Erratum: Oriented Active Solids [Phys. Rev. Lett. 123, 238001 (2019)]

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Part of the analysis of two-dimensional active, polar elastomers in this Letter was incorrect, and holds only in the limit in which the material does not move on average, i.e., $v_0 = 0$. In this Erratum, we present the analysis for the motile case and amend an incorrect statement in our conclusion (ii) that polar active elastomers, unlike solids driven in an externally imposed direction, do not suffer a transverse buckling instability. In fact, they do; the effects of rotation invariance are more subtle than we realized. As described before Eq. (4), $\hat{p}_\perp \cdot \mathbf{v} = \dot{u}_y - v_0\theta$; the second contribution is *not* eliminated by transforming to a frame moving with velocity $v_0\hat{x}$, and gives rise to contributions $\propto \partial_y u_x$ and $\partial_x u_y$ when θ is eliminated in favor of gradients of the displacement fields. Therefore, the corrected version of Eq. (4) is much closer to the dynamics of solids in an *external* rotation symmetry-breaking field [1]:

$$\dot{u}_x = a_1\partial_x u_x + a_2\partial_y u_y + b_1\partial_x^2 u_x + b_2\partial_y^2 u_x + b_3\partial_x\partial_y u_y \tag{1a}$$

$$\dot{u}_y = a_3\partial_x u_y + a_4\partial_y u_x + b_4\partial_x^2 u_y + b_5\partial_y^2 u_y + b_6\partial_x\partial_y u_x, \tag{1b}$$

where $a_3 = (v_0/2)(\beta^{-1} + 1)$ and $a_4 = (v_0/2)(\beta^{-1} - 1)$. Rotation invariance is encoded in the fact that a_3 and a_4 are not independent parameters; they arise purely kinematically (compare with the conserved current in [2,3]). In this sense, the detailed dynamics remains distinct from that of an externally driven solid, which has no such constraint.

The eigenfrequencies implied by the corrected dynamical equations (1) for a perturbation with a wave vector along an arbitrary direction are now far more complicated than those displayed in Eq. (5); for perturbations along and transverse to the ordering direction

$$\omega_\pm = \begin{array}{ll} \pm\sqrt{a_2a_4}\,q_y - \frac{i(b_2+b_5)}{2}q_y^2 & \text{for } q_x = 0 \\ \left.\begin{array}{l} -a_1q_x - ib_1q_x^2 \\ -a_3q_x - ib_4q_x^2 \end{array}\right\} & \text{for } q_y = 0 \end{array}. \tag{2}$$

Importantly, to leading order in q, $\omega_\pm \sim q$ along *all* wave vector directions: $\omega_\pm = (1/2)[(a_1 + a_3)q_x \pm \sqrt{(a_1 - a_3)^2q_x^2 + 4a_2a_4q_y^2}]$.

As discussed in [1], when $a_2a_4 > 0$, this leads to propagating modes in this system without conventional inertia because motility induces an effective nonreciprocal coupling between u_x and u_y. These modes are damped at $\mathcal{O}(q^2)$ as long as the purely active coefficients $b_2, b_4 > 0$. The conclusion that the static structure factor of displacement fluctuations scales as $1/q^2$ in all directions, when the solid is stable, remains correct. When either b_2 or b_4 is negative, the elastomer is linearly unstable with a growth rate $\propto q^2$, as we had correctly predicted.

When $a_2a_4 < 0$, the active elastomeric state suffers a transverse buckling instability with a growth rate $\propto q$ [1]. $a_2a_4 = 0$ is an exceptional point [4] which can be accessed most easily by setting $a_4 = 0$. Since rotation invariance implies that when $a_4 = 0$, $a_3 = 0$ as well, we had, in effect, explored the physics of this exceptional point in this Letter.

Our other conclusions regarding stability and the influence of nonlinearities remain correct.

[1] R. Lahiri and S. Ramaswamy, Phys. Rev. Lett. **79**, 1150 (1997).
[2] S. Ramaswamy, J. Toner, and J. Prost, Phys. Rev. Lett. **84**, 3494 (2000).
[3] A. Maitra, P. Srivastava, M. Rao, and S. Ramaswamy, Phys. Rev. Lett. **112**, 258101 (2014).
[4] M. Fruchart, Ryo Hanai, Peter B. Littlewood, and Vincenzo Vitelli, Nature (London) **592**, 363 (2021).

