the interactions (see Supplemental Material [25]). Thus, nonuniversal corrections are negligible at low temperatures. We also note that backaction effects from the charge detector [72–74] need to be considered, but are negligible when the system-detector coupling is weak.

Summary.—Despite its importance, the measurement of the topological entanglement entropy is still elusive in condensed matter systems. Using a relation between TEE and a thermodynamic entropy change in fractional QPCs, we proposed realistic approaches to measure the TEE employing charge measurement and the Maxwell relation. We illustrated our protocols for the experimentally simplest and yet nontrivial case of Abelian fractional topological order.

Our proposed setups are also applicable to extract the entropy change between UV and IR fixed points in the more general boundary sine-Gordon model describing an impurity in a Luttinger liquid with any value of the interaction parameter, within mesoscopic systems simulating this model, such as in Ref. [75].

Since the relation we applied is general, our method can be applied to more exotic systems, such as spin liquid systems [76] and, most interestingly, FQH states with non-Abelian quasiparticles. Moreover, we found that the present method works also for hierarchical FQH states with multi-edge channels [77,78], and in the limit of spatially separated edge modes, the entropy curves can even be obtained using the TBA [79].

We gratefully acknowledge support from the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation program under Grant Agreement No. 951541, ARO (W911NF-20-1-0013), and the Israel Science Foundation Grant No. 154/19. We thank Anne Anthore, Christophe Mora, Yigal Meir, and particularly Frederic Pierre for illuminating discussions.

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