Star Shaped Quivers in Four Dimensions

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We discuss 4D Lagrangian descriptions, across dimensions IR duals, of compactifications of the 6D (D, D) minimal conformal matter theory on a sphere with arbitrary number of punctures and a particular value of flux as a gauge theory with a simple gauge group. The Lagrangian has the form of a "star shaped quiver" with the rank of the central node depending on the 6D theory and the number and type of punctures. Using this Lagrangian one can construct across dimensions duals for arbitrary compactifications (any, genus, any number and type of USp punctures, and any flux) of the (D, D) minimal conformal matter gauging only symmetries which are manifest in the ultraviolet.

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Introduction.—Understanding the dynamics of strongly coupled systems is one of the most fundamental problems in theoretical physics. In the context of quantum field theory (QFT) this often takes a guise of understanding the low energy properties of an ultraviolet (UV) free system. Although some of these properties are constrained by quantities easily computed in the UV, using, e.g., ‘t Hooft anomaly matching, a general understanding is still lacking. For example, one can be interested in understanding when the low energy theory has an emergent symmetry or when two different UV systems lead to the same low energy dynamics. Thus, seeking new methods and approaches to address such problems is of utmost importance. One such approach is to utilize renormalization group flows across different dimensions. In this Letter we derive in complete detail following this method various low energy properties of a large class of supersymmetric strongly coupled theories in four dimensions.

The notion of infrared (IR) dualities across dimensions describes the situation when two ultraviolet QFTs in different dimensions flow in the IR to the same QFT. See [1] for a recent review. As usual, one typically does not rigorously derive such dualities, as these involve strong coupling physics, but rather constructs a network of self-consistent conjectures. For example, one can consider compactifications of strongly coupled CFTs on Riemann surfaces on one hand and dual QFTs defined explicitly in two dimensions less. The latter theories then are naturally labeled by the geometric data of the compactification. This geometric labeling often leads to a geometric understanding of properties, such as in-dimension IR or conformal dualities and emergence of symmetry of the lower-dimensional theories.

In this Letter we generalize some of the known across dimensions dualities [2–5] to a large network of such dualities. In particular, we construct 4D Lagrangian duals of compactifications of $(\mathrm{D}_{N+3}, \mathrm{D}_{N+3})$ minimal conformal matter theories on spheres with punctures. Such theories are expected to have conformal manifolds with S-duality groups acting on them exchanging the various punctures. Our construction has manifest symmetry under exchanging the punctures and thus describes directly the S-duality invariant locus of the conformal manifold. Moreover we obtain duals to sphere compactifications with more than two maximal punctures with puncture symmetries manifest in the UV. Using these one can construct across dimensions duals to compactifications on surfaces of arbitrary genera with no need to rely on gauging of emergent symmetries, contrary to other constructions [4,6]. Moreover, for a general genus, values of flux, and numbers of punctures, the 4D Lagrangian theory has the structure of a "*star shaped quiver theory,*" reminiscent of 3D Lagrangians for class $\mathcal{S}$ compactifications [7]. Contrary to the latter case the rank of the central gauge node depends not just on the data of the 6D theory but also on the topology of the compactification surface.

D-type conformal matter on a sphere.—We consider a sequence of 6D (1, 0) conformal theories, the $(\mathrm{D}_{N+3}, \mathrm{D}_{N+3})$ minimal conformal matter [8], compactified on a Riemann surface. These 6D theories are engineered by studying the low energy dynamics of a single $M5$-brane probing a D_{N+3} singularity in M theory. We refer the reader to [3,6,9] for discussions of facts about these 6D SCFTs we will quote

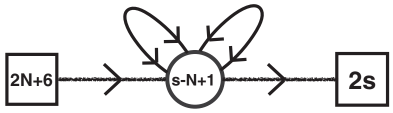

FIG. 1. The gauge theory across dimensions dual to compactification on a sphere with s minimal punctures of the (D_{N+3}, D_{N+3}) minimal conformal matter theory.

below. We make the following conjecture which we will next thoroughly explore. The theory across dimensions dual to the compactification of (D_{N+3}, D_{N+3}) minimal conformal matter on a sphere with s minimal SU(2) punctures, and certain value of flux to be discussed momentarily, is the $\mathrm{SU}(s+N-1)$ gauge theory with $2N+6$ fundamental fields Φ_i, $2s$ antifundamental fields Q^a_j ($a = 1, \ldots, s-N+1$, $j = 1, \ldots, 2s$, two fields in two index antisymmetric representation $A^{1,2}$, and a superpotential,

$$W = \sum_{I=1}^{2} h_I^{ij} Q_i^a Q_j^b A^I_{ab}, \qquad (1)$$

with h_I^{ij} being generic coupling constants. See Figs. 1 and 2. The symmetry preserved by this superpotential for general couplings is $\mathrm{U}(1)_u \times \mathrm{SU}(2N+6) \times \mathrm{SU}(2)^s$. The $\mathrm{U}(1)_u \times \mathrm{SU}(2N+6)$ part of the symmetry is a subgroup of the global $G_{6d} = \mathrm{SO}(4N+12)$ symmetry of the SCFT in six dimensions. Each SU(2) factor will be associated with a minimal puncture. The minimal punctures are defined by a 5D limit of the 6D SCFT such that the 5D effective field theory description is a $\mathrm{USp}(2N)$ gauge theory, in terms of which we consider boundary conditions at the puncture preserving $\mathrm{SU}(2) \subset \mathrm{USp}(2N)$. The theory has an $(s-3)$-dimensional conformal manifold corresponding to the complex structure moduli of the compactification surface. On this conformal manifold there are special loci where the symmetry enhances to various proper subgroups of $\mathrm{USp}(2s)$. We expect the theory to have an action of S-duality group exchanging the punctures. We can think of loci with enhanced symmetry as loci of collision of punctures. (See [10] for similar effects in class $\mathcal{S}$ [11,12].) Whenever ℓ punctures collide the symmetry is enhanced as $\mathrm{SU}(2)^\ell \to \mathrm{USp}(2\ell)$. Starting with the maximal puncture, with $\mathrm{USp}(2N)$ symmetry corresponding to the 5D gauge group, and partially closing it by Higgs branch flows, one can obtain punctures with symmetry $\mathrm{USp}(2n)$ for $n \in \{1 \cdots N\}$. In particular thus the above theory can describe spheres with any number and any types of USp punctures [13]. For example taking $s = 3N$ it describes the sphere theory with three maximal punctures. See Fig. 3. Next we discuss arguments in favor of this conjecture. The special case of $s = 2$ and $N = 1$ was discussed in [2], $s = 2N$ and general N in [3], $s = 3$ and $N = 1$ in [4], and $s = 4$ with $N = 1$ in [5].

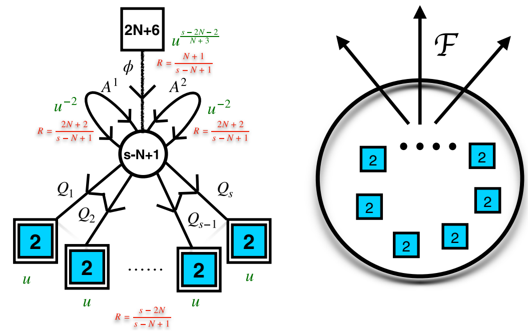

FIG. 2. The $2s$ antifundamental fields of the theory can be split into s pairs with SU(2) symmetry. Each pair is associated with a minimal puncture. In addition we have $\mathrm{U}(1)_u \times \mathrm{SU}(2N+6)$ global symmetry. The values denoted by R are 6D R-charge assignments.

The conformal manifold.—Let us analyze the conformal manifold of the theory in Fig. 1. For this we need to determine first the superconformal R symmetry as we have an Abelian factor of global symmetry, $\mathrm{U}(1)_u$. We start with the $\mathrm{SU}(s-N+1)$ gauge theory without turning on the superpotential. This gauge theory is nonanomalous and asymptotically free,

$$\mathrm{Tr} R_{\mathrm{free}} \mathrm{SU}(s-N+1)^2 = \frac{s+1-3N}{3} > 0, \qquad (2)$$

given that $s \geq 3N$. In particular for a sphere with three maximal punctures, $s = 3N$, we have an asymptotically free theory. The theory has two nonanomalous U(1) symmetries, $\mathrm{U}(1)_u$ we defined above and $\mathrm{U}(1)_v$ which can be chosen such that the fundamentals have charge $1/2N+6$ while antifundamentals have charge $-(1/2s)$. One can perform a-maximization [16] and find that the two symmetries mix with the 6D R symmetry (see Fig. 2 for the 6D R-symmetry assignment). The superconformal R charges of all the fields are above $\frac{1}{3}$ and below $\frac{2}{3}$ for all the relevant ranges of the parameters s and N. In particular the superpotential (1) is a relevant deformation. These relevant deformations are in $\mathbf{2}_A \otimes \mathbf{A}_{\mathrm{SU}(2s)}$, where $\mathbf{A}_{\mathrm{SU}(2s)}$ is the two index antisymmetric irrep of $\mathrm{SU}(2s)$ and $\mathbf{2}_A$ is the fundamental irrep of $\mathrm{SU}(2)_A$ symmetry rotating the two fields in the antisymmetric representation. We turn on first the superpotential with one antisymmetric field, say A^1, and then the other one. At each step one would need to perform a-maximization to determine the superconformal R symmetry. The first superpotential breaks the Cartan of the $\mathrm{SU}(2)_A$ and we have two Abelian symmetries which can admix with the R symmetry. The $\mathrm{SU}(2s)$ symmetry is broken to $\mathrm{USp}(2s)$ and as $\mathbf{Adj}_{\mathrm{SU}(2s)} = \mathbf{A}_{\mathrm{USp}(2s)} + \mathbf{S}_{\mathrm{USp}(2s)}$ we are left with a relevant operator in $\mathbf{A}_{\mathrm{USp}(2s)}$, no marginal operators, and symmetry $\mathrm{U}(1)^2 \times \mathrm{USp}(2s) \times \mathrm{SU}(2N+6)$. Turning on next the relevant operator in $\mathbf{A}_{\mathrm{USp}(2s)}$ we necessarily break $\mathrm{USp}(2s)$ to a subgroup, and generically to $SU(2)^s$. After the dust settles we are left with a conformal manifold of dimension $s-3$ on generic locus of which $U(1)_u \times SU(2N+6) \times SU(2)^s$ symmetry is preserved. We can, however, preserve a more general subgroup, $\otimes_{i=1}^{n} USp(2s_i)$ provided $\sum_{i=1}^{n} s_i = s$, on subloci of the conformal manifold. Insisting on turning on only deformations preserving the above symmetries we will end up with

$$s-3-\sum_{i=1}^{n}(s_i-1)=n-3, \tag{3}$$

exactly marginal deformations and marginal deformations breaking this symmetry which are in antisymmetric irrep for each factor of $\otimes_{i=1}^{n} USp(2s_i)$. The number of exactly marginal deformations preserving the symmetry is what one would expect from having n punctures [with symmetries $USp(2s_i)$]. In particular note that cases of $n=1$ or $n=2$ are special as then the dimension of the conformal manifold would have become negative. However, this is not a problem if our theory would be IR free, as happens when $s<3N$. In particular note that taking $s=2N$, $n=2$, and $s_1=s_2=N$ the model is an $SU(N+1)$ gauge theory suggested to correspond to two punctured spheres in [3]. Let us also note that the symmetries of the puncture above can be $USp(2n)$ with $n>N$. Let us refer to such punctures as *supramaximal* ones [17].

Gluings and flux.—Given the conjecture above first we can construct across dimensions dual to a sphere with three maximal punctures by taking $s=3N$. The corresponding theory is depicted in Fig. 3. Using this three punctured sphere theory we can construct across dimensions duals of compactifications on surfaces of any topology. Note that the sphere theory has a natural set of *moment map* operators [1], $M=Q\cdot\phi$, which have the following properties: they have 6D R charge equal to one; they are in the fundamental irrep of the puncture symmetry; and they are in the fundamental irrep of $SU(2N+6)$. Each component of the moment map operator M is charged under a Cartan of the 6D symmetry $G_{6D}=SO(4N+12)$. We can glue surfaces together along two maximal punctures by gauging the diagonal combination of the symmetries associated to the two punctures and turning on a superpotential. There is a choice of a superpotential,

$$W=\sum_{i\in S}M_iM_i'+\sum_{i\notin S}\Phi_i(M_i-M_i'). \tag{4}$$

Here S is a subset of the $2N+6$ components of the moment maps, M and M' are the moment maps of the two glued punctures, and Φs are chiral fields in the fundamental representation of $USp(2N)$. When S contains all the moment maps the gluing is called S gluing and when it is an empty set we call it Φ gluing. The type of gluing determines whether the fluxes corresponding to the U(1) symmetry of a component of the moment map are added (in the case of Φ gluing) or subtracted (in the case of S gluing).

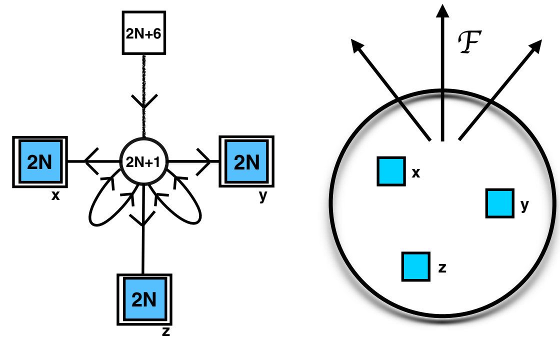

FIG. 3. An across dimensions dual of compactification on a three punctured sphere.

S gluing two spheres with $g+1$ maximal punctures together we obtain a genus g Riemann surface with vanishing flux.As the following ‘t Hooft anomaly of the puncture symmetry is

$$\mathrm{Tr}U(1)_R USp(2N)^2=\frac{1}{2}\left(\frac{s-2N}{s-N+1}-1\right)(s-N+1) = -\frac{N+1}{2}, \tag{5}$$

the gauging is not anomalous. As we glue two copies of the same theory together we also do not have a Witten anomaly. Since the charges under the U(1) symmetry of the two glued spheres are opposite, the $U(1)_u$ symmetry does not mix with the R symmetry. The conformal anomalies are then simply determined from the 6D R symmetries of the various fields,

$$a=\frac{3}{16}(g-1)N(16N+9),\qquad c=\frac{1}{8}(g-1)N(25N+18),$$

which matches perfectly with anomalies of (D_{N+3},D_{N+3}) conformal matter compactified on genus g surface with no flux [3,6,9]. Let us next analyze the dynamics of S gluing. Taking a sphere with s minimal punctures and performing a maximization, we obtain the following mixing of $U(1)_u$ with the R symmetry,

$$R_{sc}=R-q_u\frac{N+3}{3(N-s-1)(4N-s+8)}\times\left(\sqrt{2}\sqrt{38N^2-25Ns+44N+5s^2-5s+8}+6N-3s\right).$$

The anomaly $\mathrm{Tr}R_{sc}USp(2N)^2$ is less than $-(N+1/2)$ for $s<10N+2$, vanishes for $s=10N+2$, and larger than $-(N+1/2)$ for $s>10N+2$. This means that the gauging of $USp(2N)$ is relevant for $s<10N+2$, marginal for

$s = 10N + 2$, and irrelevant for $s > 10N + 2$. On the other hand the R charge of the moment maps is bigger than 1 for $s < 10N + 2$, exactly 1 for $s = 10N + 2$, and smaller than 1 for $s > 10N + 2$. This means that turning on the S-gluing superpotential coupling the moment maps is irrelevant for $s < 10N + 2$, marginal for $s = 10N + 2$, and relevant for $s > 10N + 2$. Thus for $s < 10N + 2$ we can first perform the gauging, after which the superpotential will become exactly marginal and then turn it on. For $s > 10N + 2$ we first turn on the superpotential after which the gauging becomes exactly marginal and we perform it. For $s = 10N + 2$ turning on the superpotential and the gauging together makes them exactly marginal: note that the superpotential and the gauge coupling are charged under the same anomalous symmetry with opposite signs [19]. The dynamics of S gluing always leads to an SCFT. After S gluing a pair of punctures of the two spheres the gaugings and superpotentials involved in gluing the remaining punctures become exactly marginal.

Finally, we would want also to Φ-glue two spheres together to obtain the value of flux one should turn on compactifying on a punctured sphere to obtain the theory in Fig. 1. Φ gluing two spheres with $g + 1$ maximal punctures one obtains genus g surface with the flux being twice the flux of the spheres. Here the charges of the moment maps of the glued punctures are identified and thus there is a need to perform a-maximization to determine the superconformal R symmetry. Doing so one obtains that the anomalies are consistent with the genus g surface having one unit of flux preserving $\mathrm{U}(1)\times\mathrm{SU}(2N+6)$ subgroup of the $\mathrm{SO}(4N + 12)$ symmetry of the six-dimensional theory [20].

We can compute the superconformal index [25] both for S gluing and Φ gluing and we find the following result: say for $N = 2$ and genus g building the surface from $2g - 2$ three punctured spheres,

$$\begin{aligned} S&:\ 1 + qp(3g - 3 + (g - 1)(1 + \mathbf{99} + \mathbf{45}u^2 + \overline{\mathbf{45}}u^{-2})) + \cdots, \\ \Phi&:\ 1 + qp(3g - 3 + (g - 1)(1 + \mathbf{99}) + (g - 1 + 2g - 2))\mathbf{45}u^{-2} + (g - 1 - 2g + 2)\overline{\mathbf{45}}u^2)) + \cdots, \end{aligned} \tag{6}$$

which is consistent with the S gluing having flux zero and Φ gluing having flux $1 - g$ [26,27]. In particular we observe that in S gluing the $U(1)_u \times \mathrm{SU}(2N + 6)$ symmetry of the Lagrangian enhances to $G_{6\mathrm{D}} = \mathrm{SO}(2N + 12)$ as expected. Let us also comment on the dynamics of the Φ gluing of two spheres. We need to add $2N + 6$ fields Φ in fundamental irrep of $\mathrm{USp}(2N)$ for each puncture and turn on a cubic superpotentials coupling these to moment maps. As the moment map R charges are close to one, the superpotential is always relevant and we first turn it on. The superpotentials identify all the symmetries of the two glued theories and then the gaugings becomes exactly marginal.

RG flows and dualities.—In addition to gluing surfaces along punctures we can consider closing punctures. The field theoretic procedure corresponding to closing a minimal puncture is as follows, see, e.g., Ref. [6]. First, we give a VEV (vacuum expectation value) to a component of the moment map operator, $M_i^a = \Phi_i Q^a$ which is a bifundamental of $\mathrm{SU}(2N + 6) \times \mathrm{USp}(2s)$ where i and a are the indices for the fundamentals of $\mathrm{SU}(2N + 6)$ and $\mathrm{USp}(2s)$, respectively. The VEV breaks the flavor symmetry down to $\mathrm{USp}(2(s - 1))$ and also the gauge symmetry to $\mathrm{SU}(s - N + 1) \to \mathrm{SU}(s - N)$. At the end of the RG flow, we will have the theory of a sphere with $s - 1$ minimal punctures together with extra gauge singlet fields corresponding to Goldstone modes for the broken symmetries which need to be removed by adding certain flip fields. Along the RG flow, the flavor symmetry $\mathrm{SU}(2N + 6)$ is restored as the antisymmetric fields $A^{1,2}$ decompose into two antisymmetric fields and two fundamentals for the IR $\mathrm{SU}(s - N)$ gauge symmetry and one combination of these two fundamentals provides an additional fundamental field, i.e., Φ_{2N+6}, while the other combination of the two fundamentals becomes massive due to the superpotential (1) with nonzero VEV of the field Q^s. Thus, the RG flow indeed closes a minimal puncture leaving the sphere theory with $s - 1$ minimal punctures. See Supplemental Material [21] for more details.

The theory across dimensions dual to spheres with punctures should possess conformal dualities exchanging the positions of punctures on the surface. Note that the field theory we constructed is manifestly invariant under such exchanges of the SU(2) symmetry factors and thus it flows to the locus on conformal manifold invariant under the duality. In particular, all the supersymmetric partition functions computed for this theory will be manifestly invariant under exchanging the SU(2) factors. We can take a sphere with $s = 2gN + n$ and Φ-glue pairs of punctures to form a surface of arbitrary genus g and arbitrary number of minimal punctures n. This quiver theory will be "star-shaped" and the central node is $\mathrm{SU}((2g - 1)N + s - 1)$. Moreover, we can change the value of flux by gluing in two punctured spheres. Again we obtain a Lagrangian description manifestly invariant under the dualities exchanging punctures which is reminiscent of 3D *"star-shaped"* Lagrangians of [7].

Summary.—We have constructed here explicit across-dimensions duals to *all* compactifications of a *sequence* of 6D SCFTs. In our construction of duals for general surfaces we do not need to gauge emergent symmetries. Various expected dualities are manifest. It would be interesting to understand whether similar 4D *"star-shaped"* constructions can be obtained for across dimensions duals of compactifciations of other examples of 6D SCFTs.

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