Multipartite Nonlocality in Clifford Networks

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We adopt a resource-theoretic framework to classify different types of quantum network nonlocality in terms of operational constraints placed on the network. One type of constraint limits the parties to perform local Clifford gates on pure stabilizer states, and we show that quantum network nonlocality cannot emerge in this setting. Yet, if the constraint is relaxed to allow for mixed stabilizer states, then network nonlocality can indeed be obtained. We additionally show that bipartite entanglement is sufficient for generating all forms of quantum network nonlocality when allowing for postselection, a property analogous to the universality of bipartite entanglement for generating all forms of multipartite entangled states.

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The Gottesman-Knill theorem is a classic result that enables a wide class of quantum algorithms to be efficiently simulated [1–3]. It says that circuits constructed using only gates belonging to the Clifford group can be efficiently simulated on a classical computer using the stabilizer formalism. It is interesting to investigate whether Clifford quantum information processing has similar limitations for other quantum information tasks and phenomena, such as the emergence of quantum nonlocality. It has already been shown [4–7] that such limitations do indeed exist in specific scenarios, but the role of Clifford operations in the emergence of network nonlocality has remained relatively unexplored.

The standard Bell nonlocality scenario consists of multiple parties having access to some globally shared entangled state, and they each select different measurements to perform on their respective subsystems [8]. In the network setting, the globally shared entanglement is replaced by independent sources of entanglement that get distributed according to the structure of the network [9–13]. Examples of nonlocality in networks have recently been found that appear to be fundamentally different than the nonlocality emerging in traditional Bell scenarios [14–19], although how to best articulate this difference is unclear.

To shed light on this question, we begin here by sketching a "top-down" framework for the general study of quantum nonlocality (or "nonclassicality"), inspired by the philosophy of quantum resource theories [20,21]. In this framework, different classes of nonlocality naturally emerge on a quantum network after placing different restrictions on the type of states that are "free" to distribute across the network, as well as the type of local operations that the parties are free to perform. This allows us to view different notions of nonlocality proposed in the literature under a common conceptual lens. As our main result, we show later in this Letter that quantum nonlocality can never be realized whenever the parties are restricted to Clifford operations and the free states are stabilizer states.

An operational framework for network nonlocality.— Consider an N-party quantum network whose structure is defined by a hypergraph $\mathcal{G} = (V, E)$ with a disjoint hyperedge set $E \subset 2^V$. Each vertex $v \in V$ represents a quantum system, and each hyperedge $e \subset V$ represents an independent quantum source that generates a joint state ρ_e for all $|e|$ systems in e. The network structure also specifies which quantum systems are received locally by each party. We let $A_i \subset E$ denote the collection of hyperedges connected to party A_i (see Fig. 1). In any protocol for generating network correlations, each party A_i applies a quantum channel $\mathcal{E}_i$ jointly across all its received systems, thereby connecting the previously disjoint hyperedges. The parties then measure their systems in the computational basis, and the collective output is the N-tuple $(a_1, \ldots, a_N)$ of measured

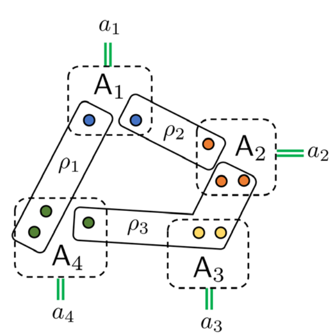

FIG. 1. A 4-party quantum network partitioned into disjoint hyperedges. Each node represents a different quantum system and each hyperedge represents a quantum state shared among the constituent nodes. Each party A_i has local control of certain systems.

values with probability distribution $p(a_1, \ldots, a_N)$. In summary, every network correlation we consider can be generated using the following three-step procedure: (i) For each hyperedge $e \in E$ in the network, a multipartite state ρ_e is distributed across the network; (ii) Each party A_i performs a local processing channel $\mathcal{E}_i$ on the systems under its control; (iii) Each party A_i measures its post-processed system in the computational basis and outputs the outcome a_i.A distribution $p(a_1, \ldots, a_N)$ built in this way will be called a *quantum network correlation*.

Since this framework is designed to study quantum nonlocality, one may want to generalize the setup and grant each party A_i an input variable x_i that controls the local processing performed. In this case, the generated correlation would be a conditional distribution $p(a_1, \ldots, a_N|x_1, \ldots, x_N)$, which is typically the object of consideration in the study of Bell nonlocality. However, in the network scenario, the distinction between correlations with inputs versus correlations without inputs is superficial. This is due to the work of Fritz [22], who showed how every correlation generated on some network with inputs can be equivalently generated on an enlarged network without inputs; essentially the local input variable x_i of party A_i becomes an independent variable shared between A_i and some new party on the enlarged network. Therefore, without loss of generality, we restrict to correlations with no inputs.

With this model in place, we can now identify different classes of quantum correlations by imposing different constraints on the distributed states ρ_e and the types of local maps $\mathcal{E}_i$. We begin with the set of local correlations defined with respect to a given network $\mathcal{G} = (V, E)$. A network local correlation is any distribution that can be generated using a shared classical variable λ_e for each hyperedge e. If $p(\lambda_e)$ denotes the probability distribution over variable λ_e, then every local correlation has the form

$$p(a_1, \ldots, a_N) = \sum_{\vec{\lambda}} p(\vec{\lambda}) \prod_{i=1}^{N} p(a_i| \cup_{e\in A_i} \lambda_e), \qquad (1)$$

where the sum is over each sequence $\vec{\lambda} = (\lambda_e)_e$ of the $|E|$ independent random variables, $p(\vec{\lambda}) = \prod_{e\in E} p(\lambda_e)$, and $p(a_i| \cup_{e\in A_i} \lambda_e)$ is a local classical channel of party A_i. If a quantum network correlation does not have this form, then it is called *quantum network nonlocal*. We claim that the set of local correlations is precisely those that can be generated via steps (i)–(iii) above under the constraint that each $\mathcal{E}_i$ satisfies the condition

$$\Delta \circ \mathcal{E}_i = \Delta \circ \mathcal{E}_i \circ \Delta, \qquad (2)$$

where $\Delta(\cdot) = \sum_x |x\rangle\langle x|(\cdot)|x\rangle\langle x|$ is the completely dephasing map in the computational basis. This condition has previously been called *resource nonactivating* [23], but in this work we will say that any completely positive trace-preserving (CPTP) map satisfying Eq. (2) is *classically simulatable*. Indeed, any correlation $p(a_1, \ldots, a_N)$ generated using classically simultable maps $\mathcal{E}_i$ can be simulated on the same network $\mathcal{G}$ using purely classical resources. Since the parties measure their qubits in the computational basis after applying $\otimes_i \mathcal{E}_i$ in step (ii), Eq. (2) says that the parties could equivalently also dephase the shared state $\otimes_e \rho_e$ prior to step (ii), thereby converting each ρ_e into a classically correlated state $\hat{\rho}_e = \sum_{\mathbf{x}} p(\mathbf{x})|\mathbf{x}\rangle\langle\mathbf{x}|$, where $|\mathbf{x}\rangle = |x_1\rangle \cdots |x_{|e|}\rangle$. Conversely, any network correlation $p(a_1, \ldots, a_N)$ generated using classical shared randomness can also be produced using classically correlated states and CPTP maps of the form $\mathcal{E}_i(\cdot) = \sum_{a_i} |a_i\rangle\langle a_i|\mathrm{Tr}[\Pi_{a_i}(\cdot)]$, where

$$\Pi_{a_i} = \sum_{e\in A_i} \sum_{\lambda_e} p(a_i| \cup \lambda_e) \bigotimes_{e\in A_i} |\lambda_e\rangle\langle\lambda_e|.$$

Another type of constraint limits each $\mathcal{E}_i$ to be a local quantum wiring map for party A_i [24–26], which can be understood as a local operations and classical communication (LOCC) transformation performed on the separated systems that A_i receives from different sources. Under this constraint, A_i is prohibited from performing entangling measurements across the different states it receives, such as in entanglement swapping. Nevertheless, these operations are still strong enough to generate nonlocal correlations when seeded with at least one entangled state ρ_e on the network. All "standard" Bell tests—such as the celebrated violations of the Clauser-Horne-Shimony-Holt (CHSH) inequality [27–29]—can be implemented under the restriction of local quantum wiring maps. In contrast, network correlations not producible by local quantum wiring maps can be defined as possessing *genuine network nonlocality* [24], much in the same way that quantum states not producible by LOCC are defined as possessing entanglement [30]. This definition is motivated by the fact that to produce genuine quantum network nonlocality, the parties must be able to truly leverage the network structure and stitch together the different quantum sources through nonclassical local interactions.

To provide a complete account of the different notions of network nonlocality proposed in the literature, we consider one final type of correlation. For a fixed subgraph of a given network, one could require that at least one of the constituent parties performs a classically simulatable operation $\mathcal{E}_i$. Nonlocal correlations that cannot be generated under this restriction are said to possess *full network nonlocality* with respect to the particular subgraph [18].

We now enjoin the main result of this Letter to the picture. Let $p(a_1, \ldots, a_n)$ be any quantum network correlation built under the constraint that all ρ_e are pure stabilizer states and the $\mathcal{E}_i$ are Clifford gates. Then $p(a_1, \ldots, a_n)$ is a network local distribution. The rest of this Letter will be devoted to explaining this result and its proof in more

TABLE I. Different classes of classical correlations emerging on a quantum network based on the operational constraints.

Operational constraint	Inaccessible type of network correlation
$\mathcal{E}_i$ are classically simultable	Network nonlocality
$\mathcal{E}_i$ are local quantum wiring maps	Genuine network nonlocality [24]
$\mathcal{E}_i$ are classically simultable for at least one party in a given subgraph	Full network nonlocality for the given subgraph [18]
$\mathcal{E}_i$ are Clifford gates and ρ_e are pure stabilizer states	Network nonlocality

detail; technical steps are postponed to the Supplemental Material [31]. Table I summarizes the different types of network correlations in the context of the operational framework outlined above.

k-network nonlocality.—Before specializing to Clifford networks, let us first sharpen the notion of local correlations to reflect even better the structure of the network. Let us say that a k network is any hypergraph with disjoint edge set in which every quantum source is connected to no more than k parties; i.e. $|\{i|e \cap A_i \neq \emptyset\}| \leq k$ for all $e \in E$. An N-party distribution $p(a_1, \ldots, a_N)$ that can be built using classically simulatable operations on some k-network will be called *k-network local*; otherwise it will be called *quantum k-network nonlocal*.

It turns out that every instance of quantum k-network nonlocality can be obtained from a quantum 2-network nonlocal correlation through postselection.

Proposition 1.—Suppose that $p(a_1, \ldots, a_N)$ is a quantum k-network nonlocal correlation for N parties. Then there exists an $(N+r)$-party correlation $\hat{p}(a_1, \ldots, a_N, c_1, \ldots, c_r)$ that is quantum 2-network nonlocal and satisfies

$$p(a_1, \ldots, a_N) = \hat{p}(a_1, \ldots, a_N | c_1 = 0, \ldots, c_r = 0). \quad (3)$$

In other words, conditioned on the new parties $\mathsf{C}_1, \ldots, \mathsf{C}_r$ having the all-zero output, the other N parties reproduce the original distribution p.

To prove this proposition one replaces every k-element hyperedge e with k bipartite edges and then uses bipartite entanglement distributed on these edges to teleport the original state ρ_e. We remark that our reduction to k-network nonlocality from 2-network nonlocality via postselection is specific to quantum networks, as it relies on quantum teleportation. It is an interesting question whether such a reduction can be found for general nonsignaling network correlations [32,33] or whether this is a uniquely quantum feature.

Clifford networks.—Let us now turn to the notion of Clifford quantum networks, which can be understood as distributed Clifford circuits [1]. We begin by recalling the definitions of stabilizers and stabilizer operations [1,34,35]. Let $\mathcal{P}_n$ denote the n-qubit Pauli group of operators, $\mathcal{P}_n = \{\pm 1, \pm i\} \times \{I, X, Y, Z\}^{\otimes n}$. Expressions like X_2 express Pauli-X applied to qubit 2 and the identity applied to all other qubits. The n-qubit Clifford group consists of all unitaries that, up to an overall phase, leave $\mathcal{P}_n$ invariant under conjugation,

$$\mathcal{C}_n = \{U | U g U^\dagger \in \mathcal{P}_n \quad \forall\ g \in \mathcal{P}_n\}/U(1). \quad (4)$$

The set of n-qubit stabilizer states is defined as

$$\mathcal{S}_n = \{U|0\rangle | U \in \mathcal{C}_n\}. \quad (5)$$

Every stabilizer pure state $|\varphi\rangle \in \mathcal{S}_n$ can be uniquely identified as the $+1$ eigenstate of n independent commuting elements from $\mathcal{P}_n$. These operators generate a group, called a stabilizer group, that we denote by $\mathrm{stab}(|\varphi\rangle)$.

Consider now a k network $\mathcal{G} = (V, E)$ in which each vertex $v \in V$ represents a qubit system. The hyperedges E again form a disjoint partitioning of the vertex set and each $e \in E$ represents a multiqubit state. Suppose that party A_i receives a total of n_i qubits from the various sources. Using the three-step framework introduced above, we consider correlations formed under the following restrictions: (ic) For each hyperedge $e \in E$, a pure stabilizer state $|\varphi_e\rangle$ is distributed; (iic) Each party A_i introduces m_i ancilla qubits, each initialized in state $|0\rangle$, and performs a Clifford unitary gate V_i on all the $n_i + m_i$ qubits held locally. Like before, step (iii) involves a measurement of each qubit in the computational basis. This generates a classical output for each party that is a sequence of bits $\mathbf{b}_i = (b_1, b_2, \ldots, b_{n_i+m_i}) \in \mathbb{Z}_2^{n_i+m_i}$, one for each qubit used by party A_i in the protocol. Letting $n = \sum_{i=1}^N n_i$ and $m = \sum_{i=1}^N m_i$, the joint probability distribution for all measurements is then given by

$$\begin{aligned} p(\mathbf{b}_1, \ldots, \mathbf{b}_N) &= p(b_1, b_2, \ldots, b_{n+m}) \\ &= \frac{1}{2^{n+m}} \langle \omega | \bigotimes_{i=1}^{n+m} (\mathbb{I} + (-1)^{b_i} Z_i) | \omega \rangle. \end{aligned} \quad (6)$$

Every distribution $p(\mathbf{b}_1, \ldots, \mathbf{b}_N)$ having this form will be called a Clifford *k-network correlation*. As an example, a triangle Clifford network is depicted in Fig. 2. Its equivalent representation as a distributed quantum circuit is also shown.

Observe that the Clifford constraint and the classical simulatable constraint are inequivalent; i.e., there are Clifford gates U (such as Hadamard) whose corresponding CPTP map $\mathcal{U}(\cdot) = U(\cdot)U^\dagger$ fails to satisfy Eq. (2) and vice versa. Yet, on the level of network nonlocality, our result shows that Clifford operations fail to offer any nonclassical advantage.

Theorem 1.—Every Clifford k-network correlation is k-network local.

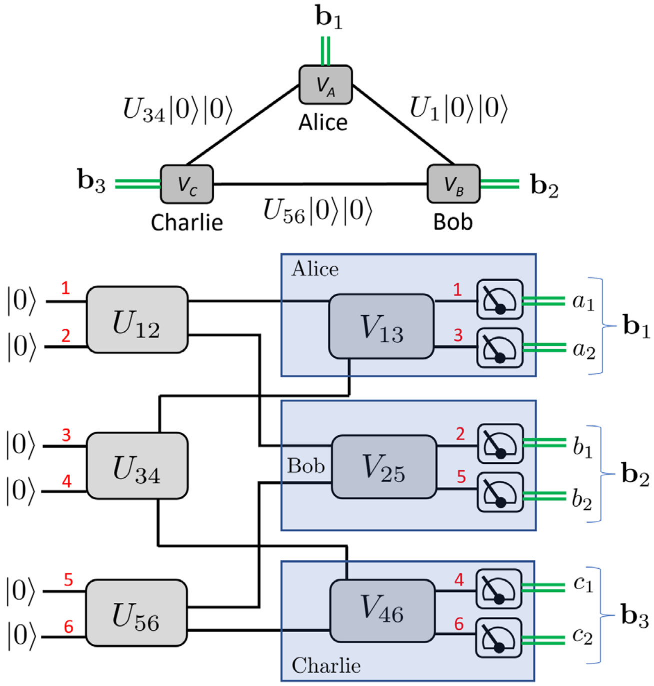

FIG. 2. The triangle network represented in the standard circuit model, with no ancilla qubits. The distribution over $(a_1, a_2, b_1, b_2, c_1, c_2)$ is a Clifford 2-network distribution if each of the U_e and V_i are Clifford gates.

The proof of this theorem begins by performing three simplifications. First, in Proposition 1 we established that every quantum k-network nonlocal distribution can be obtained from a quantum 2-network nonlocal distribution via postselection. Moreover, since teleportation is carried out using Pauli gates, the proof of Proposition 1 can be specialized to Clifford networks: if a quantum k-network nonlocal distribution is generated on a Clifford k network, then there exists a quantum 2-network nonlocal distribution generated on a Clifford 2-network. Hence, to prove Theorem 1, it suffices to show that measuring Clifford 2-network states always leads to 2-network local distributions. In other words, we can restrict $\mathcal{G}$ to just being a graph so that each $|\varphi_e\rangle$ is a bipartite stabilizer state. Second, recall the fact that every bipartite stabilizer state can be transformed into copies of $|\Phi^+\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$ and computational basis states $|0\rangle$ using local Cliffords [36]. Third, as proved in the Supplemental Material [31], the use of local ancilla provides no advantage for the purpose of generating 2-network nonlocality. Then it suffices to consider graphs $\mathcal{G}$ in which each node is connected to exactly one other node, the two nodes are held by different parties, and their connecting edge represents a maximally entangled state $|\Phi^+\rangle$ shared between them.

The next part of the proof involves constructing a local model for any graph having this structure and any choice of local stabilizer measurement. Our local model involves replacing each edge e on the graph with a random variable $\lambda_e = (\bar{X}_e, \bar{Y}_e, \bar{Z}_e)$, which is a trio of uniform independent random bits. Based on the local measurement A_i wants to perform and the values $\{\lambda_e\}_{e\in A_i}$, outputs $\mathbf{b}_i$ can be generated that jointly have the correct distribution $p(\mathbf{b}_1, \ldots, \mathbf{b}_N)$. Details of the model and a proof of its correctness are presented in the Supplemental Material [31].

Discussion.—In this Letter, we have presented a unifying framework for the general study of quantum network nonlocality. We believe this framework can help clarify what types of quantum resources are needed to generate different forms of nonlocality. It also helps draw a trifold connection between nonlocality, quantum resource theories, and quantum computation, as shown in Fig. 2. When any protocol is fully decomposed as a distributed quantum circuit, we found that some non-Clifford operation is needed to generate nonlocal correlations.

The importance of non-Clifford operations is further corroborated by the purported examples of genuine network nonlocality or full network nonlocality found in the literature [14–19]. In each of these examples some non-Clifford operation is needed to realize the nonlocality. Even more conspicuously, in the triangle network considered by Renou *et al.* in Ref. [14], nonlocality fails to emerge exactly when the parameters of their model coincided with a Clifford network. Our work formalizes the reason why this happens, and can thus be interpreted as a guide for what kind of resources are needed to generate network nonlocality.

One may feel that Theorem 1 is not that surprising given the Gottesman-Knill theorem for simulating Clifford circuits. However, let us highlight two reasons why the former stands independently of the latter, despite them sharing a kindred spirit. First, the Gottesman-Knill theorem provides a classical algorithm to correctly reproduce the outcome statistics when locally measuring a stabilizer state. However, in this algorithm, one must update the generators of the stabilizer after simulating the measurement of each qubit. This requires the communication of global information, which is forbidden in the network nonlocality model. Therefore, a completely new classical model is needed for a *distributed* simulation, which is what Theorem 1 provides.

Second, Theorem 1 breaks down when relaxing certain operational restrictions whereas the Gottesman-Knill theorem does not. Specifically, suppose we relax condition (ic) by allowing for *mixed* stabilizer states; i.e., convex combinations of stabilizer states whose purification is no longer a stabilizer state. Remarkably, now it becomes possible to violate Theorem using so-called "disguised" Bell nonlocality [13]. The construction is presented in the Supplemental Material [31] and involves extending Mermin's magic square game [38] to the network setting using a slight modification of Fritz's construction. This result highlights the strong nonconvexity that emerges in network nonlocality: local models can be constructed for all pure stabilizer states, but this fails to be possible when taking their mixture. It is tempting to think that the classical randomness in the mixed state ρ_e could just be absorbed into the classical randomness distributed on edge e. However, this does not always work since the local classical strategy for some party A_i in simulating a Clifford network distribution might depend on which state $|\varphi_e\rangle$ is seeded on edge e, even if A_i is not connected on e.

Similarly, one could consider relaxing condition (iic) by allowing for convex combinations of Clifford gates; i.e., $\mathcal{E} = \sum_k p_k U_k(\cdot) U_k^\dagger$ with each U_k being a Clifford. Then by a similar construction to the one used for mixed stabilizer states, it is possible to generate nonlocality on the network. The problem is that not every Clifford channel like $\mathcal{E}$ admits a unitary diltation that itself is Clifford, and thus Theorem 1 does not apply. In contrast, the Gottesman-Knill theorem still provides an efficient classical simulation algorithm for quantum circuits using mixed stabilizer states and random Clifford channels [39]. Ultimately, we hope that these findings and the framework described here help pinpoint the precise conditions necessary for generating nonlocal correlations on a quantum network.

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Note added.—Recently, we became aware of a similar model independently derived by Pusey in Ref. [4], although it does not directly account for network constraints and the emergence of network nonlocality [37].

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