Signatures of Quantum Gravity in the Gravitational Self-Interaction of Photons

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We propose relativistic tests of quantum gravity using the gravitational self-interaction of photons in a cavity. We demonstrate that this interaction results in a number of quantum gravitational signatures in the quantum state of the light that cannot be reproduced by any classical theory of gravity. We rigorously assess these effects using quantum parameter estimation theory and discuss simple measurement schemes that optimally extract their signatures. Crucially, the proposed tests are free of QED photon-photon scattering, are sensitive to the spin of the mediating gravitons, and can probe the locality of the gravitational interaction. These protocols provide a new avenue for studying the quantum nature of gravity in a relativistic setting.

Introduction.—The search for a theory of quantum gravity is one of the most significant open problems in contemporary physics. A key challenge in this pursuit is the absence of experimental results probing quantum gravitational effects. Remarkably, low-energy experiments may soon be capable of testing the quantum nature of the gravitational field [1–26]. In particular, proposals for observing for quantum correlations between masses induced by their gravitational interaction have seen significant development in the past five years [5,8–15,17,24–29]. The majority of these schemes consider gravitationally induced entanglement (GIE) as the key signature of a quantized gravitational field, following proposals by Bose *et al.* [2] and Marletto and Vedral [3]. More recent proposals have studied further signatures of quantum gravity that may be probed in ultracold atomic ensembles [14,15] and optomechanical systems [22,23,30,31].

While the prospect of tabletop tests of quantum gravity is extraordinary, there are significant challenges with realizing these proposals experimentally. There are the well-known technical challenges of creating and probing macroscopic superpositions of mesoscopic masses ($\sim 10^{-14}$ kg) in the presence of environmental decoherence [13,28], as well as the requirement that Casimir-Polder interactions between the masses must be suppressed [13,27]. Still, these challenges may be overcome within the next decade [32]. There are additional conceptual challenges concerning arguments that experiments that rely on the Newtonian gravitational interaction cannot truly attest to the quantum nature of the gravitational field [18,33–37]. These objections to GIE tests of quantum gravity have seen significant debate [10,12,18,33–43]—motivating the need for tests of quantum gravity beyond the Newtonian regime.

In this Letter, we propose a platform for fundamentally relativistic tests of quantum gravity, using the gravitational self-interaction of quantum light in a cavity. We demonstrate that there are multiple quantum gravitational signatures that can be extracted from the quantum state of the light, that cannot be reproduced by *any* classical theory of gravity. We provide a metrological analysis of the proposed tests and discuss challenges with their experimental realization. The proposed protocols are sensitive to the spin of the virtual gravitons that mediate the gravitational interaction between photons, are naturally free of QED photon-photon scattering, and can probe retardation effects to test the locality of the gravitational interaction. This Letter therefore provides a new approach for probing the quantum nature of the gravitational field including relativistic aspects that cannot be probed in the Newtonian regime.

Self-interaction of light due to gravity.—A sufficiently high-density electromagnetic (EM) field will experience a self-interaction mediated by the gravitational field: the energy density of light will curve spacetime, which in turn will affect the path of the light. This effect is well studied for freely propagating light, both classically [44–47] and for photons [48–53]. In the case of quantum light, the nature of this self-interaction will depend on whether or not the gravitational field is also quantized—providing a powerful avenue of probing the quantum nature of gravity with massless photons.

This gravitational self-interaction of light is well described within the framework of perturbative general relativity (GR), in which the spacetime metric is written as $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$ for a background (Minkowski) metric $\eta_{\mu\nu}$ and a perturbation $h_{\mu\nu}$. In the Lorenz gauge, this leads to the linearized Einstein field equations [54],

$$\Box \bar{h}^{\mu\nu} = -\frac{16\pi G}{c^4} T^{\mu\nu}_{(0)}, \tag{1}$$

where $\square = \nabla^2 - \partial_t^2/c^2$, $\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h_\alpha^\alpha$, and $T_{(0)}^{\mu\nu}$ is the stress-energy tensor of the matter source (the subscript indicates evaluation on the Minkowski background).

Here we consider the gravitational field sourced by an EM field, for which the stress-energy tensor is $\mu_0 T_{(0)}^{\mu\nu} = F^{\mu\lambda}F_\lambda^\nu - \eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}/4$, where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the EM field tensor in terms of the four-potential A^μ, and we have employed Einstein summation. Equation (1) then has the retarded Greens function solution:

$$h^{\mu\nu}(\boldsymbol{r}, t) = \frac{4G}{c^4} \int d\boldsymbol{r}' \frac{T_{(0)}^{\mu\nu}\left(\boldsymbol{r}', t - \frac{|\boldsymbol{r}-\boldsymbol{r}'|}{c}\right)}{|\boldsymbol{r} - \boldsymbol{r}'|}. \tag{2}$$

Note that as the EM stress energy tensor $T_{(0)}^{\mu\nu}$ is traceless, Eq. (2) implies $h_\alpha^\alpha = 0$.

The gravitational interaction energy can be treated within the linearized gravity framework starting from the interaction Lagrangian density $\mathcal{L}_{\text{int}} = h_{\mu\nu}T_{(0)}^{\mu\nu}/2$ [55], which corresponds to the interaction Hamiltonian density (Appendix A):

$$\mathcal{H}_{\text{int}}(\boldsymbol{r}) = \frac{1}{\mu_0} h^{\mu 0} F_{\mu\lambda}F_0^\lambda - \frac{1}{2}h^{\mu\nu}T_{\mu\nu}^{(0)}. \tag{3}$$

The self-interaction Hamiltonian is then obtained by substitution of Eq. (2) into $\int d^3\boldsymbol{r}\mathcal{H}_{\text{int}}(\boldsymbol{r})$, which is quartic in EM fields, nonlocal, and contains non-negligible retardation effects in general.

The quantization of the EM field gives a stress-energy tensor that is operator valued, i.e., $\hat{T}^{\mu\nu}$, which directly implies the quantization of the gravitational field via Eq. (1), i.e., $\hat{h}^{\mu\nu}$ [Fig. 1(a)]. The self-interaction term in the resulting Hamiltonian will then be quartic in bosonic creation and annihilation operators—e.g., $\hat{H}_{\text{int}} = \int d^3\boldsymbol{r}\mathcal{H}_{\text{int}}(\boldsymbol{r}) \sim \hat{a}^\dagger\hat{a}^\dagger\hat{a}\,\hat{a}$ for a single-mode field. In contrast, the self-interaction Hamiltonian for any *classical* theory of gravity—specifically, any theory for which $h^{\mu\nu}$ is not operator valued—may only be quadratic in creation or annihilation operators, at most. For example, in semiclassical gravity where $\hat{T}^{\mu\nu} \rightarrow \langle\hat{T}^{\mu\nu}\rangle$ in Eq. (2), we would have self-interaction Hamiltonian terms such as $\sim\langle\hat{a}^\dagger\hat{a}\rangle\hat{a}^\dagger\hat{a}$. Therefore, there will be distinct signatures of the quantum self-interaction that cannot be reproduced by any classical theory of gravity. We will explore these signatures both qualitatively and quantitatively in this Letter.

Crucially, the self-interaction is sensitive to the spin of the quantized gravitational mediator, unlike Newtonian interactions [41]. The self-interaction described by Eq. (3) specifically requires a spin-2 graviton—mediators with any other spin would correspond to a different tensorial structure to the gravity-light coupling, and thus give rise to a distinct interaction from $\hat{H}_{\text{int}}$ [41]. As the EM field has a traceless stress-energy tensor ($T = T_\mu^\mu = 0$), this
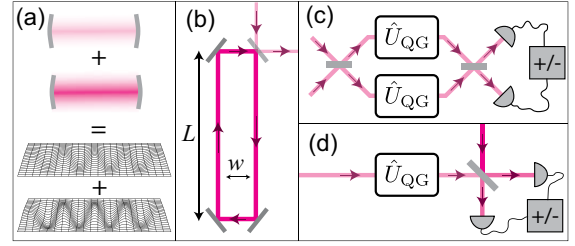

FIG. 1. (a) Superpositions of different photon numbers result in superpositions of spacetime geometries if gravity is quantum, resulting in quantum gravitational signatures in the state of the light. (b) Cavity geometry considered, where the dominant gravitational interaction is between the two long arms, $L \gg w$. (c),(d) Near-optimal measurement schemes for observing signatures of quantum gravity in (c) the interference between a pair of cavities or (d) non-Gaussianity in the phase quadratures of photons exiting a single cavity. $\hat{U}_{\text{QG}}$ represents the system in (b).

rules out mediation by spin-0 gravitons that couple solely to the trace T. This includes virtual spin-0 gravitons that appear in the canonical quantization of perturbative GR [56] and quantized scalar gravity theories, such as that of Nordström [41,57] or Brans-Dicke theory [58–60]. We note here the related proposals for realizing the quantum analog of gravitational light bending [31] and the photonic analog of the GIE tests [61], which can similarly discern between mediators of different spin.

Evading QED photon-photon scattering.—At large EM field densities, photons can also interact by exchange of virtual electron-positron pairs as described by quantum electrodynamics; for the optical frequencies we consider here this is typically many orders of magnitude stronger than the gravitational interaction [51]. To avoid this problem, we exploit two key properties of the QED photon-photon interaction: it vanishes for monochromatic plane waves [62,63] and is a local interaction. In contrast, the quantum gravitational self-interaction is *nonlocal*, and thus permits nonzero interactions in geometries where there is no QED self-interaction.

Here we consider a traveling-wave (ring) cavity of finesse $\mathcal{F}$ in a rectangular geometry [Fig. 1(b)], such that there is a gravitational self-interaction between spatially separated parts of the cavity mode. Specifically, we consider the case where the dominant interaction is between the counter-propagating light in the long arms each of length L, separated by a small distance $w \ll L$—the quantum analog of Ref. [64]. An appealing aspect of this system is that, by choice of optical polarization, both static and nonstatic gravitational perturbations can be studied. The latter case, where retardation effects will be significant, can explicitly test the locality of the interaction, a key aspect of quantum gravity inaccessible to tests with nonrelativistic sources [65]. In this Letter we will focus on the case of circularly polarized light, for which $T^{\mu\nu}$ is static; however, the qualitative features of the interaction will be unchanged for general polarizations.

Quantum gravitational Kerr effect.—First, let us consider the single-mode self-interaction of a cavity field of frequency ω_0 described by creation (annihilation) operators $\hat{a}$ ($\hat{a}^\dagger$). We will consider the multimode case where vacuum modes are included, shortly. Assuming the two long arms of the cavity are well separated such that diffraction effects can be neglected ($w \gg \lambda$), the leading contribution to the self-interaction Hamiltonian in the limit $L \gg w$ is given by (Appendix B)

$$\hat{H}_{\mathrm{QG}} = -\frac{16G}{L}\left(\frac{\hbar\omega_0}{c^2}\right)^2 \log\left(\frac{L}{w}\right)\hat{a}^\dagger\hat{a}^\dagger\hat{a}\,\hat{a}\,. \quad (4)$$

Applying this interaction Hamiltonian for the cavity interrogation time $\tau \approx 2\mathcal{F}L/c$ (assuming $L \gg w$) generates the unitary $\hat{U}_{\mathrm{QG}} = \exp(i\chi_Q\hat{a}^\dagger\hat{a}^\dagger\hat{a}\,\hat{a})$, where

$$\chi_Q \approx \frac{32G\mathcal{F}\hbar\omega_0^2}{c^5}\log\left(\frac{L}{w}\right). \quad (5)$$

Equation (4) can be understood as a gravitational Kerr effect where the graviton vacuum plays the role of the underlying nonlinear medium. Interactions of the form Eq. (4) will produce entanglement between photons, which means that observing signatures of the quantum gravitation self-interaction will also be indirect witnesses of GIE between photons.

In contrast, consider a general classical theory of gravity for which $h^{\mu\nu}$ is not operator valued, which will result in a self-interaction term of quadratic order. In a rotating-wave approximation (RWA) [66], the classical gravitational Hamiltonian must then take the form $\hat{H}_{\mathrm{int}}^C = \hbar\lambda_C\hat{a}^\dagger\hat{a}$ for some coefficient λ_C, giving the unitary $\hat{U}_C = \exp(i\chi_C\hat{a}^\dagger\hat{a})$, where $\chi_C = \lambda_C\tau$. Note that λ_C itself may depend on expectations of the quantum field as in semiclassical gravity, and may even include contributions from nongravitational effects that do not result in interactions between photons. Nevertheless, any such Hamiltonian cannot reproduce features of the quantum gravitational Kerr effect.

Fundamental detectability.—Here we consider the fundamental limit to which the quantum gravitational self-interaction can be distinguished from the self-interaction described by any classical theory of gravity. More precisely, we ask the question, how well can χ_Q be distinguished from zero, given a completely *unknown* value of χ_C? We can reformulate this as a multiparameter estimation problem by treating χ_C as a *nuisance parameter*, following the approach of Ref. [14]. Specifically, the smallest possible nonzero value of χ_Q that may be inferred from a set of M measurements each with interrogation time τ is given by the quantum Cramér-Rao bound (QCRB) [67]:

$$\chi_Q \geq \sqrt{[F_{i,j}]_{Q,Q}^{-1}/M}, \quad (6)$$

where $F_{i,j} = 4\tau^2\mathrm{Cov}(\hat{G}_i, \hat{G}_j)$ is the quantum Fisher information matrix (QFIM) with respect to the classical and quantum generators $\hat{G}_C = \hat{a}^\dagger\hat{a}$ and $\hat{G}_Q = \hat{a}^\dagger\hat{a}^\dagger\hat{a}\,\hat{a}$, respectively. The covariance is computed with respect to the state $|\Psi\rangle = \hat{U}_{\mathrm{QG}}\hat{U}_C|\psi\rangle$ for some initial state $|\psi\rangle$. Note that the existence of an inverse QFIM is not guaranteed—a noninvertible QFIM would correspond to the effect of the quantum gravitational interaction being fundamentally indistinguishable from a classical gravitational interaction.

The choice of initial state $|\psi\rangle$ is therefore crucial to the detectability of quantum gravity given ignorance of χ_C; the wrong choice of state may result in zero sensitivity to χ_Q regardless of the number of measurements. In this analysis we restrict ourselves to quantum states that may be prepared with established experimental techniques. In particular, we find that for a squeezed-vacuum state, the signatures of quantum and classical gravity theories are highly distinguishable, resulting in Heisenberg scaling of the sensitivity [68]:

$$\chi_Q \geq [96MN(N+1)]^{-1}, \quad (7)$$

where $N \equiv \langle\hat{a}^\dagger\hat{a}\rangle$ is the average number of photons in the cavity. In comparison, a significant amount of information is lost for coherent state or a squeezed coherent state, with the sensitivity scaling as $N^{-3/2}$ in the absence of the nuisance parameter, and as N^{-1} in its presence.

This result allows us to make strong statements about the fundamental detectability of χ_Q, given no information about the classical theories of gravity from which this signal is distinguished. Specifically, we can use Eq. (7) to bound the experimental requirements to infer a nonzero value of χ_Q. For example, we may rewrite Eq. (5) in terms of the circulating power in the cavity $P_{\mathrm{circ}} \approx N\hbar\omega_0 c/(2L)$ and the number of experiments $M \approx T/\tau$ that may be conducted within a total time T. For $N \gg 1$, we find that the circulating power required to satisfy Eq. (7) is approximately

$$P_{\mathrm{circ}} \gtrsim \frac{c^3}{16}\left(\frac{c\hbar^2}{12G^2\mathcal{F}L^3T\log(L/w)^2}\right)^{1/4}. \quad (8)$$

This expression shows weak dependence on the interrogation period $P_{\mathrm{circ}} \sim T^{-1/4}$, and is most strongly dependent on the cavity length L. As an example, consider experimental parameters similar to those expected of the next-generation gravitational-wave detector Cosmic Explorer [69]: a $L = 10$-km-long cavity of finesse $\mathcal{F} = 450$ with a 2 μm wavelength laser and long arm separation $w = 10$ cm. The minimum required circulating power for this system to infer the quantum signature of gravity over a year-long interrogation period is roughly 125 MW—much larger than the $\mathcal{O}(1)$ MW circulating powers expected of Cosmic Explorer. With a moderate

increase to the finesse ($\mathcal{F} = 10^3$), the required circulating power could in principle be achieved with a pump laser power of roughly 100 kW. Interestingly, this level of laser power is significantly lower than quoted requirements for directly measuring the gravitational field of the light [70], or generating observable light-matter entanglement [31]. The weak logarithmic dependence of the interaction on the separation w means the beam width σ can be made large enough to prevent damage to the mirror surfaces while satisfying $\sigma \ll w$; e.g., a beam width of 2 cm corresponds to an intensity of roughly 25 MW/cm^2 for the above laser power, which is well within the thermal tolerance of modern low-loss cavity mirrors [71]. While 100 kW continuous lasers are currently available [72,73], the requirement of a squeezed-vacuum state at this power is beyond current experiments—we will see later the need for highly squeezed states may be avoided by considering multimode signatures of the gravitational interaction.

Possible measurement schemes.—The question remains to design a measurement scheme that optimally extracts the signature of quantum gravity in the proposed cavity system, i.e., a measurement scheme for which the ideal precision is given by the right-hand side of Eq. (7). Optimal measurement schemes are not always guaranteed to exist, and are often not experimentally feasible. We instead identify two, experimentally realizable, *nearly* optimal schemes—their precision is not exactly Eq. (7), but scales the same with N (Heisenberg scaling).

In Fig. 1(c) we present a simple scheme based on a Mach-Zehnder (MZ) interferometer that provides an optimal measurement for an initial two-mode squeezed vacuum state (TMSV) [68]. In this scheme, the initial TMSV is beam split to create a pair of squeezed-vacuum states, each of which is fed into a high-finesse cavity. The gravitational self-interaction of the light leads to a relative phase between the two modes, which is then read out by interfering the light exiting the cavity and reading out the number difference between the two modes. The distinguishability of the signature of quantum gravity from classical gravity in this scheme can be simply understood by considering the evolution of the state in the number basis. For the initial TMSV state $|\psi\rangle = \sum_N c_N |N, N\rangle$, where $c_N = [-e^{i\phi} \tanh(r)]^N / \cosh(r)$, the measurement scheme can be described by the unitary operator $\hat{U}_{\mathrm{MZ}} = \hat{U}_{\mathrm{BS}}(\pi/2)\hat{U}_{\mathrm{QG}}\hat{U}_{\mathrm{BS}}(\pi/2)$, where $\hat{U}_{\mathrm{BS}}(\pi/2)$ is a 50-50 beam splitter and $\hat{U}_{\mathrm{QG}} = \exp\{i\chi_Q[(\hat{a}^\dagger)^2\hat{a}^2 + (\hat{b}^\dagger)^2\hat{b}^2]\}$ is the quantum gravitational self-interaction for the two cavity modes represented by $\hat{a}$ and $\hat{b}$. To first order in $\chi_Q \ll 1$, $\hat{U}_{\mathrm{MZ}}$ couples each $|N, N\rangle$ in the initial TMSV to $|N - 2, N + 2\rangle$ and $|N + 2, N - 2\rangle$. Higher-order terms in this expansion will result in coupling to states $|N - 2n, N + 2n\rangle$ for integer n, though these will be negligible due to the exceptionally small value of χ_Q. Number measurements on each mode will then collapse

the quantum state onto a manifold of well-defined N, with a small probability $\sim \chi_Q^2$ of measuring a nonzero number difference between the modes.

In contrast, consider the same scheme for a classical theory of gravity. First we assume the two cavities experience precisely the same self-interaction—i.e., $H_{\mathrm{int}}^C = \hbar\lambda_C(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b})$—the number distribution of each mode measured at the detector is unchanged from the number distribution of the original TMSV state. More generally, we allow for the case of a classical theory of gravity that additionally has a term of the form $H_{\mathrm{int}}^C \sim \hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}$. Such a term could also be contributed by nongravitational effects that result in an asymmetry between the two cavities. In this case, each $|N, N\rangle$ in the initial TMSV will be coupled to states $|N - n, N + n\rangle$ for integer n by the measurement scheme, with dominant coupling for $n = \pm 1$. This is clearly distinguishable from the signature of quantum gravity, in which the dominant coupling is for $n = \pm 2$. The main limitation with this approach is the requirement of ultrahigh efficiency detectors operating at the single-photon level, which for the high power required is beyond reason for modern detection technology.

An appealing alternative measurement scheme uses homodyne detection to observe signatures of quantum gravity in high-order cumulants of the phase quadratures [15], which can be implemented straightforwardly with a single cavity and homodyne detection [Fig. 1(c)]. This approach relies on the fact that the quartic self-interaction term described by a quantum theory of gravity will generate non-Gaussianity of the optical field, that cannot be reproduced by any classical theory of gravity [15]. As a result, a nonzero value of the fourth-order cumulant in a phase quadrature of the light is a clear signature of the quantum nature of gravity. This scheme was initially proposed in the context of massive ultracold atomic systems [15], in which observing this signature is considerably more challenging due to the inability to directly measure the phase quadratures of massive particles. For this scheme it will again be optimal to use a high-power squeezed-vacuum state of light, to maximize the sensitivity of the state to the gravitational self-interaction [15].

While our proposed tests are free of QED photon-photon interactions, radiation pressure on the cavity mirrors can also result in a quantum interaction between photons. Fortunately, radiation pressure can be reduced by rigidly coupling the mirrors to Earth—seismic noise generates only a classical phase shift and does not mimic signatures of photon-photon interactions. However, there may be other challenges associated with microscopic material surface effects at high laser intensities that are poorly understood and merit further investigation in the context of the proposed tests.

Third harmonic generation.—In addition to the gravitational Kerr nonlinearity, there are additional signatures of quantum gravity that arise when we include multiple modes of the quantized optical field in our calculation.

Specifically, we may have a four-wave mixing process between photons in the pump mode (ω_0) that can populate higher frequency resonances $n\omega_0$, where $n \in [2, 3, \dots]$. For our system, where only a single frequency is initially populated, the dominant four-wave mixing process is third-harmonic generation, i.e., $H_{\text{int}} \sim \hat{a}_{\omega_0} \hat{a}_{\omega_0} \hat{a}_{\omega_0} \hat{a}_{3\omega_0}^\dagger$. In contrast, any classical gravitational theories can only have non-energy-conserving terms arising such as $H_{\text{int}} \sim \hat{a}_{\omega_0} \hat{a}_{3\omega_0}^\dagger$, which in the interaction picture will oscillate with frequency $2\omega_0$, and can thus be discarded in a RWA. Therefore, the detection of photons of frequency $3\omega_0$ exiting the cavity would be a clear signature of quantum gravity. This could in principle be achieved with modern photodetectors that operate at the single-photon level, by discarding the pump mode using a wavelength selective beam splitter. The key benefit of this experiment is that it can be implemented with coherent light—with the QCRB for the sensitivity scaling as $N^{-3/2}$ for both coherent and squeezed-vacuum states of the pump mode.

Conclusions.—Near-future experiments seeking to observe GIE between masses may soon provide the first evidence to support the quantum nature of gravity. However, such nonrelativistic experiments can only probe the quantization of the Newtonian interaction and cannot attest to other aspects of quantum gravity using quantum light, such as the locality of the gravitational interaction or the spin of the graviton. This Letter may provide a new pathway for probing relativistic aspects of quantum gravity; together with near-term GIE experiments, the proposed tests may provide a more detailed insight to the true nature of the gravitational field.

While the proposed tests provide a powerful avenue to probing quantum gravity, they will certainly require improvements to current experimental capabilities. In particular, reading out signatures of quantum gravity from photon statistics will require photon detection efficiencies well beyond current experimental capabilities—though there are clear routes to reducing these requirements by using interaction-based readouts [74,75], for example. Reducing technological requirements should be the focus of future investigations that may, for example, consider alternate measurement schemes or more complicated geometries. Regardless, we expect the experimental requirements will become increasingly realistic in the future, given the outstanding rate at which photonic quantum technologies are advancing. In the near term, the experiments proposed here can also be adapted to be precision tests of QED, by using a standing wave of light rather than a traveling wave, such that the QED photon-photon interaction is nonvanishing.

Appendix A: Derivation of the gravitational interaction Hamiltonian.—Here we provide a detailed derivation of the self-interaction Hamiltonian $\hat{H}_{\text{int}}$ given in the main text. We start with the complete action describing the coupled light-gravity system:

$$S = S_{\text{EM}} + S_{\text{grav}}. \tag{A1}$$

Here S_{EM} is the Maxwell action,

$$S_{\text{EM}} = -\frac{1}{4\mu_0} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}, \tag{A2}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the EM field tensor, and $g \equiv \det(g_{\mu\nu})$. The Einstein-Hilbert action S_{grav} is given by

$$S_{\text{grav}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R, \tag{A3}$$

where $R \equiv g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar. We proceed by expanding the gravitational metric around a Minkowski background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, in the $[-, +, +, +]$ convention. Next, we expand the Lagrangian density of the Maxwell action to linear order in the gravitational perturbation $h_{\mu\nu}$:

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4\mu_0} F_{\mu\nu} g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} + \mathcal{O}(h^2) \tag{A4}$$

$$= \mathcal{L}_{(0)} + \mathcal{L}_{(1)} + \mathcal{O}(h^2), \tag{A5}$$

where we have decomposed the Lagrangian density into a free (flat-spacetime) component $\mathcal{L}_{(0)} = -F_{\mu\nu} F^{\mu\nu}/(4\mu_0)$, and the first-order correction,

$$\mathcal{L}_{(1)} = -\frac{h}{8\mu_0} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\mu_0} h^{\mu\beta} \eta^{\nu\lambda} F_{\lambda\beta} F_{\mu\nu}, \tag{A6}$$

$$= \frac{1}{2} h^{\mu\nu} T_{\mu\nu}^{(0)}, \tag{A7}$$

where $h \equiv \eta^{\mu\nu} h_{\mu\nu}$, $T_{\mu\nu}^{(0)}$ is the stress-energy tensor of the free EM field on a Minkowski background:

$$T_{\mu\nu}^{(0)} = \frac{1}{\mu_0} \left(F_{\mu\lambda} F_\nu^\lambda - \frac{1}{4} \eta_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right), \tag{A8}$$

noting the symmetry properties $h^{\alpha\beta} = h^{\beta\alpha}$ and $F_{\mu\nu} = -F_{\nu\mu}$. This interaction can alternatively be derived by a variational argument, and is the standard linear coupling term between matter and the linearized gravitational field [55,76].

We then obtain the corresponding interaction Hamiltonian via a Legendre transform of the linearized Lagrangian. As the interaction depends on derivatives of

the EM field via $T_{\mu\nu}$, the resulting interaction term is not simply $-\mathcal{L}_{(1)}$ and contains additional terms due to corrections to the conjugate momenta of the EM field:

$$\Pi^\alpha \equiv \frac{\partial\mathcal{L}}{\partial(\partial_0 A_\alpha)} = \Pi^\alpha_{(0)} + \Pi^\alpha_{(1)}, \qquad (A9)$$

where $\Pi^\alpha_{(0)}$ is the free field result, and

$$\Pi^\alpha_{(1)} = \frac{1}{2} h^{\mu\nu} \frac{\partial T^{(0)}_{\mu\nu}}{\partial(\partial_0 A_\alpha)}$$
$$= \frac{1}{\mu_0}\left(\frac{h}{2} F^{\alpha 0} + h^{\mu 0}\eta^{\alpha\lambda}F_{\mu\lambda} - h^{\mu\alpha}\eta^{0\lambda}F_{\mu\lambda}\right) \qquad (A10)$$

is the first-order correction due to the interaction with the gravitational field. Then, working in the interaction picture, the interaction Hamiltonian density is given by

$$\mathcal{H}_{(1)}(\mathbf{r}) = \Pi^\alpha_{(1)}\partial_0 A_\alpha - \mathcal{L}_{(1)}. \qquad (A11)$$

For the system under consideration, we have $h = 0$ by virtue of the vanishing trace of the EM stress-energy tensor. Furthermore, for the gravitational field generated by a traveling wave, we have

$$h^{\mu\alpha}F_{\mu 0} = h^{i\alpha}E_i = 0, \qquad (A12)$$

as a consequence of the explicit form of the gravitational field solution—$h^{i\alpha}$ only has nonzero components longitudinal to the direction of propagation [see Eq. (B2)], to which the electric field E_i is transverse. Therefore, for our system the interaction Hamiltonian density can be simplified to (noting $\partial_0 A_\alpha = F_{0\alpha}$)

$$\mathcal{H}_{(1)}(\mathbf{r}) = \frac{1}{\mu_0} h^{\mu 0}\eta^{\alpha\lambda}F_{\mu\lambda}F_{0\alpha} - \frac{1}{2} h^{\mu\nu}T^{(0)}_{\mu\nu}. \qquad (A13)$$

Appendix B: Self-interaction Hamiltonian for rectangular ring-cavity.—For the rectangular ring-cavity setup described in the main text, the dominant contribution to the gravitational self-interaction energy will be the interaction between the two long arms, provided the longer arms of the cavity are significantly longer than their separation $L \gg w$ (where w is the length of the shorter arms). Labeling the two long arms as L and R, we can then approximately decompose the stress-energy tensor as $T^{\mu\nu} \approx T^{\mu\nu}_L + T^{\mu\nu}_R$, ignoring the contributions from the short arms. The nonzero elements of these tensors are $T^{00}_L = T^{33}_L = T^{03}_L = T^{30}_L = \mathcal{H}_{(0)}$ and $T^{00}_R = T^{33}_R = -T^{03}_R = -T^{30}_R = \mathcal{H}_{(0)}$, taking the light along the left arm to be traveling in the positive z direction. By linearity of Eq. (1), its solution can then be written as

$$h^{\mu\nu} = h^{\mu\nu}_L + h^{\mu\nu}_R, \qquad (B1)$$

where

$$h^{\mu\nu}_{L,R} = h_p(\mathbf{r})\begin{pmatrix} 1 & 0 & 0 & \pm 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm 1 & 0 & 0 & 1 \end{pmatrix}, \qquad (B2)$$

$$h_p(\mathbf{r}) = \frac{4G}{c^4}\int_{L,R} d^3\mathbf{r}' \frac{\mathcal{H}_{(0)}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}. \qquad (B3)$$

Here the notation $\int_{L,R}$ denotes integrating over the left or right arm of the cavity, respectively. Note we have considered here a time-independent Hamiltonian density corresponding to a circular polarization of light for simplicity, though this can be simply extended to the time-dependent case (linear or elliptical polarizations). Then, noting each arm does not self-interact, $T^{\mu\nu}_i h^i_{\mu\nu} = 0$ for $i = L, R$ [68], we have

$$-\frac{1}{2}\int d^3\mathbf{r} h^{\mu\nu}(\mathbf{r})T^{(0)}_{\mu\nu}(\mathbf{r})$$
$$\approx -\frac{1}{2}\int d^3\mathbf{r}[h^{\mu\nu}_L(\mathbf{r})T^R_{\mu\nu}(\mathbf{r}) + h^{\mu\nu}_R(\mathbf{r})T^L_{\mu\nu}(\mathbf{r})] \qquad (B4)$$

$$= -2\sum_{i=L,R}\int_i d^3\mathbf{r} h_i(\mathbf{r})\mathcal{H}_{(0)}(\mathbf{r}). \qquad (B5)$$

A similar calculation follows for the other term in the interaction Hamiltonian density, $h^{\mu 0}F_{\mu\lambda}F^\lambda_0$, where we have to explicitly consider the form of the electric and magnetic fields:

$$\mathbf{E} \approx E_L[\mathbf{e_z}] + E_R[-\mathbf{e_z}], \qquad \mathbf{B} \approx B_L[\mathbf{e_z}] + B_R[-\mathbf{e_z}],$$

where the argument $[\mathbf{n}]$ indicates direction of propagation in terms of a normalized vector $\mathbf{n}$, with $\mathbf{e_i}$ being the unit vectors for $i = x, y, z$. For circularly polarized light, the EM fields are of the form:

$$\mathbf{E}_L = \{E^x_L, E^y_L, 0\}, \qquad \mathbf{B}_L = \{-E^y_L/c, E^x_L/c, 0\}, \qquad (B6)$$

$$\mathbf{E}_R = \{E^x_R, E^y_R, 0\}, \qquad \mathbf{B}_R = \{E^y_R/c, -E^x_R/c, 0\}, \qquad (B7)$$

which, using $\mathcal{H}_{(0)} = (|E|^2 + |B|^2)/2$, gives

$$\int d^3\mathbf{r} h^{\mu 0}F_{\mu\lambda}F^\lambda_0 \approx -2\sum_{i=L,R}\int_i d^3\mathbf{r} h_i(\mathbf{r})\mathcal{H}_{(0)}(\mathbf{r}). \qquad (B8)$$

This is precisely the same expression as Eq. (B5), though in general (e.g., for other polarizations) the two may differ.

Combining Eqs. (B5), (B8), and (B3) we find the self-interaction energy to be given by the Hamiltonian:

$$H_{\text{int}} \approx -16G\left(\int_L d^3\boldsymbol{r} \int_R d^3\boldsymbol{r}' \frac{\mathcal{H}_{(0)}(\boldsymbol{r})\mathcal{H}_{(0)}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} \right.$$
$$\left. + \int_R d^3\boldsymbol{r} \int_L d^3\boldsymbol{r}' \frac{\mathcal{H}_{(0)}(\boldsymbol{r})\mathcal{H}_{(0)}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\right). \quad \text{(B9)}$$

Equation (4) can then be derived by expanding the energy density as $\mathcal{H}_{(0)}(\boldsymbol{r}) = (\hbar\omega_0/c^2)|f(\boldsymbol{r})|^2 \hat{a}^\dagger \hat{a}$ and computing the spatial integrals in the limit $\lambda \ll w \ll L$. Details of this calculation are provided in the Supplemental Material [68], with the final result:

$$\hat{H}_{\text{int}} \approx -16G\left(\frac{\hbar\omega_0}{c^2}\right)^2 \left(\frac{1}{L}\right)\log\left(\frac{L}{w}\right)\hat{a}^\dagger \hat{a}^\dagger \hat{a}\,\hat{a}. \quad \text{(B10)}$$

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[1] *Experimental Search for Quantum Gravity*, 1st ed., edited by Sabine Hossenfelder, FIAS Interdisciplinary Science Series (Springer International Publishing, Basel, Switzerland, 2017).

[2] Sougato Bose, Anupam Mazumdar, Gavin W. Morley, Hendrik Ulbricht, Marko Toroš, Mauro Paternostro, Andrew A. Geraci, Peter F. Barker, M. S. Kim, and Gerard Milburn, Spin Entanglement Witness for Quantum Gravity, Phys. Rev. Lett. **119**, 240401 (2017).

[3] C. Marletto and V. Vedral, Gravitationally Induced Entanglement between Two Massive Particles is Sufficient Evidence of Quantum Effects in Gravity, Phys. Rev. Lett. **119**, 240402 (2017).

[4] Tanjung Krisnanda, Margherita Zuppardo, Mauro Paternostro, and Tomasz Paterek, Revealing Nonclassicality of Inaccessible Objects, Phys. Rev. Lett. **119**, 120402 (2017).

[5] Chiara Marletto and Vlatko Vedral, When can gravity path-entangle two spatially superposed masses?, Phys. Rev. D **98**, 046001 (2018).

[6] H. Pino, J. Prat-Camps, K. Sinha, B. Prasanna Venkatesh, and O. Romero-Isart, On-chip quantum interference of a superconducting microsphere, Quantum Sci. Technol. **3**, 025001 (2018).

[7] Richard Howl, Lucia Hackermüller, David Edward Bruschi, and Ivette Fuentes, Gravity in the quantum lab, Adv. Phys. **3**, 1383184 (2017).

[8] Daniel Carney, Philip C. E. Stamp, and Jacob M. Taylor, Tabletop experiments for quantum gravity: A user's manual, Classical Quantum Gravity **36**, 034001 (2019).

[9] M. Carlesso, A. Bassi, M. Paternostro, and H. Ulbricht, Testing the gravitational field generated by a quantum superposition, New J. Phys. **21**, 093052 (2019).

[10] Marios Christodoulou and Carlo Rovelli, On the possibility of laboratory evidence for quantum superposition of geometries, Phys. Lett. B **792**, 64 (2019).

[11] Tanjung Krisnanda, Guo Yao Tham, Mauro Paternostro, and Tomasz Paterek, Observable quantum entanglement due to gravity, npj Quantum Inf. **6**, 12 (2020).

[12] Ryan J. Marshman, Anupam Mazumdar, and Sougato Bose, Locality and entanglement in table-top testing of the quantum nature of linearized gravity, Phys. Rev. A **101**, 052110 (2020).

[13] Thomas W. van de Kamp, Ryan J. Marshman, Sougato Bose, and Anupam Mazumdar, Quantum gravity witness via entanglement of masses: Casimir screening, Phys. Rev. A **102**, 062807 (2020).

[14] Simon A. Haine, Searching for signatures of quantum gravity in quantum gases, New J. Phys. **23**, 033020 (2021).

[15] Richard Howl, Vlatko Vedral, Devang Naik, Marios Christodoulou, Carlo Rovelli, and Aditya Iyer, Non-Gaussianity as a signature of a quantum theory of gravity, PRX Quantum **2**, 010325 (2021).

[16] T. Weiss, M. Roda-Llordes, E. Torrontegui, M. Aspelmeyer, and O. Romero-Isart, Large Quantum Delocalization of a Levitated Nanoparticle Using Optimal Control: Applications for Force Sensing and Entangling via Weak Forces, Phys. Rev. Lett. **127**, 023601 (2021).

[17] Sougato Bose, Anupam Mazumdar, Martine Schut, and Marko Toroš, Mechanism for the quantum natured gravitons to entangle masses, Phys. Rev. D **105**, 106028 (2022).

[18] C. Anastopoulos and B. L. Hu, Quantum superposition of two gravitational cat states, Classical Quantum Gravity **37**, 235012 (2020).

[19] Alessio Belenchia, Dionigi M. T. Benincasa, Stefano Liberati, Francesco Marin, Francesco Marino, and Antonello Ortolan, Testing Quantum Gravity Induced Nonlocality via Optomechanical Quantum Oscillators, Phys. Rev. Lett. **116**, 161303 (2016).

[20] M. Derakhshani, C. Anastopoulos, and B. L. Hu, Probing a gravitational cat state: Experimental possibilities, J. Phys. Conf. Ser. **701**, 012015 (2016).

[21] Jonas Schmöle, Mathias Dragosits, Hans Hepach, and Markus Aspelmeyer, A micromechanical proof-of-principle experiment for measuring the gravitational force of milligram masses, Classical Quantum Gravity **33**, 125031 (2016).

[22] Abdulrahim Al Balushi, Wan Cong, and Robert B. Mann, Optomechanical quantum Cavendish experiment, Phys. Rev. A **98**, 043811 (2018).

[23] Haixing Miao, Denis Martynov, Huan Yang, and Animesh Datta, Quantum correlations of light mediated by gravity, Phys. Rev. A **101**, 063804 (2020).

[24] Jules Tilly, Ryan J. Marshman, Anupam Mazumdar, and Sougato Bose, Qudits for witnessing quantum-gravity-induced entanglement of masses under decoherence, Phys. Rev. A **104**, 052416 (2021).

[25] Martine Schut, Jules Tilly, Ryan J. Marshman, Sougato Bose, and Anupam Mazumdar, Improving resilience of quantum-gravity-induced entanglement of masses to decoherence using three superpositions, Phys. Rev. A **105**, 032411 (2022).

[26] Julen S. Pedernales, Kirill Streltsov, and Martin B. Plenio, Enhancing Gravitational Interaction between Quantum Systems by a Massive Mediator, Phys. Rev. Lett. 128, 110401 (2022).

[27] Hadrien Chevalier, A. J. Paige, and M. S. Kim, Witnessing the nonclassical nature of gravity in the presence of unknown interactions, Phys. Rev. A 102, 022428 (2020).

[28] Simone Rijavec, Matteo Carlesso, Angelo Bassi, Vlatko Vedral, and Chiara Marletto, Decoherence effects in nonclassicality tests of gravity, New J. Phys. 23, 043040 (2021).

[29] Marko Toros, Thomas W. van de Kamp, Ryan J. Marshman, M. S. Kim, Anupam Mazumdar, and Sougato Bose, Relative acceleration noise mitigation for nanocrystal matter-wave interferometry: Applications to entangling masses via quantum gravity, Phys. Rev. Res. 3, 023178 (2021).

[30] Akira Matsumura and Kazuhiro Yamamoto, Gravity-induced entanglement in optomechanical systems, Phys. Rev. D 102, 106021 (2020).

[31] Dripto Biswas, Sougato Bose, Anupam Mazumdar, and Marko Toros, Gravitational optomechanics: Photon-matter entanglement via graviton exchange, arXiv:2209.09273.

[32] Yair Margalit, Or Dobkowski, Zhifan Zhou, Omer Amit, Yonathan Japha, Samuel Moukouri, Daniel Rohrlich, Anupam Mazumdar, Sougato Bose, Carsten Henkel, and Ron Folman, Realization of a complete Stern-Gerlach interferometer: Toward a test of quantum gravity, Sci. Adv. 7, 22777 (2021).

[33] C. Anastopoulos and Bei-Lok Hu, Comment on "A spin entanglement witness for quantum gravity" and on "Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity", arXiv:1804.11315.

[34] Charis Anastopoulos, Michalis Lagouvardos, and Konstantina Savvidou, Gravitational effects in macroscopic quantum systems: A first-principles analysis, Classical Quantum Gravity 38, 155012 (2021).

[35] Charis Anastopoulos and Ntina Savvidou, Quantum information in relativity: The challenge of QFT measurements, Entropy 24, 4 (2021).

[36] Eduardo Martín-Martínez and T. Rick Perche, What gravity mediated entanglement can really tell us about quantum gravity, arXiv:2208.09489.

[37] Charis Anastopoulos and Bei-Lok Hu, Gravity, quantum fields and quantum information: Problems with classical channel and stochastic theories, Entropy 24, 490 (2022).

[38] Alessio Belenchia, Robert M. Wald, Flaminia Giacomini, Esteban Castro-Ruiz, Časlav Brukner, and Markus Aspelmeyer, Quantum superposition of massive objects and the quantization of gravity, Phys. Rev. D 98, 126009 (2018).

[39] Alessio Belenchia, Robert M. Wald, Flaminia Giacomini, Esteban Castro-Ruiz, Časlav Brukner, and Markus Aspelmeyer, Information content of the gravitational field of a quantum superposition, Int. J. Mod. Phys. D 28, 1943001 (2019).

[40] Daine L. Danielson, Gautam Satishchandran, and Robert M. Wald, Gravitationally mediated entanglement: Newtonian field versus gravitons, Phys. Rev. D 105, 086001 (2022).

[41] Daniel Carney, Newton, entanglement, and the graviton, Phys. Rev. D 105, 024029 (2022).

[42] Nick Huggett, Niels Linnemann, and Mike Schneider, Quantum gravity in a laboratory?, arXiv:2205.09013.

[43] Emily Adlam, Tabletop experiments for quantum gravity are also tests of the interpretation of quantum mechanics, Found. Phys. 52, 115 (2022).

[44] Richard C. Tolman, Paul Ehrenfest, and Boris Podolsky, On the gravitational field produced by light, Phys. Rev. 37, 602 (1931).

[45] John Archibald Wheeler, Geons, Phys. Rev. 97, 511 (1955).

[46] Dennis Rätzel, Martin Wilkens, and Ralf Menzel, Gravitational properties of light—The gravitational field of a laser pulse, New J. Phys. 18, 023009 (2016).

[47] Jan van Holten, The gravity of light-waves, Universe 4, 110 (2018).

[48] Bruce M. Barker, Manjit S. Bhatia, and Suraj N. Gupta, Gravitational scattering of light by light, Phys. Rev. 158, 1498 (1967).

[49] P. J. Westervelt and L. F. Karr, Gravitational light-light and light-particle scattering: Comparison between the classical (general-relativistic) and quantum field-theoretic results, Il Nuovo Cimento B Series 10 66, 129 (1970).

[50] S. Vilasi, G. Sparano, and G. Vilasi, On the gravity of light, Classical Quantum Gravity 28, 195014 (2011).

[51] Gert Brodin, Daniel Eriksson, and Mattias Marklund, Graviton mediated photon-photon scattering in general relativity, Phys. Rev. D 74, 124028 (2006).

[52] Dennis Rätzel, Martin Wilkens, and Ralf Menzel, The effect of entanglement in gravitational photon-photon scattering, Europhys. Lett. 115, 51002 (2016).

[53] Michalis Lagouvardos and Charis Anastopoulos, Gravitational decoherence of photons, Classical Quantum Gravity 38, 115012 (2021).

[54] Éanna É. Flanagan and Scott A. Hughes, The basics of gravitational wave theory, New J. Phys. 7, 204 (2005).

[55] Michele Maggiore, Gravitational Waves. Vol. 1: Theory and Experiments (Oxford University Press, Oxford, 2007).

[56] Suraj N. Gupta, Quantization of Einstein's gravitational field: Linear approximation, Proc. Phys. Soc. London Sect. A 65, 161 (1952).

[57] Gunnar Nordström, Zur theorie der gravitation vom standpunkt des relativitätsprinzips, Ann. Phys. (Berlin) 347, 533 (1913).

[58] C. Brans and R. H. Dicke, Mach's principle and a relativistic theory of gravitation, Phys. Rev. 124, 925 (1961).

[59] Narayan Banerjee and Barun Majumder, A question mark on the equivalence of Einstein and Jordan frames, Phys. Lett. B 754, 129 (2016).

[60] Sridip Pal, Quantized Brans-Dicke theory: Phase transition, strong coupling limit, and general relativity, Phys. Rev. D 94, 084023 (2016).

[61] Stefan Aimet, Hadrien Chevalier, and M. S. Kim, Gravity mediated entanglement between light beams as a table-top test of quantum gravity, arXiv:2210.12713.

[62] Julian Schwinger, On gauge invariance and vacuum polarization, Phys. Rev. 82, 664 (1951).

[63] Mattias Marklund, Gert Brodin, Lennart Stenflo, and Padma K. Shukla, Dynamics of radiation due to vacuum nonlinearities, Phys. Scr. T 107, 239 (2004).

[64] D Kramer, The gravitational field of two counter-propagating beams of light, Classical Quantum Gravity 15, L73 (1998).

[65] Marios Christodoulou, Andrea Di Biagio, Markus Aspelmeyer, Časlav Brukner, Carlo Rovelli, and Richard Howl, Locally mediated entanglement through gravity from first principles, Phys. Rev. Lett. **130,** 100202 (2023).

[66] The validity of a RWA is justified here by the long cavity interrogation time, $\tau \gg \omega_0^{-1}$.

[67] Jing Liu, Haidong Yuan, Xiao-Ming Lu, and Xiaoguang Wang, Quantum Fisher information matrix and multiparameter estimation, J. Phys. A **53,** 023001 (2019).

[68] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.130.240203, which includes comments on the gravitational self-interaction of a single plane wave, derivation of the single-mode interaction Hamiltonian, and quantum and classical Fischer information calculations.

[69] B. P. Abbott *et al.*, Exploring the sensitivity of next generation gravitational wave detectors, Classical Quantum Gravity **34,** 044001 (2017).

[70] Felix Spengler, Dennis Rätzel, and Daniel Braun, Perspectives of measuring gravitational effects of laser light and particle beams, New J. Phys. **24,** 053021 (2022).

[71] Lei S. Meng, Jason K. Brasseur, and David K. Neumann, Damage threshold and surface distortion measurement for high-reflectance, low-loss mirrors to $100+$ MW/cm^2 cw laser intensity, Opt. Express **13,** 10085 (2005).

[72] High Power CW Fiber Lasers, $1 - 100+$ kW: IPG Photonics (IPG Photonics Corporation, 2023), https://www.ipgphotonics.com/en/products/lasers/high-power-cw-fiber-lasers.

[73] E. A. Shcherbakov, V. V. Fomin, A. A. Abramov, A. A. Ferin, D. V. Mochalov, and V. P. Gapontsev, Industrial grade 100 kw power cw fiber laser, in *Advanced Solid-State Lasers Congress* (Optica Publishing Group, Paris, France, 2013), p. ATh4A.2.

[74] Samuel P. Nolan, Stuart S. Szigeti, and Simon A. Haine, Optimal and Robust Quantum Metrology Using Interaction-Based Readouts, Phys. Rev. Lett. **119,** 193601 (2017).

[75] Simon A. Haine, Using interaction-based readouts to approach the ultimate limit of detection-noise robustness for quantum-enhanced metrology in collective spin systems, Phys. Rev. A **98,** 030303(R) (2018).

[76] Sean M. Carroll, *Spacetime and Geometry: An Introduction to General Relativity* (Addison Wesley, Reading, MA, 2004).