Is Light Neutralino Thermal Dark Matter in the Phenomenological Minimal Supersymmetric Standard Model Ruled Out?

Rahool Kumar Barman,[1,*] Geneviève Bélanger,[2,†] Biplob Bhattacherjee,[3,‡]
Rohini M. Godbole,[3,§] and Rhitaja Sengupta[3,∥]
[1]*Department of Physical Sciences, Oklahoma State University, Stillwater, Oklahoma 74078, USA*
[2]*LAPTh, Université Savoie Mont Blanc, CNRS, 9 Chemin de Bellevue, 74940 Annecy, France*
[3]*Centre for High Energy Physics, Indian Institute of Science, Bangalore 560012, India*

(Received 26 July 2022; revised 2 May 2023; accepted 25 May 2023; published 6 July 2023)

We explore the parameter space of the phenomenological minimal supersymmetric standard model with a light neutralino thermal dark matter ($m_{\tilde{\chi}_1^0} \leq m_h/2$) that is consistent with current collider and astrophysical constraints. We consider both positive and negative values of the higgsino mass parameter (μ). Our investigation shows that the recent experimental results from the LHC as well as from direct detection searches for dark matter by the LUX-ZEPLIN Collaboration rule out the Z-funnel region for the $\mu > 0$ scenario. The same results severely restrict the h-funnel region for positive μ; however, the allowed points can be probed easily with few more days of data from the LUX-ZEPLIN experiment. In the $\mu < 0$ scenario, we find that very light higgsinos in both the Z and h funnels might survive the present constraints from the electroweakino searches at the LHC, and dedicated efforts from experimental collaborations are necessary to make conclusive statements about their present status.

DOI: 10.1103/PhysRevLett.131.011802

The R-parity conserving scenario of the minimal supersymmetric extension of the standard model (MSSM) has been among the most favorable choices for exploring physics beyond the standard model (BSM). The R-parity conserving MSSM scenario alleviates the "naturalness" problem [1,2] in the standard model (SM), while also providing a SM-like Higgs boson (h) with mass $m_h \sim 125$ GeV and a stable lightest supersymmetric particle (LSP), typically the neutralino $\tilde{\chi}_1^0$, which can be a cold dark matter (DM) candidate. The case of the light neutralino $m_{\tilde{\chi}_1^0} \leq m_h/2$ is of special interest since it is kinematically feasible for the SM Higgs boson to decay invisibly through $h \to \tilde{\chi}_1^0\tilde{\chi}_1^0$, thus providing an additional signature for DM in the Higgs sector. Several studies have explored the prospect of a light neutralino DM in the constrained MSSM and the phenomenological MSSM (pMSSM) considering the various experimental constraints at the time [3–24]. Collider experiments, like ATLAS and CMS, have made available the latest results from searches of heavy Higgs bosons [25], direct searches of charginos and neutralinos [26–29], as well as the invisible decay of the SM Higgs boson [30]. The XENON-1T, PICO-60, PandaX-4T, and LUX-ZEPLIN (LZ) collaborations have also published limits on the DM direct detection (DD) cross sections, both spin-dependent (SD) and spin-independent (SI) [31–36]. Among these, the results from the LZ Collaboration are the most stringent ones for the SI DD cross sections [35]. In lieu of these new and improved results, it becomes crucial to revisit the MSSM parameter space containing light neutralino DM, which can also contribute to the invisible decay of the Higgs boson.

In this Letter, we study the current status of the light neutralino DM in the MSSM for both positive and negative values of the higgsino mass parameter μ. It is worth noting that the supersymmetric explanation for the discrepancy between the experimentally measured value of the $(g-2)_\mu$ and the SM prediction [37] typically requires $\mu > 0$. Because of the prevalent uncertainties in estimating the hadronic contributions in the SM prediction, we prefer an agnostic attitude toward the sign of μ. As the Large Hadron Collider (LHC) is gearing up for Run-3 and will start collecting data soon, a careful study of the overall status of this scenario is very timely to identify the interesting regions of the parameter space that can be a focal point of the LHC searches at Run-3.

We consider the pMSSM parameter space with ten free parameters defined at the electroweak scale. Our focus is the light neutralino sector with $m_{\tilde{\chi}_1^0} \leq m_h/2$ such that it can contribute to the invisible decay mode of the Higgs boson. The input parameters that capture the physics of the Higgs and electroweakino sectors are M_1, the bino mass; M_2, the wino mass (M_1 and M_2 are collectively referred to as the gaugino masses); μ, the higgsino mass; $\tan\beta$, the ratio of

---

the Higgs vacuum expectation values; M_A, the pseudoscalar mass; $M_{\tilde{Q}_{3l}}$, $M_{\tilde{t}_R}$, $M_{\tilde{b}_R}$, the mass of the third generation squarks; A_t, the trilinear coupling of the stop; and M_3, the mass of the gluino. We perform a random scan over ten input parameters for the pMSSM in the following range:

$$\begin{gathered}
30\ \mathrm{GeV} < M_1 < 100\ \mathrm{GeV}, \qquad 1\ \mathrm{TeV} < M_2 < 3\ \mathrm{TeV}, \\
100\ \mathrm{GeV} < |\mu| < 2\ \mathrm{TeV}, \qquad 2 < \tan\beta < 50, \\
100\ \mathrm{GeV} < M_A < 5\ \mathrm{TeV}, \qquad 3\ \mathrm{TeV} < M_{\tilde{Q}_{3L}} < 20\ \mathrm{TeV}, \\
3\ \mathrm{TeV} < M_{\tilde{t}_R} < 20\ \mathrm{TeV}, \qquad 3\ \mathrm{TeV} < M_{\tilde{b}_R} < 20\ \mathrm{TeV}, \\
-20\ \mathrm{TeV} < A_t < 20\ \mathrm{TeV}, \qquad 2\ \mathrm{TeV} < M_3 < 5\ \mathrm{TeV},
\end{gathered}$$

while we fix

$$\begin{gathered}
M_{\tilde{Q}_{1,2L}} = M_{\tilde{u}_{1,2R}} = M_{\tilde{d}_{1,2R}} = 5\ \mathrm{TeV}, \qquad A_{u/d/c/s/b} = 0, \\
M_{\tilde{L}_{1,2,3L}} = M_{\tilde{e}_{1,2,3R}} = 2\ \mathrm{TeV}, \qquad A_{e/\mu/\tau} = 0.
\end{gathered}$$

Since we are interested in light neutralino with $m_{\tilde{\chi}_1^0} \le m_h/2$, it shall dominantly have bino ($\tilde{B}$) component. Therefore, we scan M_1 in the low mass region. The coupling of Z and h bosons to a pair of $\tilde{\chi}_1^0$ also depends on its higgsino ($\tilde{H}$) and wino ($\tilde{W}$) components. Therefore, in order to circumvent the overabundance of $\tilde{\chi}_1^0$ as the DM candidate, we require it to have some $\tilde{H}$ or $\tilde{W}$ component. We are primarily interested in the higgsinolike next-to-lightest supersymmetric partner in the present work due to the existing stronger limits on winolike next-to-lightest supersymmetric partners. Hence, M_2 is scanned above 1 TeV, while $|\mu|$ is varied starting from a comparatively lower value of 100 GeV, considering both positive and negative values of μ. Previous studies have shown that the DM relic density and direct detection constraints allow only the Z- and h-funnel regions [23,24,38]. We have performed a dedicated scan where we dynamically tune the M_1 parameter to keep $m_{\tilde{\chi}_1^0}$ within $m_Z/2 \pm 5$ GeV and $m_h/2 \pm 3$ GeV to populate the funnel regions sufficiently. Additionally, we extract the pole mass of the top quark, M_t, randomly from a Gaussian distribution with a central value of 173.21 GeV and a standard deviation of 0.55 GeV [39]. In total, until this point, we scan over a sample of size 2×10^8 points.

We use FeynHiggs 2.18.1 [40–47] to generate the supersymmetry spectra corresponding to the various sets of input parameters [48], and to calculate the Higgs boson mass and decays in the Higgs sector. We assume that the lightest CP-even Higgs boson of MSSM is the SM-like Higgs boson, observed by the ATLAS and CMS collaborations, with a combined measured mass $m_h = 125.09 \pm 0.21(\mathrm{stat}) \pm 0.11(\mathrm{syst})$ GeV [49]. We conservatively allow for a theoretical uncertainty of 3 GeV and consider the mass range 122 GeV $< m_h <$ 128 GeV. Our scan starts from very low $\tan\beta$ values where satisfying the observed Higgs boson mass will require high stop masses and A_t. In order to ensure that the latter is not so large as to lead to color and charged breaking minima [50–52], we require $|X_t| \lesssim \sqrt{6 m_{\tilde{t}_1} m_{\tilde{t}_2}}$ [51], where $X_t = A_t - \mu/\tan\beta$ and $m_{\tilde{t}_{1,2}}$ represents the masses of the two stops. We observe that the color and charge breaking condition has little effect on our parameter space.

We apply limits on the partial decay width of the invisible decay of Z boson from new physics, $\Gamma_{\mathrm{inv}}^{\mathrm{new}} < 2$ MeV [53]; chargino mass, $m_{\chi_1^\pm} > 103$ GeV [54]; and cross section of associated production of neutralinos in final states with jets $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times \mathrm{Br}(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 + \mathrm{jets}) + \sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0) \times \mathrm{Br}(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 + \mathrm{jets}) < 0.1$ pb [54], as obtained from experiments at the Large Electron-Positron Collider (LEP).

We also include the flavor physics constraints on various observables, like the branching fractions of processes $b \to s\gamma$, $B_s \to \mu^+\mu^-$, and $B \to \tau\nu$, which are required to satisfy $3.00 \times 10^{-4} < \mathrm{Br}(b \to s\gamma) < 3.64 \times 10^{-4}$ [55], $1.66 \times 10^{-9} < \mathrm{Br}(B_s \to \mu^+\mu^-) < 4.34 \times 10^{-9}$ [56], and $0.78 < [\mathrm{Br}(B \to \tau\nu)]_{\mathrm{obs}}/[\mathrm{Br}(B \to \tau\nu)]_{\mathrm{SM}} < 1.78$ [57], respectively. We use MicrOMEGAS 5.2.13 [58–63] to calculate both the LEP and flavor physics observables.

Additionally, we apply the limits from signal strength measurements of the SM Higgs boson implemented in HiggsSignal 2.6.2 [64–66], as well as limits from heavy Higgs searches at the colliders using the HiggsBounds 5.10.0 [67–71] package. The recent search of heavy Higgs bosons decaying to τ leptons at ATLAS [25] excludes a large part of high $\tan\beta$ region for $M_A \lesssim 1$ TeV. The parameter space must also satisfy the recent limit on the invisible branching fraction of the SM Higgs boson, $\mathrm{Br}(h \to \mathrm{invisible}) < 0.11$ [72]. We refer to all the constraints related to the Higgs bosons together as the "Higgs constraints" hereafter for simplicity, including the SM-like Higgs mass constraint, constraints from HiggsSignal 2.6.2, HiggsBounds 5.10.0, and the invisible decay of the SM-like Higgs boson.

The LSP, $\tilde{\chi}_1^0$, is a viable DM candidate in the MSSM, having a thermal freeze-out production in the early Universe. In the standard cosmology, we require the relic density of the LSP (Ω_{LSP}) to be equal to the observed DM relic density as measured by the Planck Collaboration $\Omega_{\mathrm{DM}}^{\mathrm{obs}} h^2 = 0.120 \pm 0.001$ [73], which, assuming a 2σ interval, can vary from 0.118 to 0.122. Lifting up the requisite that the neutralino LSP forms 100% of the observed DM relic owing to the possibility of multicomponent DM, we can modify the relic density constraint to $\Omega_{\mathrm{LSP}} \lesssim 0.122$. MicrOMEGAS 5.2.13 is used to compute the relic density of $\tilde{\chi}_1^0$.

In addition to the relic density constraint, we take into consideration the results from the current DD experiments. These experiments constrain the spin-dependent DM neutron (SDn) and DM proton as well as the spin-independent DD cross sections of the lightest neutralino LSP ($\tilde{\chi}_1^0$) as a function of its mass. We use MicrOMEGAS 5.2.13 to compute these cross sections and then compare them with the 90%

confidence level upper limits quoted by the XENON-1T (SI [31] and SDn [32]), PICO-60 (DM proton [33]), PandaX-4T (SI [34] and SDn [36]), and LZ (SI,[35]) experiments. The DD limits, typically derived by assuming that a single DM candidate constitutes the entire relic, will weaken in the scenario where the neutralino DM is underabundant by a factor of $\xi = \Omega_{\rm LSP}/0.120$.

Moreover, we must consider the results from direct electroweakino searches at the LHC. We use the SModelS 2.2.1 [74–81] package to implement the electroweakino search constraints on our scanned parameter space. This version of SModelS includes results from the recent search for electroweakinos in the leptonic final states at CMS [26] and ATLAS [27] and in the hadronic final states at ATLAS [28], all of which play significant roles in excluding a large range of $m_{\tilde{\chi}_1^\pm}$, $m_{\tilde{\chi}_2^0}$, and $m_{\tilde{\chi}_3^0}$, especially with the ATLAS analysis extending the sensitivity to high masses with the hadronic final states.

We apply the constraints on our scanned parameter space in three steps: first, “Before LZ” with constraints from LEP, flavor, Higgs constraints, relic density, and the DD experiments XENON-1T, PICO-60, and PandaX-4T, then with the constraint from the LZ experiment, denoted as “After LZ,” and last, we add the electroweakino constraints from the LHC. We identify interesting regions of the parameter space surviving all the constraints and perform dedicated scans over these regions again with an additional sample of size $\sim 10^8$, which makes the total size of our scan 3×10^8. The upper panel of Fig. 1 shows the scaled (with ξ) SI DD cross sections for the allowed parameter space with $\mu > 0$ after applying all the constraints from “Before LZ” in yellow and “After LZ” in light green and the electroweakino constraints in dark green. We observe that the recent LZ experiment, with only 60 days of data, has played a crucial role in excluding the Z-funnel region for positive μ. To ascertain the robustness of this result we allow for a 20% theoretical uncertainty on the relic density. We find that the scaled SI DD cross section can be reduced below the present LZ limit. However, all such points are excluded by the present electroweakino searches. In the h funnel we find a region of parameter space surviving all present constraints. In this region the SI DD cross section is just below the present limit and is well above the full 1000 days projections, thus the region will be probed with just a few more days of data from the LZ experiment [82]. In fact this strong impact of the recent LZ result stands true for any BSM model with a light Majorana fermionic DM having couplings with the h boson in the framework of a simplified model (see Ref. [83]). Figure 2 illustrates the impact of electroweakino searches that restrict $M_{\tilde{\chi}_2^0} \gtrsim 850$ GeV.

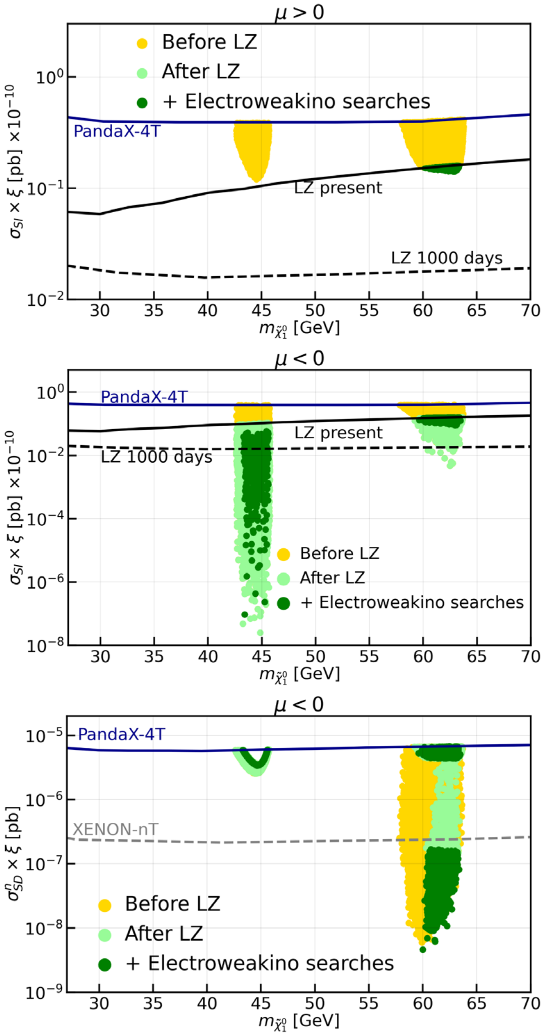

FIG. 1. Scaled SI DM-nucleon cross section ($\sigma_{SI} \times \xi$) for $\mu > 0$ (upper) and $\mu < 0$ (center), along with scaled SD DM-neutron cross section ($\sigma_{\rm SDn} \times \xi$) for $\mu < 0$ (lower) as a function of the mass of the LSP neutralino DM in the region of parameter space satisfying LEP, flavor, Higgs constraints, relic density, DM DD constraints from the XENON-1T, PICO-60, and PandaX-4T experiments: “Before LZ” in yellow circles, “After LZ” in light green circles, overlayed with additional constraints from electroweakino searches at the LHC (in dark green circles). The current experimental limits from PandaX-4T and LZ are shown (solid lines) as well as projections for LZ 1000 days and XENON-nT (dashed lines).

Let us now investigate the $\mu < 0$ scenario. The center and lower panels of Fig. 1, respectively, illustrate the scaled SI and SDn DD cross sections for the allowed parameter space with $\mu < 0$, where the colors have the same meaning as described for Fig. 1 upper panel. We observe that the recent LZ result and electroweakino searches have excluded most of the h-funnel region, leaving only a marginally allowed region where the $M_{\tilde{\chi}_2^0}$ is either very small (∼140–155 GeV) or larger than 850 GeV, as seen from the bottom panel of Fig. 2. We expect this region to be probed in the near future by the LZ experiment with its full 1000-day exposure. On the other hand, the Z funnel is not affected much by the LZ limits and a large fraction lies well below the future reach of LZ. However, the entire Z-funnel region is well within the projected reach on $\sigma_{\rm SDn}$ of

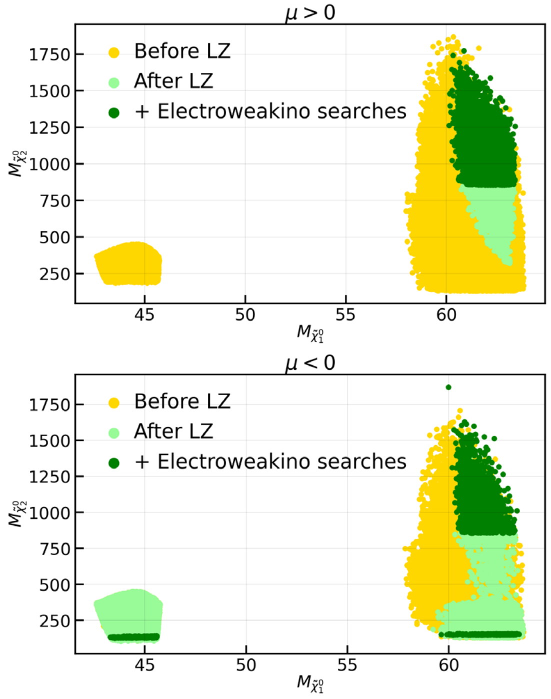

FIG. 2. Allowed parameter space for $\mu > 0$ (top) and $\mu < 0$ (bottom) after satisfying the LEP, flavor, Higgs constraints, relic density DM DD constraints from the XENON-1T, PICO-60, and PandaX-4T experiments: "Before LZ" in yellow circles, "After LZ" in light green circles, overlayed with additional constraints from electroweakino searches at the LHC (in dark green circles) in the $m_{\tilde{\chi}_1^0}$-$m_{\tilde{\chi}_2^0}$ plane.

XENON-nT (bottom panel of Fig. 1). In fact XENON-nT will probe the parameter space with light $\tilde{\chi}_2^0$ in both the Z and h funnels. Indeed the SD DD cross section is proportional to the square of the coupling of the LSP with the Z boson, which depends on the higgsino component of $\tilde{\chi}_1^0$. Therefore the lighter higgsinos have higher values of the SDn DD cross section than the heavier ones, which creates the two separate dark green patches in the h funnel of the lower plot of Fig. 1.

Figure 2 reveals a major difference between the $\mu > 0$ and the $\mu < 0$ scenarios. The LZ limit excludes lighter higgsinos ($\lesssim$300 GeV) for $\mu > 0$, while they survive when $\mu < 0$. This happens due to a cancellation [84] between the contributions of the two CP-even neutral Higgs bosons (h and H) involving diagrams with down-type quarks to the SI DD cross section when $\mu < 0$ in the MSSM. On the other hand these two contributions interfere constructively for $\mu > 0$, increasing the SI DD cross sections. Among the light higgsinos allowed by LZ, the electroweakino searches allow points in a very narrow region of parameter space in both the Z and h funnels for $\mu < 0$, many of which have very small R values [85], as shown in Fig. 1 of Supplemental Material [86]. It is interesting to note that if future DD experiments discover a light DM in the Z funnel, it will mostly indicate a negative value of μ. An observation of DM signal in the h funnel from DD experiments would require an additional observation of a signal in the collider experiments to shed light on the sign of μ—observation of light higgsinos will hint toward negative μ; however, heavy higgsinos will not be able to lift this ambiguity. Moreover, we find that the allowed points in each of these regions are clustered around specific $\tan\beta$ ranges: low $\tan\beta$ values of 3–10 in the h funnel for $\mu > 0$, while for $\mu < 0$, $\tan\beta \sim 3$–18 in the Z funnel, ~3–6 for heavy higgsinos in the h funnel, and ~16–50 for light higgsinos in the h funnel.

Representative benchmarks from each of the allowed regions of the parameter space are presented in the Supplemental Material [86]. These benchmarks have very small uncertainty in the Higgs boson mass [$\lesssim\mathcal{O}(1)$ GeV], and have SModelS R values below 0.5. They are also allowed when tested with CheckMATE 2 [90], another package that implements the constraints from electroweakino searches. We find that the Tevatron limits for light charginos [91] are also not sensitive to these benchmarks. To estimate the prospects for probing the region with light charginos and neutralinos at the LHC, we perform an analysis of the low mass higgsinolike electroweakinos in the leptonic $3l + \not{E}_T$ final state at $\sqrt{s} = 14$ TeV using the XGBOOST [92] framework. We study the process $pp \to \tilde{\chi}_1^\pm \tilde{\chi}_2^0/\tilde{\chi}_1^\pm \tilde{\chi}_3^0$, $\tilde{\chi}_1^\pm \to W^\pm \tilde{\chi}_1^0$, $\tilde{\chi}_2^0/\tilde{\chi}_3^0 \to f\bar{f}\tilde{\chi}_1^0$ with $m_{\tilde{\chi}_1^\pm} = 125.1$ GeV, $m_{\tilde{\chi}_2^0} = 129.9$ GeV, $m_{\tilde{\chi}_3^0} = 133.5$ GeV, and $m_{\tilde{\chi}_1^0} = 44.6$ GeV (benchmark 2 from Table I in Supplemental Material [86]) where f is an SM fermion, considering 11 possible SM backgrounds for this process. The XGBOOST model, trained with 21 kinematic variables, is used to discriminate the signal benchmark from each background class by computing the significance of observing the signal over the background events. At the $\sqrt{s} = 14$ TeV LHC with 137 fb^{-1} of integrated luminosity ($\mathcal{L}$), we expect to observe 763 signal events and 987 background events for a threshold of 0.9 on our XGBOOST output. Adding a 20% (50%) systematic uncertainty translates to a significance (using the formula in Ref. [93]) of 3.1 (1.3). We present our results for $\sqrt{s} = 14$ TeV to make it easier to translate to the case of Run-3 ($\sqrt{s} = 13.6$ TeV) and HL-LHC ($\sqrt{s} = 14$ TeV) as the cross sections are not expected to change much. We find that the result sensitively depends on the systematic uncertainty, which can have a significant impact for light electroweakinos.

In summary, this Letter shows that the current experiments, especially the recent results from electroweakino searches at the LHC and dark matter DD measurements at the LZ, have severely constrained the $\mu > 0$ scenario, with the Z funnel being completely excluded, and only very heavy higgsinos allowed in the h funnel. For the $\mu < 0$ scenario, the allowed parameter space consists of either higgsinos heavier than ~850 GeV in the h funnel or restricted to a narrow region of light higgsinos having mass of 120–155 GeV in the Z and h funnels, for a light neutralino thermal DM in the pMSSM with ten free parameters. Light right-handed staus, still allowed by the LHC, can have a mild affect on the relic density of the lightest neutralino, which is investigated in detail in our future work (Ref. [83]). Moreover, presence of light staus can affect the collider constraints on higgsinos in regions of the parameter space where the latter decay into the former with significant branching fractions.

The current status of light higgsinos in the mass range of 125–150 GeV from the present electroweakino constraints is not completely clear in the pMSSM, since the experimental results are presented for simplified models. Therefore, experimental collaborations must focus on this region and provide definitive answers, covering all possibilities of light higgsinos (including multiple decay modes), which can be an interesting target for Run-3 of LHC. Such light higgsinos are also motivated by naturalness arguments. To conclude, at present, we are at a very exciting juncture where the experiments lined up in the near future might exclude the possibility of a light neutralino thermal DM in the MSSM altogether, or we might be very close to starting to observe the first direct hints of new physics at the LHC.

We thank Sabine Kraml for the useful discussion and help related to the SModelS package. The work of G. B. and R. M. G. was funded in part by the Indo-French Centre for the Promotion of Advanced Research, Grant No. 6304-2. R. M. G. wishes to acknowledge the support of Indian National Science Academy under the award of INSA Senior Scientist. B. B. and R. S. thank Prabhat Solanki and Camellia Bose for useful discussions. R. K. B. thanks the U.S. Department of Energy for the financial support under Award No. DE-SC0016013.

---

[*] Corresponding author.
rahool.barman@okstate.edu
[†] Corresponding author.
belanger@lapth.cnrs.fr
[‡] Corresponding author.
biplob@iisc.ac.in
[§] Corresponding author.
rohini@iisc.ac.in
[‖] Corresponding author.
rhitaja@iisc.ac.in

[1] E. Gildener and S. Weinberg, Symmetry breaking and scalar bosons, Phys. Rev. D **13**, 3333 (1976).

[2] L. Susskind, Dynamics of spontaneous symmetry breaking in the Weinberg-Salam theory, Phys. Rev. D **20**, 2619 (1979).

[3] K. Griest and H. E. Haber, Invisible decays of Higgs bosons in supersymmetric models, Phys. Rev. D **37**, 719 (1988).

[4] A. Djouadi, P. Janot, J. Kalinowski, and P. M. Zerwas, SUSY decays of Higgs particles, Phys. Lett. B **376**, 220 (1996).

[5] G. Belanger, F. Boudjema, F. Donato, R. Godbole, and S. Rosier-Lees, SUSY Higgs at the LHC: Effects of light charginos and neutralinos, Nucl. Phys. **B581**, 3 (2000).

[6] G. Belanger, F. Boudjema, A. Cottrant, R. M. Godbole, and A. Semenov, The MSSM invisible Higgs in the light of dark matter and g-2, Phys. Lett. B **519**, 93 (2001).

[7] D. Hooper and T. Plehn, Supersymmetric dark matter: How light can the LSP be?, Phys. Lett. B **562**, 18 (2003).

[8] G. Belanger, F. Boudjema, A. Cottrant, A. Pukhov, and S. Rosier-Lees, Lower limit on the neutralino mass in the general MSSM, J. High Energy Phys. 03 (2004) 012.

[9] H. K. Dreiner, S. Heinemeyer, O. Kittel, U. Langenfeld, A. M. Weber, and G. Weiglein, Mass bounds on a very light neutralino, Eur. Phys. J. C **62**, 547 (2009).

[10] L. Calibbi, T. Ota, and Y. Takanishi, Light neutralino in the MSSM: A playground for dark matter, flavor physics and collider experiments, J. High Energy Phys. 07 (2011) 013.

[11] H. K. Dreiner, J. S. Kim, and O. Lebedev, First LHC constraints on neutralinos, Phys. Lett. B **715**, 199 (2012).

[12] B. Ananthanarayan, J. Lahiri, P. N. Pandita, and M. Patra, Invisible decays of the lightest Higgs boson in supersymmetric models, Phys. Rev. D **87**, 115021 (2013).

[13] L. Calibbi, J. M. Lindert, T. Ota, and Y. Takanishi, Cornering light neutralino dark matter at the LHC, J. High Energy Phys. 10 (2013) 132.

[14] G. Bélanger, G. Drieu La Rochelle, B. Dumont, R. M. Godbole, S. Kraml, and S. Kulkarni, LHC constraints on light neutralino dark matter in the MSSM, Phys. Lett. B **726**, 773 (2013).

[15] T. Han, Z. Liu, and S. Su, Light neutralino dark matter: Direct/indirect detection and collider searches, J. High Energy Phys. 08 (2014) 093.

[16] G. Belanger, D. Ghosh, R. Godbole, and S. Kulkarni, Light stop in the MSSM after LHC Run 1, J. High Energy Phys. 09 (2015) 214.

[17] K. Hamaguchi and K. Ishikawa, Prospects for Higgs- and Z-resonant neutralino dark matter, Phys. Rev. D **93**, 055009 (2016).

[18] J. Cao, Y. He, L. Shang, W. Su, and Y. Zhang, Testing the light dark matter scenario of the MSSM at the LHC, J. High Energy Phys. 03 (2016) 207.

[19] R. K. Barman, G. Belanger, B. Bhattacherjee, R. Godbole, G. Mendiratta, and D. Sengupta, Invisible decay of the Higgs boson in the context of a thermal and nonthermal relic in MSSM, Phys. Rev. D **95**, 095018 (2017).

[20] G. Pozzo and Y. Zhang, Constraining resonant dark matter with combined LHC electroweakino searches, Phys. Lett. B **789**, 582 (2019).

[21] P. Athron *et al.* (GAMBIT Collaboration), Combined collider constraints on neutralinos and charginos, Eur. Phys. J. C **79**, 395 (2019).

[22] K. Wang and J. Zhu, Funnel annihilations of light dark matter and the invisible decay of the Higgs boson, Phys. Rev. D **101**, 095028 (2020).

[23] R. Kumar Barman, G. Belanger, and R. M. Godbole, Status of low mass LSP in SUSY, Eur. Phys. J. Special Topics **229**, 3159 (2020).

[24] M. Van Beekveld, W. Beenakker, M. Schutten, and J. De Wit, Dark matter, fine-tuning and $(g-2)_\mu$ in the pMSSM, SciPost Phys. **11**, 049 (2021).

[25] G. Aad *et al.* (ATLAS Collaboration), Search for Heavy Higgs Bosons Decaying into Two Tau Leptons with the ATLAS Detector using pp Collisions at $\sqrt{s} = 13$ TeV, Phys. Rev. Lett. **125**, 051801 (2020).

[26] A. M. Sirunyan *et al.* (CMS Collaboration), Search for supersymmetry in final states with two oppositely charged same-flavor leptons and missing transverse momentum in proton-proton collisions at $\sqrt{s} = 13$ TeV, J. High Energy Phys. 04 (2021) 123.

[27] G. Aad *et al.* (ATLAS Collaboration), Search for chargino–neutralino pair production in final states with three leptons and missing transverse momentum in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector, Eur. Phys. J. C **81**, 1118 (2021).

[28] G. Aad *et al.* (ATLAS Collaboration), Search for charginos and neutralinos in final states with two boosted hadronically decaying bosons and missing transverse momentum in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, Phys. Rev. D **104**, 112010 (2021).

[29] CMS Collaboration, Search for electroweak production of charginos and neutralinos at $\sqrt{s} = 13$ TeV in final states containing hadronic decays of WW, WZ, or WH and missing transverse momentum, Phys. Lett. B **842**, 137460 (2023).

[30] G. Aad *et al.* (ATLAS Collaboration), Search for invisible Higgs-boson decays in events with vector-boson fusion signatures using 139 fb^{-1} of proton-proton data recorded by the ATLAS experiment, J. High Energy Phys. 08 (2022) 104.

[31] E. Aprile *et al.* (XENON Collaboration), Dark Matter Search Results from a One Ton-Year Exposure of XENON1T, Phys. Rev. Lett. **121**, 111302 (2018).

[32] E. Aprile *et al.* (XENON Collaboration), Constraining the Spin-Dependent WIMP-Nucleon Cross Sections with XENON1T, Phys. Rev. Lett. **122**, 141301 (2019).

[33] C. Amole *et al.* (PICO Collaboration), Dark matter search results from the complete exposure of the PICO-60 C_3F_8 bubble chamber, Phys. Rev. D **100**, 022001 (2019).

[34] Y. Meng *et al.* (PandaX-4T Collaboration), Dark Matter Search Results from the PandaX-4T Commissioning Run, Phys. Rev. Lett. **127**, 261802 (2021).

[35] J. Aalbers *et al.*, First dark matter search results from the LUX-ZEPLIN (LZ) experiment, arXiv:2207.03764.

[36] Z. Huang *et al.* (PandaX Collaboration), Constraints on the axial-vector and pseudo-scalar mediated WIMP-nucleus interactions from PandaX-4T experiment, Phys. Lett. B **834**, 137487 (2022).

[37] B. Abi *et al.* (Muon g-2 Collaboration), Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm, Phys. Rev. Lett. **126**, 141801 (2021).

[38] M. Carena, J. Osborne, N. R. Shah, and C. E. M. Wagner, Supersymmetry and LHC missing energy signals, Phys. Rev. D **98**, 115010 (2018).

[39] K. Olive, Review of particle physics, Chin. Phys. C **38**, 090001 (2014).

[40] S. Heinemeyer, W. Hollik, and G. Weiglein, FeynHiggs: A program for the calculation of the masses of the neutral CP even Higgs bosons in the MSSM, Comput. Phys. Commun. **124**, 76 (2000).

[41] S. Heinemeyer, W. Hollik, and G. Weiglein, The masses of the neutral CP—even Higgs bosons in the MSSM: Accurate analysis at the two loop level, Eur. Phys. J. C **9**, 343 (1999).

[42] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, and G. Weiglein, Towards high precision predictions for the MSSM Higgs sector, Eur. Phys. J. C **28**, 133 (2003).

[43] M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, and G. Weiglein, The Higgs boson masses and mixings of the complex MSSM in the Feynman-diagrammatic approach, J. High Energy Phys. 02 (2007) 047.

[44] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, and G. Weiglein, High-Precision Predictions for the Light CP-Even Higgs Boson Mass of the Minimal Supersymmetric Standard Model, Phys. Rev. Lett. **112**, 141801 (2014).

[45] H. Bahl and W. Hollik, Precise prediction for the light MSSM Higgs boson mass combining effective field theory and fixed-order calculations, Eur. Phys. J. C **76**, 499 (2016).

[46] H. Bahl, S. Heinemeyer, W. Hollik, and G. Weiglein, Reconciling EFT and hybrid calculations of the light MSSM Higgs-boson mass, Eur. Phys. J. C **78**, 57 (2018).

[47] H. Bahl, T. Hahn, S. Heinemeyer, W. Hollik, S. Paßehr, H. Rzehak, and G. Weiglein, Precision calculations in the MSSM Higgs-boson sector with FeynHiggs 2.14, Comput. Phys. Commun. **249**, 107099 (2020).

[48] The input parameters are read and written in the Supersymmetry Les Houches Accord (SLHA) file as on-shell parameters by the FeynHiggs code. We find no significant changes in our result when we use a different spectrum generator, like `SoftSUSY-4.1.17`, which provides the output SLHA with $\overline{DR}$ parameters.

[49] G. Aad *et al.* (ATLAS, CMS Collaboration), Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments, Phys. Rev. Lett. **114**, 191803 (2015).

[50] J. E. Camargo-Molina, B. O'Leary, W. Porod, and F. Staub, Stability of the CMSSM against sfermion VEVs, J. High Energy Phys. 12 (2013) 103.

[51] D. Chowdhury, R. M. Godbole, K. A. Mohan, and S. K. Vempati, Charge and color breaking constraints in MSSM after the Higgs discovery at LHC, J. High Energy Phys. 02 (2014) 110; 03 (2018) 149(E).

[52] N. Blinov and D. E. Morrissey, Vacuum stability and the MSSM Higgs mass, J. High Energy Phys. 03 (2014) 106.

[53] S. Schael *et al.* (ALEPH, DELPHI, L3, OPAL, SLD Collaborations, LEP Electroweak Working Group, SLD Electroweak and Heavy Flavour Groups), Precision electroweak measurements on the Z resonance, Phys. Rep. **427**, 257 (2006).

[54] G. Abbiendi *et al.* (OPAL Collaboration), Search for chargino and neutralino production at $\sqrt{s} = 192$-209 GeV at LEP, Eur. Phys. J. C **35**, 1 (2004).

[55] Y. Amhis *et al.* (HFLAV Collaboration), Averages of b-hadron, c-hadron, and τ-lepton properties as of summer 2016, Eur. Phys. J. C **77**, 895 (2017).

[56] V. Khachatryan *et al.* (CMS, LHCb Collaborations), Observation of the rare $B_s^0 \to \mu^+\mu^-$ decay from the combined analysis of CMS and LHCb data, Nature (London) **522**, 68 (2015).

[57] K. Hara *et al.* (Belle Collaboration), Evidence for $B^- - > \tau^- \bar{\nu}$ with a semileptonic tagging method, Phys. Rev. D **82**, 071101 (2010).

[58] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov, MicrOMEGAS: Version 1.3, Comput. Phys. Commun. **174**, 577 (2006).

[59] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov, MicrOMEGAS 2.0: A program to calculate the relic density of dark matter in a generic model, Comput. Phys. Commun. **176**, 367 (2007).

[60] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov, Dark matter direct detection rate in a generic model with MicrOMEGAS 2.2, Comput. Phys. Commun. **180**, 747 (2009).

[61] G. Belanger, F. Boudjema, P. Brun, A. Pukhov, S. Rosier-Lees, P. Salati, and A. Semenov, Indirect search for dark matter with MicrOMEGAS2.4, Comput. Phys. Commun. **182**, 842 (2011).

[62] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov, MicrOMEGAS_3: A program for calculating dark matter observables, Comput. Phys. Commun. **185**, 960 (2014).

[63] G. Belanger, A. Mjallal, and A. Pukhov, Recasting direct detection limits within MicrOMEGAS and implication for non-standard dark matter scenarios, Eur. Phys. J. C **81**, 239 (2021).

[64] P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak, and G. Weiglein, HiggsSignals: Confronting arbitrary Higgs sectors with measurements at the Tevatron and the LHC, Eur. Phys. J. C **74**, 2711 (2014).

[65] O. Stål and T. Stefaniak, Constraining extended Higgs sectors with HiggsSignals, Proc. Sci. EPS-HEP2013 (**2013**) 314 [arXiv:1310.4039].

[66] P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak, and G. Weiglein, Probing the standard model with Higgs signal rates from the Tevatron, the LHC and a future ILC, J. High Energy Phys. 11 (2014) 039.

[67] P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein, and K. E. Williams, HiggsBounds: Confronting arbitrary Higgs sectors with exclusion bounds from LEP and the tevatron, Comput. Phys. Commun. **181**, 138 (2010).

[68] P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein, and K. E. Williams, HiggsBounds 2.0.0: Confronting neutral and charged Higgs sector predictions with exclusion bounds from LEP and the tevatron, Comput. Phys. Commun. **182**, 2605 (2011).

[69] P. Bechtle, O. Brein, S. Heinemeyer, O. Stal, T. Stefaniak, G. Weiglein, and K. Williams, Recent developments in HiggsBounds and a preview of HiggsSignals, Proc. Sci. CHARGED2012 (**2012**) 024 [arXiv:1301.2345].

[70] P. Bechtle, O. Brein, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein, and K. E. Williams, HiggsBounds − 4: Improved tests of extended Higgs sectors against exclusion bounds from LEP, the tevatron and the LHC, Eur. Phys. J. C **74**, 2693 (2014).

[71] P. Bechtle, S. Heinemeyer, O. Stal, T. Stefaniak, and G. Weiglein, Applying exclusion likelihoods from LHC searches to extended Higgs sectors, Eur. Phys. J. C **75**, 421 (2015).

[72] ATLAS Collaboration, Combination of searches for invisible Higgs boson decays with the ATLAS experiment, Technical Report No. ATLAS-CONF-2020-052, CERN, Geneva, 2020.

[73] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. **641**, A6 (2020).

[74] S. Kraml, S. Kulkarni, U. Laa, A. Lessa, W. Magerl, D. Proschofsky-Spindler, and W. Waltenberger, SModelS: A tool for interpreting simplified-model results from the LHC and its application to supersymmetry, Eur. Phys. J. C **74**, 2868 (2014).

[75] F. Ambrogi, S. Kraml, S. Kulkarni, U. Laa, A. Lessa, V. Magerl, J. Sonneveld, M. Traub, and W. Waltenberger, SModelS v1.1 user manual: Improving simplified model constraints with efficiency maps, Comput. Phys. Commun. **227**, 72 (2018).

[76] J. Dutta, S. Kraml, A. Lessa, and W. Waltenberger, SModelS extension with the CMS supersymmetry search results from Run 2, Lett. High Energy Phys. **1**, 5 (2018).

[77] J. Heisig, S. Kraml, and A. Lessa, Constraining new physics with searches for long-lived particles: Implementation into SModelS, Phys. Lett. B **788**, 87 (2019).

[78] F. Ambrogi *et al.*, SModelS v1.2: Long-lived particles, combination of signal regions, and other novelties, Comput. Phys. Commun. **251**, 106848 (2020).

[79] C. K. Khosa, S. Kraml, A. Lessa, P. Neuhuber, and W. Waltenberger, SModelS database update v1.2.3, Lett. High Energy Phys. **2020**, 158 (2020).

[80] G. Alguero, S. Kraml, and W. Waltenberger, A SModelS interface for pyhf likelihoods, Comput. Phys. Commun. **264**, 107909 (2021).

[81] G. Alguero, J. Heisig, C. Khosa, S. Kraml, S. Kulkarni, A. Lessa, H. Reyes-González, W. Waltenberger, and A. Wongel, Constraining new physics with SModelS version 2, J. High Energy Phys. 08 (2022) 068.

[82] Nonstandard cosmological scenario can impact the results of the current study. We are performing a detailed study of the possibility for a nonstandard cosmological scenario in a future work [83].

[83] R. K. Barman, G. Belanger, B. Bhattacherjee, R. Godbole, and R. Sengupta, Current status of the light neutralino dark matter in the Z and h funnels in MSSM (to be published).

[84] Note that the contribution of h/H bosons to the SI DD cross section involves the product of couplings $g_{h\tilde{\chi}_1^0\tilde{\chi}_1^0} \times g_{hqq}$ and $g_{H\tilde{\chi}_1^0\tilde{\chi}_1^0} \times g_{Hqq}$, where q can be both up- and down-type quarks. The magnitudes and signs of $g_{h\tilde{\chi}_1^0\tilde{\chi}_1^0}$, $g_{H\tilde{\chi}_1^0\tilde{\chi}_1^0}$ couplings depend on μ and $\tan\beta$. The g_{Huu} and g_{Hdd} couplings depend on $\tan\beta$ with a relative sign difference between them.

[85] A smaller R value indicates that the parameter space point lies way outside the current limit, whereas an R value close to 1 indicates that it lies just on the border.

[86] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.131.011802 for the analyses sensitive for light higgsinos and benchmarks, details of the XGBOOST analysis, and prospects at future lepton colliders, which includes Refs. [87–89].

[87] D. M. Asner *et al.*, ILC Higgs White Paper, in *Community Summer Study 2013: Snowmass on the Mississippi* (2013), p. 10, arXiv:1310.0763.

[88] F. An *et al.*, Precision Higgs physics at the CEPC, Chin. Phys. C **43**, 043002 (2019).

[89] M. Carena, A. de Gouvea, A. Freitas, and M. Schmitt, Invisible Z boson decays at e^+e^- colliders, Phys. Rev. D **68**, 113007 (2003).

[90] D. Dercks, N. Desai, J. S. Kim, K. Rolbiecki, J. Tattersall, and T. Weber, CheckMATE 2: From the model to the limit, Comput. Phys. Commun. **221**, 383 (2017).

[91] M. P. Giordani, Beyond the standard model physics at the tevatron, J. Phys. Conf. Ser. **53**, 329 (2006).

[92] XGBOOST Documentation. https://xgboost.readthedocs.io/en/stable/.

[93] A. Adhikary, N. Chakrabarty, I. Chakraborty, and J. Lahiri, Probing the $H^{\pm}W^{\mp}Z$ interaction at the high energy upgrade of the LHC, Eur. Phys. J. C **81**, 554 (2021).