Editors' Suggestion

Trivial Andreev Band Mimicking Topological Bulk Gap Reopening in the Nonlocal Conductance of Long Rashba Nanowires

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We consider a one-dimensional Rashba nanowire in which multiple Andreev bound states in the bulk of the nanowire form an Andreev band. We show that, under certain circumstances, this trivial Andreev band can produce an apparent closing and reopening signature of the bulk band gap in the nonlocal conductance of the nanowire. Furthermore, we show that the existence of the trivial bulk reopening signature in nonlocal conductance is essentially unaffected by the additional presence of trivial zero-bias peaks in the local conductance at either end of the nanowire. The simultaneous occurrence of a trivial bulk reopening signature and zero-bias peaks mimics the basic features required to pass the so-called "topological gap protocol." Our results therefore provide a topologically trivial minimal model by which the applicability of this protocol can be benchmarked.

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Majorana bound states (MBSs) are predicted to appear in the cores of vortices or at boundaries of topological superconductors [1–5]. The non-Abelian statistics of MBSs make them highly promising candidates for fault tolerant topological quantum computing [6–11]. However, so far, despite significant efforts, there has been no conclusive experimental observation of MBSs. The most heavily investigated platform purported to host MBSs are hybrid semiconductor-superconductor devices. These devices consist of a semiconductor nanowire with strong Rashba spin-orbit interaction (SOI), e.g., InSb or InAs, that has been brought into proximity with a superconductor, e.g., NbTiN or Al [12–16]. Although the presence of zero-bias peaks (ZBPs) in local conductance measurements initially appeared to be promising evidence for MBSs in such devices [12–15,17], it was subsequently realized that the same signature could be produced by trivial effects, e.g., Andreev bound states (ABSs) [18–40].

Trivial mechanisms that mimic the expected experimental signatures of the topological superconducting phase have significantly complicated the search for MBSs. Several auxiliary features have been suggested to provide further clarity for the origin of a ZBP. Examples include oscillations around zero energy due to the overlap of the MBSs in short nanowires [41–45], the flip of the lowest band spin polarization [46,47], correlated ZBPs at either end of the nanowire, the superconducting diode effect [48,49], and a quantized conductance peak with height $2e^2/h$ [50–54]. Although oscillations and correlated ZBPs have been experimentally observed [12–15,17,55,56], these signatures can also be explained by trivial mechanisms [28,57,58].

Separately, it has been proposed that nonlocal conductance measurements in three-terminal devices, e.g., as shown in Fig. 1(a), can detect the bulk gap closing and reopening that is associated with the phase transition to topological superconductivity, potentially providing a

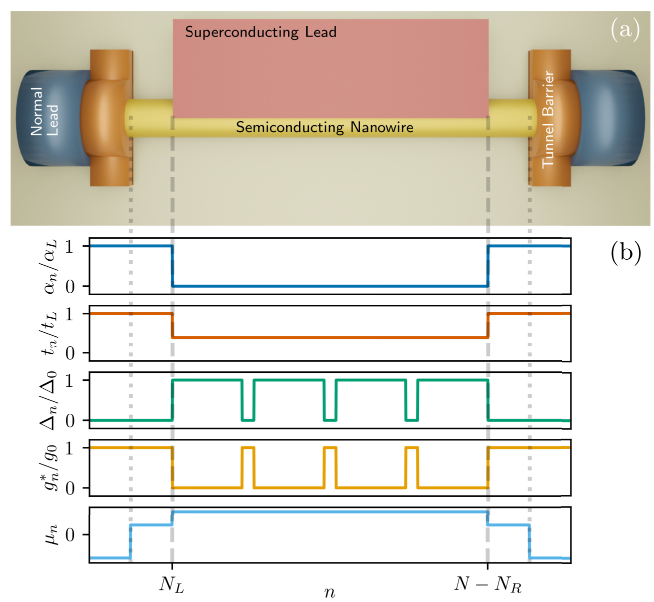

FIG. 1. Schematic sketch of a three-terminal device and a typical set of parameter profiles supporting the formation of an Andreev band. (a) A grounded superconducting lead (red) is attached to a semiconducting nanowire (yellow). Normal leads (blue), connected to the ends of the nanowire and tunnel barriers (orange), control the transparency of the interface between the normal leads and the nanowire. Experimentally several different device architectures exist but the basic features for theoretical modeling remain the same in all cases. (b) Typical parameter profile used to model ABSs forming an Andreev band and the ZBPs at either end of the nanowire.

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signature for the bulk topology of the nanowire [59–68]. In particular, it is important that the length of the proximitized region in a device is much longer than the localization length of the induced superconductivity in the nanowire; otherwise a trivial bulk reopening signature (BRS) can arise simply due to the avoided crossing of close to zero-energy ABSs [37]. When arising due to a topological phase transition, the BRS in nonlocal conductance provides an upper bound for the size of the topological energy gap [67,69]. Based on these ideas, a so-called "topological gap protocol" has been proposed [67]. The basic features required to pass this protocol are correlated ZBPs at either end of the nanowire in combination with a BRS. Recently, state-of-the-art experimental devices were reported to have passed this protocol [69].

In this Letter, we consider trivial mechanisms that can mimic the basic features of the topological gap protocol in nanowire devices, where the length of the proximitized nanowire is significantly longer than the localization length of the induced superconductivity. While trivial origins of ZBPs have been discussed extensively in the literature [20–31,33–38], trivial mechanisms that mimic the BRS are much less understood. First, we show that it is possible for multiple ABSs to form a band inside the superconducting gap. In particular, when approximately periodically spatially distributed and at similar energies, the states within the Andreev band can have a finite support throughout the nanowire. As a result we find that this Andreev band can result in a nonlocal conductance signal reminiscent of a BRS. Furthermore, we combine this trivial BRS with known mechanisms for trivial ZBPs at each end of the nanowire and show that a trivial BRS and correlated ZBPs can occur independently. Finally, we discuss the consequences for future experimental probes of topological superconducting phases.

Model.—The real-space Hamiltonian of the one-dimensional Rashba nanowire, brought into proximity with a superconductor and subject to an external magnetic field, is given by [3,4,37]

$$H = \sum_{n=1}^{N}\left(\sum_{\nu,\nu'} c^{\dagger}_{n,\nu}\left[(-t_{n+\frac{1}{2}}\delta_{\nu\nu'} + i\alpha_{n+\frac{1}{2}}\sigma^{z}_{\nu\nu'})c_{n+1,\nu'} + \frac{1}{2}\left\{(t_{n+\frac{1}{2}} + t_{n-\frac{1}{2}} - \mu_n)\delta_{\nu\nu'} + \Delta_{Z,n}\sigma^{x}_{\nu\nu'}\right\}c_{n,\nu'}\right] + \Delta_n c^{\dagger}_{n,\downarrow}c^{\dagger}_{n,\uparrow} + \text{H.c.}\right), \quad (1)$$

where $c^{\dagger}_{n,\nu}(c_{n,\nu})$ creates (annihilates) an electron at site n with spin $\nu = \uparrow, \downarrow$ in a one-dimensional chain with a total number of N sites. The Pauli matrices $\sigma^{l}_{\nu\nu'}$, with $l \in \{x, y, z\}$, act in spin space. All parameter profiles, namely hopping $t_{n\pm(1/2)}$, the proximity induced superconducting gap Δ_n, the chemical potential μ_n, the Rashba SOI strength α_n, and the Zeeman energy $\Delta_{Z,n}$, are assumed to be position dependent, indicated by the index n. A typical parameter profile is shown in Fig. 1(b) and the full mathematical expressions used can be found in the Supplemental Material (SM) [70].

Throughout we will distinguish between "interior" and "exterior" ABSs depending on whether a given ABS occurs in the bulk or at the ends of the nanowire, respectively. The position distinguishing the bulk and ends of the nanowires is indicated by gray dashed lines in Fig. 1(b). Interior ABSs arise due to "interior normal sections" that are modeled by a vanishing local proximity gap and increase in g factor at certain positions within the bulk of the nanowire. We consider distributions of interior normal sections over the full length of the nanowire that allow the formation of an Andreev band within the superconducting gap (see below). For simplicity ABSs are created using normal sections, but a modification of g factor alone is sufficient to create the Andreev band that results in a trivial BRS (see SM [70]). Separately, in order to enable the nanowire to host zero-energy exterior ABSs at its ends we also model normal sections on the left and right end, which we call "exterior normal sections," consisting of N_L and N_R sites, respectively.

The Andreev band.—We first develop a mechanism for the formation of a trivial band formed from Andreev bound states inside the superconducting gap based on the interplay of multiple ABSs. We will later consider the impact of this trivial band on nonlocal transport and trivial ZBPs. If individual ABSs are distributed in a quasiperiodic way and if, in addition, the separation between the ABSs is of the order of the superconducting coherence length, then the individual ABSs partially overlap and hybridize to form a band of Andreev states. In contrast to the individual ABSs, which are well localized, the states within this band can have a finite support throughout the nanowire. As such, we will call this band of extended Andreev states an "Andreev band" due its strong similarity with the well-studied Shiba band [78–80]. We emphasize that, unlike previous proposals for topological phases due to inhomogeneous superconductivity [81,82], the system we consider here remains trivial for all values of magnetic field. In particular, this is ensured because the bulk g factor is zero apart from in the normal sections that form the Andreev band and/or the SOI vanishes in the bulk of the nanowire.

Since the states in the Andreev band are extended they can connect the left and right normal lead and hence are visible in nonlocal differential conductances. The Andreev band emerges around the energy of the individual ABSs that form it and the bandwidth is determined by the overlap of the ABSs, which is related to the separation length between the ABSs. We note that, within our minimal model, the band width of the Andreev band is normally smaller than the size of the bulk superconducting gap such

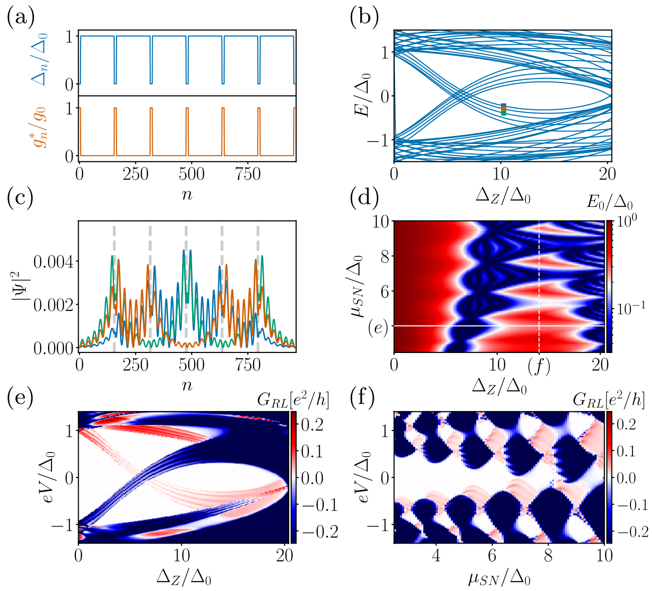

FIG. 2. Formation of the Andreev band. (a) Spatial profiles of the induced superconducting gap and of the g factor at zero Zeeman field in terms of their maximal zero-field values Δ_0 and g_0, respectively. The combination of the two profiles leads to the formation of an Andreev band inside the superconducting gap. (b) Energy spectrum of the Rashba nanowire, with Andreev band, as a function of Zeeman field. (c) Probability densities of the extended trivial Andreev states at the positions marked by the colored squares in panel (b). Gray dashed vertical lines indicate the position of the normal sections. Because of the hybridization of ABSs that form in the normal sections, the Andreev states extend throughout the nanowire. (d) Lowest energy as a function of Zeeman field and chemical potential. (e)[(f)] Nonlocal differential conductance G_{RL} as a function of Zeeman field [chemical potential] calculated at the chemical potential [Zeeman field] indicated by the white horizontal [vertical] line in panel (d). Parameters: $a = 4$ nm; $(N_L = N_R = N_{B,L} = N_{B,R}, N_S, N_N) = (5, 150, 10)$; $M = 6$; $(t_L = t_R = t_{SN}, \Delta_0, \Delta_Z^c, \alpha_L = \alpha_R, \gamma_L = \gamma_R, \mu_{\mathrm{Lead},L} = \mu_{\mathrm{Lead},R}) \approx (158, 0.6, 12.2, 0, 5, 5)$ meV; $T = 0$ mK; see also SM [70].

that there is usually a finite gap between bulk superconducting states and Andreev band states.

Trivial bulk reopening signature.—To study the transport consequences of the delocalized states in the Andreev band, we first consider a profile with periodically distributed variations in the induced proximity gap and g factor along the nanowire, as shown in Fig. 2(a). ABSs created as a result of this profile have the majority of their weight in the normal sections and hybridize to form highly extended states, as shown by the probability densities in Fig. 2(c). We note that, even though we do not consider zero-energy exterior ABSs here, short exterior normal sections are present in the model to provide tunnel barriers for the differential conductance computation, which we perform with the Python package Kwant [83].

Because of the variation in the g factor between the normal and superconducting sections, the energies of states that form the Andreev band have a different slope as a function of Zeeman field than states with the majority of their weight in the superconducting sections, as shown in Fig. 2(b). Here, the Zeeman field is defined as $\Delta_Z = g_0 \mu_B B/2$, where μ_B is the Bohr magneton, B the magnetic field, and g_0 is the g factor in the normal sections. Importantly, the larger g factor means that the Andreev band states cross zero energy considerably before the closing of the bulk superconducting gap and therefore mimic a topological BRS in the energy spectrum. We note that for our model, the slope of the energies as a function of the Zeeman field is nonlinear since we consider the superconducting gap to be a function of the Zeeman field and states leak into the regions with reduced g factor, such that the average Zeeman field experienced by the ABSs is reduced.

The zero-energy crossing of the Andreev band states results in gapped regions of phase space that are entirely surrounded by regions that are essentially gapless, i.e., with a very small lowest energy state E_0; see Fig. 2(d) and also SM [70]. This behavior of the bulk spectrum is the same when there is a topological region. Nonlocal conductance, however, measures the transport gap since it is sensitive only to states that connect left and right leads. As shown in Figs. 2(e) and 2(f), the extended nature of the states that form the Andreev band means that these are indeed visible in the nonlocal conductance. Therefore, the zero-energy crossing of the Andreev band states can mimic the transport gap closing and reopening for regions of phase space, even though, by design, the system remains entirely trivial.

Combination of trivial effects.—We now combine the trivial BRS due to an Andreev band with trivial ZBPs due to exterior ABSs at the ends of the nanowire; such zero-energy ABSs have previously been shown to be abundant in Rashba nanowires [20–38,40]. Together, these trivial features mimic the key transport signatures of the topological gap protocol, namely, the exterior ABSs result in ZBPs on either end of the nanowire and the Andreev band results in a trivial BRS. To generate the ZBPs we tune the system to a certain resonance condition for SOI strength and the length of the normal sections in order to pin the exterior ABSs to zero energy; see Refs. [25,37] for more details. However, the particular mechanism causing ZBPs at the ends of the nanowire is not the main subject of this Letter and this mechanism can be exchanged for any other that results in ZBPs, as long as the formation of the Andreev band is not affected. As previously, we set the Rashba SOI strength to zero in the superconducting sections of the nanowire, which ensures the system is always in the trivial phase. In fact, the Rashba SOI is in our model only nonzero in the normal sections at the ends of the system to provide a control knob in the simulation for the zero-energy pinning of the exterior ABSs.

In Fig. 3(a), we show the energy spectrum of a system that combines the trivial BRS due to the Andreev band and

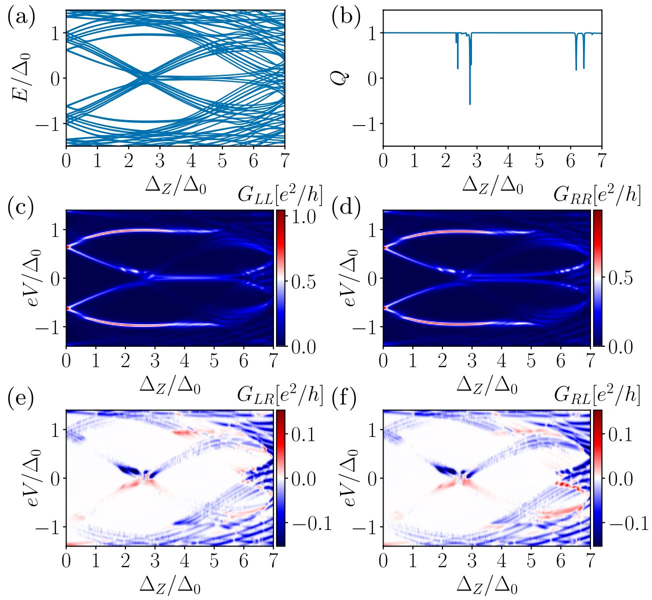

FIG. 3. Combination of the Andreev band with zero-energy states. (a) Energy spectrum showing the Andreev band crossing zero energy at the Zeeman field strength at which exterior ABSs are pinned to zero energy. (b) The topological visibility indicates the trivial nature of the system for all Zeeman fields. (c),(d) Local differential conductance calculated on the left and on the right end of the nanowire, showing pronounced ZBPs. (e),(f) Nonlocal conductances showing that the BRS is essentially unaffected by the presence of the ZBPs. Parameters: $a = 5$ nm; $(N_L, N_R, N_S, N_N, N_{B,L} = N_{B,R}) = (90, 94, 140, 30, 7)$; $M = 5$; $(t_L = t_R, t_{SN}, \mu_L = \mu_R, \mu_{SN}, \Delta_0, \Delta_Z^c, \alpha_L, \alpha_R, \gamma_L = \gamma_R, \mu_{\mathrm{Lead},L} = \mu_{\mathrm{Lead},R}) = (100, 20, 0, 2, 0.25, 1.75, 13.75, 13.20, 10, 40)$ meV; $T = 40$ mK. See also SM [70].

trivial states with almost zero energy. Here, we tune the right exterior ABS slightly away from zero energy in order to show that the exterior ABSs are independent of each other. In addition to the spectrum, we also calculate the topological visibility Q [84,85], which is positive over the majority of the range of Zeeman field strengths, as expected since our system is always in the trivial phase; see Fig. 3(b). We note that when Andreev band states cross zero energy then the unitary property of the reflection matrix breaks down and Q is ill-defined; see the SM [70].

Finally, the differential conductance matrix elements are shown in Figs. 3(c)–3(f). The trivial ZBPs due to the ABSs localized at both ends of the nanowire are clearly visible in the local differential conductances [see Figs. 3(c) and 3(d)] but they do not appear in the nonlocal conductance, as expected for the ZBP signatures predicted for well-separated MBSs in long nanowires [37,66]. The Andreev band is visible in the nonlocal conductance and is also weakly pronounced in the local conductance. Hence, the Andreev band results in signature that mimics a topological BRS also in the case where trivial ZBPs occur at either end of the nanowire.

Experimental relevance and outlook.—Our mechanism for a trivial BRS requires that a few ABSs occur at similar energies and are approximately equally spatially distributed within the bulk of the nanowire. Local conductance measurements on experimental state-of-the-art Rashba nanowire devices reveal many subgap states, indicating that many ABSs remain prevalent even in high quality devices [69]. We also note that, due to the long localization length of bound states in current nanowires (normally at least several 100 nm [69]), the distribution only needs to include a small number of ABSs, even in nanowires with lengths of several micrometers. More precisely we find the Andreev band in systems with a length of $L \lesssim 10$ localization lengths can tolerate sizeable deviations from a periodic distribution; see the SM [70] for a systematic analysis. The prevalence of ABSs in current devices and the small number of ABSs required means that it is likely there are regions of parameter space in current devices where a trivial BRS can occur. Because of their independence, the probability of the combination of trivial ZBPs and trivial BRS occurring simultaneously can be approximated from the number of occurrences of each individual signature.

We want to emphasize that the mechanism described above is much more general than the trivial BRS due to avoided crossing of ABSs in short nanowires described previously [37]. The Andreev band mechanism for a trivial BRS is relevant for systems longer than the localization length. Additionally, the Andreev band contains many states that contribute to the trivial BRS and is therefore much more reminiscent of a bulk signature.

Our results suggest a road map to conclusively observe MBSs in nanowire devices. First, it remains crucial to determine the magnitude of the necessary parameters, such as SOI, in the presence of a superconductor where metallization effects can drastically reduce their values [86]. With this in mind, it might be desirable to switch device architecture or material platform to increase the phase space for MBSs and/or associated energy scales [87–89]. Second, the idea of combining multiple indicators, as suggested by the topological gap protocol, can reduce the probability that trivial mechanisms simultaneously result in similar signatures as a topological phase transition for large continuous regions of phase space. In addition to local and nonlocal conductance, further auxiliary features (see the introduction [41–54]) could also be used to increase confidence that the topological phase transition is really being observed. Reducing disorder further lowers the probability of many trivial mechanisms for MBS indicators, including the trivial BRS discussed here. Finally, as discussed above, the trivial BRS becomes less likely when the length of the system is increased. Practically, this means that nanowires should be made with a length that is substantially longer than the superconducting coherence length, such that the probability of a trivial BRS in nonlocal conductance is suppressed; see SM [70].

Conclusions.—We have shown here that, when approximately equally distributed throughout the nanowire and at similar energies, multiple ABSs can hybridize to form an Andreev band within the proximity induced superconducting gap of a Rashba nanowire. This Andreev band can cross zero energy and therefore mimics a topological bulk closing and reopening signature. The trivial BRS discussed here can be easily combined with well-known mechanisms for trivial ZBPs since we find both phenomena are essentially independent of each other. Our simple model contains the basic features of the topological gap protocol [67,69] and thus can serve as a basis for the applicability of this protocol. In particular, such a mechanism can provide a benchmark as to whether ZBPs in local conductance and a BRS in nonlocal conductance alone are sufficient to distinguish a trivial and topological phase, especially when these signatures occur in only a small region of phase space.

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