Distilling Nonlocality in Quantum Correlations

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Nonlocality, as established by the seminal Bell's theorem, is considered to be the most striking feature of correlations present in spacelike separated events. Its practical application in device independent protocols, such as secure key distribution, randomness certification, etc., demands identification and amplification of such correlations observed in the quantum world. In this Letter we study the prospect of nonlocality distillation, wherein, by applying a natural set of free operations (called wirings) on many copies of weakly nonlocal systems, one aims to generate correlations of higher nonlocal strength. In the simplest Bell scenario, we identify a protocol, namely, logical OR-AND wiring, that can distill nonlocality to a significantly high degree starting from arbitrarily weak quantum nonlocal correlations. As it turns out, our protocol has several interesting facets: (i) it demonstrates that a set of distillable quantum correlations has nonzero measure in the full eight-dimensional correlation space, (ii) it can distill quantum Hardy correlations by preserving its structure, (iii) it shows that (nonlocal) quantum correlations sufficiently close to the local deterministic points can be distilled by a significant amount. Finally, we also demonstrate efficacy of the considered distillation protocol in detecting postquantum correlations.

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Introduction.—One of the most celebrated nonclassical aspects of quantum mechanics was pioneered by Bell in 1964 [1] (see also [2]). Bell's theorem mandates departure of quantum theory from the *locally causal* world view which subsequently has been confirmed in several milestone experiments led by Clauser, Aspect, Zeilinger, and others [3–12]. Unlike other nonclassical features, such as entanglement and coherence, study of nonlocality can be conducted in a device-independent setting where only the input-output statistics of the device matters and one does not need to know the inner design or working mechanisms of the device [13]. Along with foundational implications, Bell nonlocality has also been identified as the necessary resource for several important protocols [14–25], which, thus, makes the question of refinement or distillation of this resource practically indispensable. Study of nonlocality distillation has two major implications—(i) practical: where one aims to distill nonlocal correlations observed in the quantum world which can be then applied to make information flow networks efficient and secure, and (ii) foundational: where the goal is to identify postquantum correlations, which, in turn, helps to understand the speciality of quantum theory among other possibilities allowed within the framework of generalized probabilistic theories. Interestingly, in Ref. [26], Forster *et al.* proposed a nonlocality distillation protocol that can extract nonlocality in a stronger form starting with many copies of weakly nonlocal systems; this work has inspired a number of subsequent works consisting of interesting results on nonlocality distillation [27–41].

The research conducted so far on nonlocality distillation is mainly focused on distilling postquantum correlations [27–32,34–39]. Only a few protocols are known that successfully distill some quantum correlations [26,39]. The difficulty arises due to the top-down approach considered in earlier works where one starts with some parametric family of generic no-signaling (NS) correlations, and after obtaining a successful distillation protocol the aim is to check whether for some range of the parameter values the considered NS correlations allow quantum realization or not. For the simplest bipartite case, the well-known analytical criterion by Tsirelson-Landau-Masanes [42–44] and the Navascues-Pironio-Acin (NPA) criterion [45], and, in the general case, a hierarchy of semidefinite programming conditions [46] can serve this purpose. Only in some fortunate cases sophisticated choices of the parametric class of NS correlations might lead to a desirable subset of quantum realizable correlations. However, the approach has a severe pitfall when more input-output scenarios are considered, as the recent mathematical breakthrough by Slofstra and the subsequent results establish that the set of quantum correlations is not topologically closed [47–49]. There are only a few results that report distillation of nonlocal correlations within the quantum setup [26,39], albeit the nonlocal strength of the distilled correlation is low. Therefore the aspects of analytical and quantitative study for distillation of quantum nonlocal correlations remain open.

In this Letter, we propose a generic framework for nonlocality distillation which overcomes limitations of the thus far proposed protocols. In contrast to the previously reported results, we intend to find out efficient distillation protocol(s) for quantum correlations. To this aim we consider the bottom-up approach. Instead of generic NS signaling correlations we start with weak nonlocal correlations which are quantum, and then try to obtain a nonlocality distillation protocol. The set of quantum correlations being closed under *wirings* [40,41] assures the resulting distilled correlations to be quantum. Interestingly, we identify a simple protocol and come up with a generic approach that successfully distills nonlocality in a large class of weakly nonlocal quantum correlations. Toward this goal, first we consider a variant of nonlocality test proposed by Lucien Hardy [50]. Success probability in Hardy's test qualifies as a measure of nonlocality for Hardy's correlations [51]. Given two copies of a quantum Hardy correlation, we show that there exists a simple wiring that can distill Hardy nonlocality. We call this wiring logical OR-AND protocol, where OR ($\vee$) and AND ($\wedge$) functions on 2-bits z_1, z_2 are defined as $\vee(z_1, z_2) = \max\{z_1, z_2\}$ and $\wedge(z_1, z_2) = \min\{z_1, z_2\}$, respectively. The OR-AND protocol allows an immediate n-copy generalization (see Fig. 1), which can provide a substantial distillation of Hardy's success with sufficiently large copies of initial correlations. Further, we show that the OR-AND wiring when applied to a broader class of quantum correlations yields an interesting result: an arbitrarily small violation of the Clauser-Horne-Shimony-Holt (CHSH) [3] inequality can be amplified to a significantly higher degree. Finally, by applying our protocol we demonstrate that nonlocal correlations arbitrarily close to the extreme points of the set of local correlations are always distilled, which, in turn, establishes that a set of distillable quantum correlations has nonzero measure in the full eight dimensions of the correlation space. We also study distillation of post quantum correlations, and show that the OR-AND protocol becomes efficient there too. In particular, we find correlations whose post-quantum signature is established through OR-AND distillation, while the known information principles, such as nontrivial communication complexity [52] and information causality [53,54], fail to serve the purpose. In the end, we discuss the novelty of the approach followed here in comparison to the existing methods on nonlocality distillation.

Preliminaries.—A Bell experiment involves spatially separated parties performing local measurements on the respective parts of a composite system shared among them. The simplest case considers two parties, Alice and Bob, with their respective inputs to the box denoted as x and y, and outputs from the box denoted as a and b, respectively, where $x, y, a, b \in \{0, 1\}$; and this is generally called the

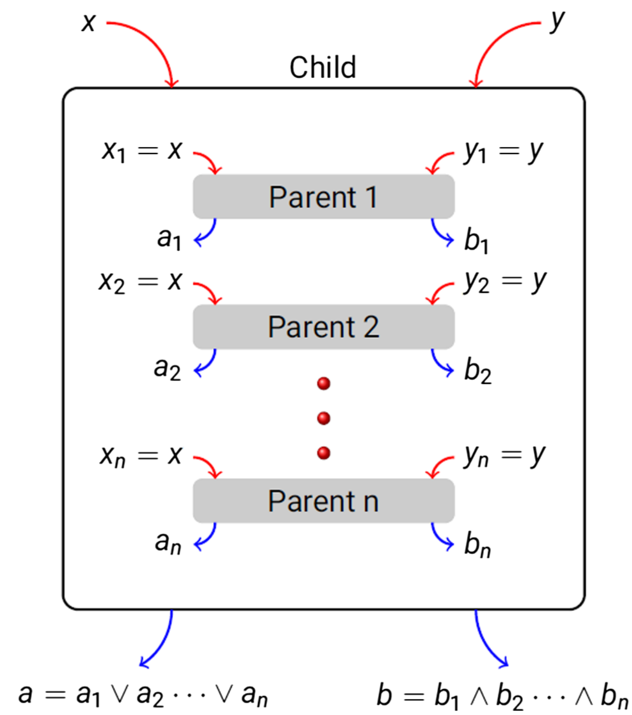

FIG. 1. Multicopy OR-AND wiring. Given n-number of parent correlations $\{\mathrm{P}_{NS}[i]\}_{i=1}^{n} \subset \mathcal{NS}$, the OR-AND wiring produce a child correlation $\mathrm{P}_{NS}^{(n)} \in \mathcal{NS}$. The outcome a on Alice's side for the child box is obtained as $a = a_1 \vee \cdots \vee a_n = \max\{a_1, \ldots, a_n\}$ for the input $x_1 = \cdots = x_n = x$, where x_i and a_i are the input and output of the ith parent. On Bob's side, $y_1 = \cdots = y_n = y$ and $b = b_1 \wedge \cdots \wedge b_n = \min\{b_1, \ldots, b_n\}$.

2-2-2 Bell scenario. Correlation generated by a box P is the set of joint input-output probabilities, i.e., $\mathrm{P} \equiv \{p(ab|xy)\}$. A set of boxes satisfying the no-signaling condition forms an eight-dimensional polytope $\mathcal{NS}$ having 24 vertices [55]: 8 nonlocal vertices [Popescu-Rohrlich (PR) boxes [56]] given by $\mathrm{P}_{NL}^{\alpha\beta\gamma} \equiv \{p(ab|xy) = \frac{1}{2}\delta_{(a\oplus b, xy\oplus\alpha x\oplus\beta y\oplus\gamma)}\}$, where $\alpha, \beta, \gamma \in \{0, 1\}$, and 16 local deterministic vertices given by $\mathrm{P}_{L}^{\alpha_1\alpha_2\beta_1\beta_2} \equiv \{p(ab|xy) = \delta_{(a,\alpha_1 x\oplus\alpha_2)}\delta_{(b,\beta_1 y\oplus\beta_2)}\}$, where $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \{0, 1\}$. A correlation is termed as local *if and only if* it allows a decomposition of the form $p(ab|xy) = \sum p(\lambda)p(a|x,\lambda)p(b|y,\lambda)$, where λ is some classical variable shared between Alice and Bob following a distribution $\{p(\lambda)\}$ [1] (see also [2]). A collection of all local correlations forms a subpolytope $\mathcal{L}$, within $\mathcal{NS}$, with 16 local deterministic boxes as their vertices. Correlations obtained from local quantum measurements performed on some bipartite quantum state are called quantum correlations. A set of all quantum correlations $\mathcal{Q}$ forms a convex set (but not a polytope) lying strictly in between the local and NS polytope, i.e., $\mathcal{L} \subsetneq \mathcal{Q} \subsetneq \mathcal{NS}$ [57]. No signaling correlations that do not belong to the set $\mathcal{L}$ are called nonlocal as they do not allow a local-causal description [1]. We consider one of the eight symmetries for the nonlocal correlations witnessed by the Bell CHSH inequality

$$\mathcal{B} \equiv \langle 00\rangle - \langle 01\rangle - \langle 10\rangle - \langle 11\rangle \leq 2, \tag{1}$$

where $\langle xy\rangle \coloneqq \sum_{a,b}(-1)^{a\oplus b}p(ab|xy)$. Then, only one PR-box violates the inequality and there are exactly eight extremal local boxes that saturate the local bound. These nine boxes, which form an eight dimensional simplex [58], are as follows:

$$\{\mathrm{P}^{110}_{NL}\} \Rightarrow \mathcal{B}=4, \qquad \left\{\begin{matrix}\mathrm{P}^{0001}_{L_1}, \mathrm{P}^{0100}_{L_2}, \mathrm{P}^{0111}_{L_3},\\ \mathrm{P}^{1101}_{L_4}, \mathrm{P}^{1111}_{L_5}, \mathrm{P}^{1000}_{L_6},\\ \mathrm{P}^{0010}_{L_7}, \mathrm{P}^{1010}_{L_8}\end{matrix}\right\} \Rightarrow \mathcal{B}=2. \tag{2}$$

The CHSH value $\mathcal{B} < 2$ for all the remaining extremal nonlocal or local boxes. The choice of this symmetry is due to the simple OR-AND function description of our distillation protocol. One can always apply local reversible relabelings to switch to any of the eight symmetries (then the protocol become suitably relabelled OR-AND). From here on, we will simply refer to the nine boxes in Eq. (2) by dropping their superscripts.

While violation of the Bell-CHSH inequality is a paradigmatic test for certifying nonlocality of a correlation, Hardy provided a simpler nonlocality argument for the same [50]. He showed that if the following four conditions

$$p_{\text{Hardy}} \equiv p(0,0|A_0,B_0) > 0,$$
$$p(0,0|A_0,B_1) = p(0,0|A_1,B_0) = p(1,1|A_1,B_1) = 0, \tag{3}$$

are satisfied, then the resulting correlation is necessarily nonlocal. The probability p_{Hardy} in Eq. (3) quantifies strength of nonlocality of Hardy correlations. While the maximum value of p_{Hardy} in the no-signaling set is $1/2$ (achieved with PR box), its optimal value in quantum mechanics turns out to be $(5\sqrt{5}-11)/2 \approx 0.09$ [59] (see also [60,61]), and it is achieved on a pure two qubit state with projective measurements. The correlation yielding the maximum Hardy nonlocality in quantum theory reads

$$\mathrm{H}^{\max}_{Q} = (5\sqrt{5}-11)\mathrm{P}_{NL} + \frac{1}{2}(7-3\sqrt{5})\sum_{i=1}^{4}\mathrm{P}_{L_i} + (\sqrt{5}-2)\mathrm{P}_{L_5}, \tag{4}$$

and it has been shown to be an extreme point of the set $\mathcal{Q}$ [57].

Distillation of quantum nonlocality.—In the following parts of this Letter, we first present our results on distillation of quantum nonlocal correlations. For ease of readability, here we present our main results and discuss the important proofs. The detailed analyses of some of the more technical aspects is deferred to the Supplemental Material [62]. We start by presenting our results on nonlocality distillation of Hardy's correlations, by applying the OR-AND protocol.

Theorem 1.—The OR-AND wiring preserves the structure of quantum Hardy correlations and can efficiently distill the strength of success probability in Hardy's test of nonlocality.

Proof.—Within the considered CHSH symmetry [i.e., Eq. (2)], any NS correlation can be represented as a convex mixture of the nonlocal vertex along with 8 local vertices. A quantum Hardy correlation demands nonzero weights for exactly 5 different local vertices along with the nonlocal vertex [38]. This, in turn, ensures that the correlation matrix has precisely 3 zero elements corresponding to probabilities taking the value zero in the Hardy's test. It turns out that, then any quantum Hardy correlation H_Q, can be represented as

$$\mathrm{H}_Q = c_0\mathrm{P}_{NL} + \sum_{i=1}^{5} c_i\mathrm{P}_{L_i}; \qquad c_i > 0 \quad \forall\ i, \quad \text{and} \quad \sum_{i=0}^{5} c_i = 1, \tag{5}$$

with success in Hardy's test $p_{\text{Hardy}} = (c_0/2)$. On applying the OR-AND wiring to 2 copies of (parents) H_Q one obtains a resulting (child) Hardy correlation $\mathrm{H}^{(2)}_Q$ with nonlocality strength $p^{(2)}_{\text{Hardy}} = [(c_0/2)+c_1]^2 - c_1^2$. Similarly, on applying OR-AND protocol to n copies of H_Q we obtain a (child) Hardy correlation $\mathrm{H}^{(n)}_Q$ with nonlocality strength $p^{(n)}_{\text{Hardy}} = [(c_0/2)+c_1]^n - c_1^n$. The protocol serves the purpose of an effective n copy distillation as long as $p^{(k)}_{\text{Hardy}} > p^{(k-1)}_{\text{Hardy}}$, for all $k \in \{2,3,\ldots,n\}$. For example, let us consider the class of quantum correlations

$$\tilde{\mathrm{H}}_Q(\lambda) \coloneqq \lambda\mathrm{H}^{\max}_Q + (1-\lambda)\mathrm{P}_{L_1}, \qquad \lambda \in (0,1], \tag{6}$$

for which the Hardy success probability $p_{\text{Hardy}}(\lambda) = \lambda k_1 = (\lambda/2)(5\sqrt{5}-11)$. Then, on applying the OR-AND protocol to two copies of (parent) $\tilde{\mathrm{H}}_Q(\lambda)$ yields the child $\tilde{\mathrm{H}}^{(2)}_Q(\lambda)$ with Hardy success probability $p^{(2)}_{\text{Hardy}}(\lambda) = [\lambda(k_1+2k_2)+2(1-\lambda)]\lambda k_1$. Then it follows that $p^{(2)}_{\text{Hardy}}(\lambda) > p_{\text{Hardy}}(\lambda)$ for $\lambda \in (0,\phi^{-1})$, where ϕ is the golden ratio, i.e., $\phi = [(1+\sqrt{5})/2]$. On considering sufficiently large copies (n) of very weakly nonlocal correlations $\tilde{\mathrm{H}}_Q$ with $p_{\text{Hardy}}(\lambda) \to 0$, we find that the OR-AND wiring results in a Hardy correlation with a considerably large nonlocal strength $p^{(n)}_{\text{Hardy}} = 0.041$ (see Supplemental Material [62]). ■

In the Supplemental Material [62] we further shown that arbitrarily weak quantum Hardy correlation can be distilled up to 0.0433. Since a correlation with Hardy success p_{Hardy}

yields CHSH value $2 + 4p_{\text{Hardy}}$ [51], in order to measure the efficacy of distillation let us define the Tsirelson gain in percentage as follows:

$$\Delta\mathcal{T} := \frac{1}{2(\sqrt{2}-1)}(\mathcal{B}_{\text{Child}} - \mathcal{B}_{\text{Parent}}) \times 100\%. \qquad (7)$$

Manifestly, the gain will be 100% when by wiring a nonlocal correlation with arbitrary small CHSH violation, the distilled correlation achieves Tsirelson's bound—the maximum CHSH value in $\mathcal{Q}$ [63]. We then obtain that under the OR-AND wiring a quantum Hardy correlation can yield Tsirelson gain at-most 20.9%.

While applying the multicopy OR-AND protocol it turns out that the optimal distillation of Hardy's success is obtained with a threshold number of initial boxes, and the success gets decreased if more numbers of initial boxes are considered. Our next proposition provides an exact expression for the optimal number of the initial Hardy correlation required for maximal distillation.

Proposition 2.—A no-signaling Hardy correlation of the form of Eq. (5) yields Hardy success $p^{(n)}_{\text{Hardy}} = [(c_0/2) + c_1]^n - c_1^n$, when its n copy is wired under OR-AND protocol. The optimal value of distilled Hardy success is given by $p^{\text{opt}}_{\text{Hardy}} = \max\{p^{(\text{N})}_{\text{Hardy}}, p^{(\text{N}+1)}_{\text{Hardy}}\}$, where

$$\text{N} := \left\lfloor \log\left(\frac{\log c_1}{\log \frac{c_0}{2} + c_1}\right) \Big/ \log\left(\frac{\frac{c_0}{2} + c_1}{c_1}\right)\right\rfloor. \qquad (8)$$

Proof provided in Supplemental Material [62]. Next, we consider quantum nonlocal correlation, not necessarily in Hardy form, to establish an even higher Tsirelson gain through the OR-AND wiring.

Theorem 2.—Starting with a quantum correlation with arbitrarily small CHSH nonlocality OR-AND wiring can yield Tsirelson gain up to (≈)39.75%.

A proof of Theorem 2 follows similar arguments as Theorem 1. However, for the sake of completeness a detailed proof is provided in Supplemental Material [62].

Theorem 2 has many interesting implications for information processing tasks, wherein higher CHSH violation is desirable for higher performance of the protocols. For instance, the amount of certifiable randomness as obtained in [17] monotonically scales with the degree of violation of CHSH inequality. On the other hand, the authors in [22] come up with a conflicting interest Bayesian game where the payoff in correlated equilibrium strategy increases linearly with the amount of CHSH violation (see also [23]). More recently, the authors in [24] proposed a communication task where preshared entanglement between sender and receiver is shown to enhance the communication utility of a perfect classical communication channel. As it turns out payoff of this task is also a linear function of the value of CHSH expression [25].

By now, an observant reader might have already noticed that the local box P_{L_1} plays crucial role in the proof of Theorem 1 and Theorem 2. We will now use this observation to prove a generic result as formalized in the following theorem.

Theorem 3.—CHSH nonlocality of any no-signaling correlation of the form $\tilde{\text{C}}(\lambda) = \lambda\text{C} + (1-\lambda)P_{L_1}$, where $0 < \lambda \leq 1$ and $\text{C} \in \text{ConvexHull}\{\text{P}_{NL}, \text{P}_{L_i} | i \in \{1, \ldots, 8\}\}$ can be distilled through OR-AND wiring by choosing the values of λ sufficiently small. Furthermore, 2-copy OR-AND distillation is successful for all the $\tilde{C}(\lambda)$ correlation boxes whenever $\lambda < \frac{2}{3}c_0$; where c_0 is the P_{NL} fraction in C.

Proof.—Given two correlations $\chi_1, \chi_2 \in \mathcal{NS}$, let $\mathcal{W}[\chi_1, \chi_2]$ denote the resulting correlation obtained under OR-AND wiring, where $\mathcal{W}[\chi, \chi] \equiv \chi^{(2)}$. We, therefore, have

$$\tilde{\text{C}}^{(2)}(\lambda) = \lambda^2\text{C}^{(2)} + \lambda(1-\lambda)\{\mathcal{W}[\text{C}, P_{L_1}] + \mathcal{W}[P_{L_1}, \text{C}]\} + (1-\lambda)^2\mathcal{W}[P_{L_1}, P_{L_1}].$$

A straightforward calculation yields $\mathcal{W}[\text{C}, P_{L_1}] = \text{C} = \mathcal{W}[P_{L_1}, \text{C}]$, which further result in

$$\tilde{\text{C}}^{(2)}(\lambda) = \lambda^2\text{C}^{(2)} + 2\lambda(1-\lambda)\text{C} + (1-\lambda)^2 P_{L_1}.$$

While the CHSH value of the box P_{L_1} is 2, let the CHSH value of the boxes C and $\text{C}^{(2)}$ be denoted as $\mathcal{K}(>2)$ and $\mathcal{K}^{(2)}$, respectively. Then, the CHSH value of the correlations $\tilde{\text{C}}(\lambda)$ and $\tilde{\text{C}}^{(2)}(\lambda)$ can be expressed as

$$\mathcal{B}(\lambda) = \lambda\mathcal{K} + 2(1-\lambda),$$
$$\mathcal{B}^{(2)}(\lambda) = \lambda^2\mathcal{K}^{(2)} + 2\lambda(1-\lambda)\mathcal{K} + 2(1-\lambda)^2.$$

A successful distillation demands $\mathcal{B}^{(2)}(\lambda) > \mathcal{B}(\lambda)$, implying $(\mathcal{K}-2) + (\mathcal{K}^{(2)} - 2\mathcal{K} + 2)\lambda > 0$, which can be guaranteed by choosing the values for λ accordingly. This completes the first part of the proof. A bit more calculation yields the quantitative bound $\lambda < \frac{2}{3}c_0$, on the radius of the eight-dimensional Ball centered at point P_{L_1} assuring 2-copy OR-AND distillation such that any nonlocal no-signaling correlation, be it quantum or post-quantum, chosen from a nonzero-measure sector of the Ball can be distilled (in the full eight dimensions). We provide the detailed calculation in the Supplemental Material [62]. ■

Theorem 3 has profound topological implications. It establishes that the sets of no-signaling as well as quantum correlations allowing nonlocality distillation have nonzero measure in the full eight-dimensional correlation space. Furthermore, it should be mentioned that the correlation box P_{L_1} appearing in Theorem 3 is not any special local deterministic box: the result holds also for all the remaining 15 local deterministic boxes on suitable relabeling of the OR-AND wiring.

While the studies in nonlocality distillation of quantum correlations are mostly limited to the 2-2-2 Bell scenario, Theorem 3 opens up an avenue to study the same in a general N-M-K scenario that involves N spatially separated parties each performing M different measurements with K outcomes. It is not hard to find the extreme local boxes in such a general scenario. Now if for such an extreme box P_L we obtain a wiring W such that $W[P_L, X] = X = W[X, P_L]$ for any N-M-K no-signaling correlation X, then it results in a generalization of Theorem 3 in the N-M-K scenario. This consequently will imply that quantum correlations allowing nonlocality distillation have nonzero measure even in this general scenario.

Distillation of post-quantum nonlocality.—We now proceed to show that OR-AND wiring also has an important proviso in ruling out (unphysical) post-quantum correlations. Several techniques are there to establish postquantumness of a given correlation. For instance, isotropic no-signaling correlations yielding CHSH value more than $4\sqrt{2/3}$ violate the principle of nontrivial communication complexity [52] (see also [64–66]), thus demarcating such correlations as unphysical. Furthermore, any NS correlating with CHSH value more than Tsirelson bound violates the principle of information causality [53,54]. It has also been shown that a correlation might not violate these principles by its own, but after distillation the resulting correlation violates such a principle, which, in turn, establish unphysicality of the original correlation [27] (see also [39]).

Interestingly, the OR-AND wiring becomes useful to establish the postquantum nature of a correlation. Let us consider the following NS correlation

$$\mathrm{H}_{NS} = 0.1\mathrm{P}_{NL} + 0.85\mathrm{P}_{L_1} + 0.01(\mathrm{P}_{L_2} + \mathrm{P}_{L_3} + 2\mathrm{P}_{L_4} + \mathrm{P}_{L_5}), \qquad (9)$$

which exhibits Hardy's nonlocality with success probability $p_{\mathrm{Hardy}} = 0.05$. However, correlation $\mathrm{H}_{NS}^{(8)}$ obtained by distilling 8 copies of H_{NS} has the Hardy success $p_{\mathrm{Hardy}}^{\mathrm{opt}} = 0.15797$, which ensures that the boxes $\mathrm{H}_{NS}^{(8)}$ and H_{NS} are postquantum. As it turns out the correlation H_{NS} neither violates known necessary condition for respecting the principal of nontrivial communication complexity nor for the principle of information causality. However, the considered example violates the macroscopic locality principle [67] (see [62]). That being said, we do point out that checking membership to the NPA hierarchy can become computationally expensive, particularly at higher orders of the hierarchy, while the distillation criteria is far more tractable [62].

Discussion.—Distillation, the process of extracting a desirable substance in pure form from a source of impure mixture through heating and other means, has an ancient history. Quite interestingly, during the recent past, the idea finds novel applications in quantum information theory, where one aims to obtain fewer number of higher resourceful states starting with larger number of lesser resourceful states under free operations [68]. Some canonical examples are (i) for our present study, the resource theory of Bell-nonlocal boxes [69,70], or (ii) the well-known protocols for quantum entanglement distillation, where many copies of mixed entangled states are distilled into pure form under local quantum operations and classical communications [71].

In this Letter, we have established a generic approach for distillation of nonlocal correlations arising in quantum mechanics. This problem is of utmost importance as Bell nonlocal correlations are ubiquitous in device independent protocols—more the nonlocality more the utility. Interestingly, we come up with an elegant protocol, the OR-AND wiring, that distills nonlocality in quantum correlations with high efficiency. In the simplest bipartite scenario, in stark distinction with the results reported prior to our work [26–41], our protocol establishes that, within the set of full eight-dimensional correlation space, the distillable quantum as well as no-signaling nonlocal correlations form subsets of nonzero measures; i.e., sector of open balls of a specified radius centered at local deterministic correlations. Moreover, by considering correlations arbitrarily close to local deterministic points, applying our protocol, with optimal number of copies, one can distill nonlocality by a significant amount both for the quantum as well as postquantum nonsignaling correlations. As for the future, it would be interesting to explore the full potential of our generic framework proposed here in distilling quantum nonlocal correlations. In particular, obtaining some bound on the relative volume of the quantum correlations in the correlation space that can be distilled under OR-AND wiring would be interesting. Furthermore a generalization of this protocol for higher input-output as well as in a multiparty scenario might be of great use.

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