Line of Fixed Points in Gross-Neveu Theories

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In the limit of many fermion flavors it is demonstrated that the sextic Gross-Neveu theory in three dimensions displays a line of interacting UV fixed points, characterized by an exactly marginal sextic interaction. We determine the conformal window of UV-complete theories, universal scaling dimensions, and the phase diagram using renormalization group methods. Massless theories arise naturally, and the generation of mass proceeds without the breaking of a discrete symmetry. Striking similarities with critical scalar theories at large N are highlighted, and implications from the viewpoint of conformal field theory and the AdS/CFT conjecture are indicated.

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Introduction.—The large N limit, where N denotes the number of particle species or flavors, is an important tool in quantum and statistical physics [1]. Large N limits often enable rigorous control over fluctuations and critical points [2–4] including beyond perturbation theory [5,6], and offer wide-ranging applications from proofs of nonperturbative renormalizability [7] and new quantum critical points [8,9] or symmetry breaking mechanisms [10], to equivalences [11,12], dualities [13,14], and the AdS/CFT conjecture [15], which further relates large N field theories to higher-dimensional duals [16].

The 3D Gross-Neveu theory of interacting fermions, originally introduced for the study of chiral symmetry breaking [17], is an important toy model in particle and condensed matter physics. Its four-fermion coupling $\sim(\bar\psi_a\psi_a)^2$ makes the theory perturbatively nonrenormalizable; yet nonperturbatively, it develops an interacting fixed point [18], including at large N [7], which renders the theory UV complete and predictive at all scales [19,20]. Besides offering a benchmark for 3D conformal field theories and chiral symmetry breaking, it has also been shown to be dual to Vassiliev's higher spin theories under the $\mathrm{AdS}_4/\mathrm{CFT}_3$ conjecture [16,21], and serves as a toy model for asymptotic safety of gravity [22].

In this Letter, we investigate large N Gross-Neveu theories in the absence of chiral symmetry [23]. The generation of mass is no longer protected by symmetry, and the impact of chirally odd interactions such as $\sim(\bar\psi_a\psi_a)^3$ on the short distance physics needs to be clarified. Our method of choice is functional renormalization [24–27], which, in combination with the large N limit, enables a nonperturbative study of fixed points, scaling dimensions, and UV-IR connecting trajectories [28–32] in a purely fermionic formulation [33]. We thereby determine all UV-complete Gross-Neveu theories, with or without chiral symmetry. We also uncover striking similarities between large N fermionic and large N bosonic theories, and indicate links with conformal field theory and higher spin gauge theories.

Sextic Gross-Neveu theory.—We consider $U(N)$ symmetric classical actions for N Dirac fermions ψ_a in three Euclidean dimensions, given by

$$S_f=\int_x\left\{\bar\psi_a(\partial\!\!\!/+M)\psi_a+\frac{G}{2}(\bar\psi_a\psi_a)^2+\frac{H}{3!}(\bar\psi_a\psi_a)^3\right\}. \quad (1)$$

In addition to the kinetic term and a mass term M, we observe a four-fermion interaction with coupling G, and a six-fermion interaction with coupling H. In the limit $M=H=0$ the theory reduces to the standard Gross-Neveu model [17] with manifest invariance under

$$\psi\to\gamma^5\psi,\qquad \bar\psi\to-\bar\psi\gamma^5,\qquad \bar\psi\psi\to-\bar\psi\psi, \quad (2)$$

which corresponds to chiral symmetry (parity) in even (odd) dimensions. In the chiral limit, interactions include even powers of the bilinear $(\bar\psi_a\psi_a)$, the leading one being the four-fermion (4F) interaction with coupling G. Owing to its negative canonical mass dimension, $[G]=-1$, the theory is perturbatively nonrenormalizable by power counting, yet nonperturbatively renormalizable due to the existence of an interacting ultraviolet fixed point [7,18]. The 4F coupling becomes a relevant interaction while all higher-order chirally invariant interactions remain irrelevant. The discrete symmetry, Eq. (2), further entails that the theory is fundamentally massless, although mass can be generated dynamically in the infrared.

Without chiral symmetry, mass terms $M \neq 0$ and new interactions such as odd powers in $(\bar{\psi}_a\psi_a)$ become available, the leading one being the six-fermion (6F) coupling H. It is the central purpose of this work to establish that theories with $H \neq 0$ can be defined fundamentally, despite of the fact that negative mass dimension of the 6F coupling $[H] = -3$ implies that the theory is power-counting nonrenormalizable in perturbation theory.

*Renormalization group.—*To achieve our claims, we investigate the theory, Eq. (1), nonperturbatively with the help of Wilson's renormalization group (RG) [24–27] based on the successive integrating out of momentum modes from a path integral representation of the theory. The scale dependence of the quantum effective action Γ_k is given by an exact identity [25],

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\{[\Gamma_k^{(2)} + R_k]^{-1} \cdot \partial_t R_k\}, \tag{3}$$

where k denotes the RG momentum scale with $t = \ln k$, $\Gamma_k^{(2)}$ denotes the matrix of second functional derivatives, and the supertrace STr stands for a sum over all momenta and fields, also accounting for relative minus signs through fermionic degrees of freedom. As a function of k, the flow, Eq. (3), interpolates between the microscopic action in the ultraviolet $(1/k \to 0)$ and the full quantum effective action in the infrared $(k \to 0)$. Within a few constraints, the infrared cutoff function $R_k(q)$ can be chosen freely [34–36]. It ensures that the propagation of momentum modes is suppressed for $q^2 \ll k^2$ and that the flow remains finite and well-defined for all scales. Further, Wetterich's RG, Eq. (3), reproduces standard perturbation theory at weak coupling and is related to Polchinski's RG (based on an UV cutoff) [24] by a duality transform [29,32]. We use Eq. (3) to identify interacting UV fixed points in the fermionic theory, Eq. (1). For similar studies in scalar and supersymmetric theories, see [20,30–32,37,38].

*Fermionic flow.—*The theory, Eq. (1), is investigated by writing its effective action as

$$\Gamma_k[\bar{\psi}, \psi] = \int d^3x \{\bar{\psi}_a \partial\!\!\!/ \psi_a + V_k(\bar{\psi}_a\psi_a)\}, \tag{4}$$

where the function V_k accounts for all nonderivative interactions. In the large N limit, the functional flow, Eq. (3), does not generate derivative interactions other than those already present in the initial action [29]. Consequently, the anomalous dimension vanishes, and Eq. (4) is exact under the flow, Eq. (3), and valid for all scales. The flow for the function V_k is obtained by inserting the ansatz Eq. (4) into Eq. (3) and projecting onto constant fields [33,39]. Introducing dimensionless variables $z = \bar{\psi}_a\psi_a/k^2$ and $v(z;t) = V_k(\bar{\psi}_a\psi_a)/k^3$ gives

$$\partial_t v + 3v - 2zv' = -3A \int_0^\infty dy \frac{y^{5/2}(1+r)r'(y)}{y(1+r)^2 + (v')^2}, \tag{5}$$

where $y = q^2/k^2$, $v' = \partial_z v$, and $R_k(q) = q\!\!\!/ r(y)$ with $r(y)$ the cutoff shape function [34,35]. The terms on the left-hand side account for the canonical scaling of v and z, while the integral on the right-hand side, a remnant of the operator trace in Eq. (3), is due to quantum fluctuations. We remove the constant $A = 4N S_3/(2\pi)^3$ with S_3 the surface area of a unit 3-sphere by rescaling it into $v \to v/A$ and $z \to z/A$, implying that couplings are now measured in units of perturbative loop factors, in accord with naïve dimensional analysis [40].

*Running couplings and line of fixed points.—*Fixed points are the t-independent solutions $v_*(z)$ of $\partial_t v = 0$. The free theory of noninteracting fermions corresponds to $v'_* = 0$. To find interacting fixed points, we expand $v(z) = \sum_{n=1}^\infty \lambda_n z^n/n!$ in terms of couplings λ_n describing the $2n$ fermion $(2nF)$ interactions, with $\lambda_{1,2,3}$ the dimensionless counterparts of M, G, H in Eq. (1). Their flows $\partial_t\lambda_n$ follow from Eq. (5) by projection. For concreteness, we use an optimized regulator $r(y) = (1/\sqrt{y} - 1)\theta(1-y)$ that permits simple analytical expressions [34–36,41]; key results are independent of this choice.

Starting with the dimensionless mass, we find that its flow is driven by the mass and the 4F coupling,

$$\partial_t\lambda_1 = -\lambda_1\left(1 - \frac{2\lambda_2}{(1+\lambda_1^2)^2}\right). \tag{6}$$

We observe that the fermion mass is natural [42] in that quantum fluctuations or interactions cannot switch it on if it has been set to zero initially, even if chiral symmetry is absent. Contributions proportional to the chirally odd 6F coupling do not arise because the exact flow, Eq. (5), does not involve v'' on its right-hand side. Turning to the 4F coupling, we observe that its flow is driven by the mass and the 4F and 6F couplings

$$\partial_t\lambda_2 = \lambda_2 + \frac{(2 - 6\lambda_1^2)\lambda_2^2}{(1+\lambda_1^2)^3} + \frac{2\lambda_1\lambda_3}{(1+\lambda_1^2)^2}. \tag{7}$$

Fluctuations and interactions can switch on 4F interactions even if they were set to vanish initially. However, any dependence on the chirally odd interactions drop out as soon as the mass term vanishes, leading to

$$\partial_t\lambda_2|_{\lambda_1^*=0} = \lambda_2(1 + 2\lambda_2). \tag{8}$$

Besides the free fixed point we observe an interacting one for the 4F coupling $\lambda_2^* = -\frac{1}{2}$. It coincides with the well-known ultraviolet fixed point in the chirally symmetric limit [7,18,20,22,43–46].

We now turn to the 6F coupling, which is the lowest-order chirally odd interaction. Its RG flow is driven by the mass and the 4F, 6F, and 8F couplings

$$\partial_t\lambda_3 = 3\lambda_3 + 6\lambda_2\lambda_3\frac{1 - 3\lambda_1^2}{(1+\lambda_1^2)^3} + 2\lambda_1\frac{\lambda_4 - 12\frac{\lambda_2^3(1-\lambda_1^2)}{(1+\lambda_1^2)^2}}{(1+\lambda_1^2)^2}. \tag{9}$$

Fluctuations can switch on 6F interactions even if they were absent initially. However, provided the mass term vanishes, the beta function simplifies into

$$\partial_t \lambda_3|_{\lambda_1^*=0} = 3(1+2\lambda_2)\lambda_3. \tag{10}$$

Most notably, $\partial_t \lambda_3$ vanishes identically, and independently of λ_3, provided that the mass and the 4F coupling take their respective fixed point values. In other words, quantum fluctuations have turned the perturbatively irrelevant 6F coupling into an exactly marginal one, which takes the role of a new fundamental parameter.

For all higher-order interactions, solving $\partial_t \lambda_{n\geq 4} = 0$ gives the $2nF$ couplings at a fixed point as functions of the 6F coupling, $\lambda^*_{n\geq 4} = P_n(\lambda_3^*)$, with P_n polynomials of degree $n-2$ with vanishing constant term for n odd. Then, all $2nF$ interactions with $n > 3$ are found to be irrelevant nonperturbatively without offering new fundamentally free parameters. We conclude that the theory displays a line of critical points parameterized by λ_3^*, which reduces to known results in the chiral limit.

Global fixed points.—Next, we show that the line of interacting fixed points is limited to a finite range in λ_3^* values. To that end, we integrate Eq. (5) at a fixed point directly, without first resorting to a polynomial expansion. Using the method of characteristics [28,31,32,37] leads to solutions of the form $z = z(v')$ with

$$z = 4\lambda_3^*(v')^2 - v'\left[\frac{2+3(v')^2}{1+(v')^2} + 3v' \arctan v'\right], \tag{11}$$

whose validity is confirmed by direct inspection. Further, inverting the solution into $v'(z)$ and expanding it in a power series for small z, we recover all $2nF$ fixed point couplings determined previously. The virtue of the closed expression, Eq. (11), is its validity even beyond the radius of convergence of polynomial expansions.

As a physical requirement, we demand fixed points with Eq. (11) to exist "globally," meaning for all fields z. Interestingly, this requirement is not empty: if and only if $|\lambda_3^*|$ remains below a critical vale $\lambda_3^{\rm crit}$, the fixed point $v(z)$ exists for all values of the field z [47] (similar restrictions are known from other large N critical theories at strong coupling [20,30–32,37,38]). This can also be appreciated from recursively solving all $\partial_t \lambda_{n\geq 1} = 0$ to find λ^*_{n+1} as functions of λ_1^*. Resumming the expressions shows that global solutions do not exist if $\lambda_1^* \neq 0$, leading to the same constraint. All in all, we conclude that

$$\lambda_1^* = 0, \quad \lambda_2^* = -\frac{1}{2}, \quad 0 \leq |\lambda_3^*| < \lambda_3^{\rm crit} \tag{12}$$

characterizes the complete set of interacting and globally well-defined fixed points. While the values λ_2^* and $\lambda_3^{\rm crit}$ are nonuniversal ($\lambda_3^{\rm crit} = (3\pi/8)$, here), we have checked by varying the cutoff shape function in Eq. (5) that the existence of a line with an upper bound is universal [31,47]. Chiral symmetry is only available if $\lambda_3^* = 0$. Figure 1 illustrates two examples for viable fixed points.

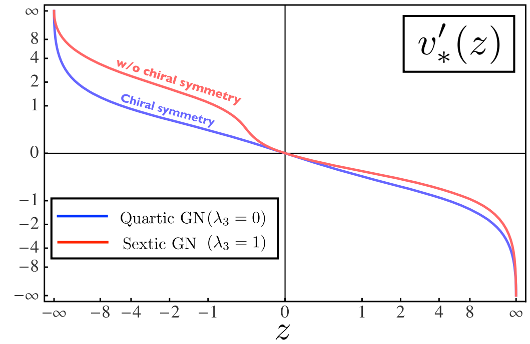

FIG. 1. Shown are ultraviolet fixed point solutions $v'_*(z)$ for all fields, comparing UV-complete quartic and sextic Gross-Neveu theories (axes scaled as $x \to [x/(2+|x|)]$).

Scaling dimensions and fundamental parameters.—In the vicinity of a fixed point, Eqs. (4) and (5) can be expanded in a basis of scaling operators $\mathcal{O}_n$ that scale $\sim k^{-3+\vartheta_n}$ with universal exponents ϑ_n. In the free theory that is an infrared fixed point, we have $\mathcal{O}_n \sim (\bar\psi\psi)^n$ and exponents are given by (minus) their canonical mass dimensions,

$$\vartheta_n^{(\rm IR)} = 2n - 3 \qquad (n \geq 1), \tag{13}$$

with the mass being the sole relevant perturbation $\vartheta_1^{(\rm IR)}$. In turn, the fixed points, Eq. (12), are all UV. For the universal scaling dimensions of small perturbations we find

$$\vartheta_n^{(\rm UV)} = n - 3 \qquad (n \geq 1). \tag{14}$$

The relevant perturbations $\vartheta_{1,2}^{(\rm UV)}$ and the marginal $\vartheta_3^{(\rm UV)}$ relate to the mass, the 4F and the 6F interactions, respectively. Fluctuations have shifted scaling dimensions substantially away from canonical values, Eq. (13),

$$\frac{\vartheta_n^{(\rm IR)} - \vartheta_n^{(\rm UV)}}{\vartheta_n^{(\rm IR)}} = \frac{n+1}{2n-1}, \tag{15}$$

a testament to the theory being strongly coupled in the UV. We note that even though scaling dimensions are the same along the line of fixed points, the underlying discrete symmetries are not. Further, at the endpoints of the critical line ($|\lambda_3^*| = \lambda_3^{\rm crit}$) scale symmetry is broken spontaneously, akin to the Bardeen-Moshe-Bander phenomenon [5,10,48], and scaling dimensions deviate from the values given in Eq. (14) (see [47] for a detailed analysis). We conclude that the perturbatively nonrenormalizable theories, Eq. (1), are well-defined due to the line of fixed points, Eq. (12), and

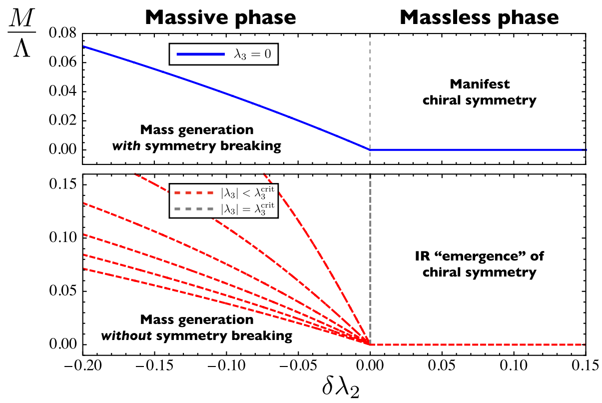

FIG. 2. The dynamical generation of a fermion mass M takes the form of a second order quantum phase transition in $\delta\lambda_2(\Lambda)$, irrespective of whether the underlying Lagrangian is chirally invariant (upper panel) or not (lower panel).

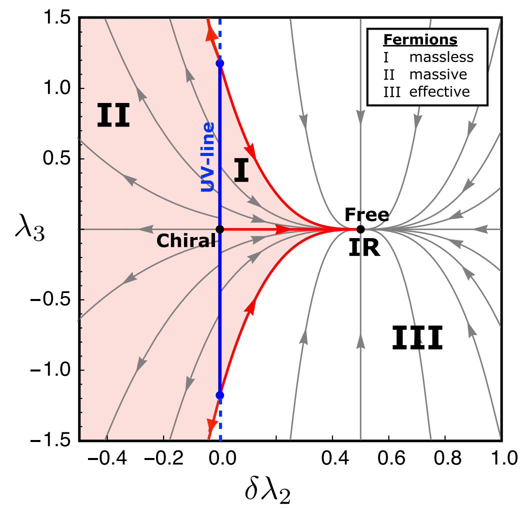

FIG. 3. Phase diagram of the fermionic theory, Eq. (1), in the $(\delta\lambda_2, \lambda_3)$ plane with $\delta\lambda_1 = 0$. For $\lambda_3 \neq 0$ chiral symmetry is absent fundamentally and "emerges" at the IR fixed point.

fundamentally characterized by the exactly marginal coupling λ_3^* and the relevant perturbations $\delta\lambda_1 = \lambda_1 - \lambda_1^*$ and $\delta\lambda_2 = \lambda_2 - \lambda_2^*$ at the high scale Λ.

Generation of mass.—The absence of chiral symmetry on the level of the fundamental action implies that mass can be generated explicitly as soon as $\delta\lambda_1 \neq 0$ at the high scale. If so, this relevant perturbation triggers an RG flow towards the IR, for all couplings, which invariably entails a massive fermionic theory.

As soon as $\delta\lambda_1 = 0$, however, mass is not generated explicitly. This is noteworthy in that it shows that chiral symmetry, which removes all $2nF$ interactions $\lambda_{n=\mathrm{odd}}$, is not necessary to ensure massless fermions. Rather, the significantly milder constraint $\delta\lambda_1 = 0$ is already sufficient, courtesy of λ_1 being natural, Eq. (6).

Still, mass can be generated dynamically through strong interactions. Parametrically, this takes the form of a second order quantum phase transition controlled by the microscopic parameter $\delta\lambda_2$, with $\delta\lambda_2 < 0$ ($\delta\lambda_2 > 0$) leading to a massive (massless) phase. In the presence of chiral symmetry, Eq. (2), strong 4F interactions are responsible for the generation of mass (Fig. 2, upper panel). In the absence of chiral symmetry, 6F interactions additionally enhance the mass with growing $|\lambda_3|$, until it becomes indeterminate for finite sextic coupling $|\lambda_3| \to \lambda_3^{\mathrm{crit}}$ right at the endpoint of the conformal window (Fig. 2, lower panel). We emphasize that mass is generated without the breaking of a discrete symmetry. Moreover, for $\delta\lambda_2 > 0$, we observe the "emergence" of chiral symmetry in the IR, courtesy of the fully attractive free fixed point. Lastly, mass can also be generated explicitly as soon as $\delta\lambda_1 \neq 0$, which then takes the form of a crossover as a function of $\delta\lambda_2$.

Fermionic phase diagram.—We are now in a position to discuss the full phase diagram of the theory. Figure 3 shows the UV line of chirally nonsymmetric fixed points, Eq. (12), where $\lambda_3^* \neq 0$, its endpoints, and the chirally symmetric UV and IR fixed points, all in the $(\delta\lambda_2, \lambda_3)$ plane and for $\delta\lambda_1 = 0$. Arrows point from the UV to the IR. Trajectories emanating from the blue line correspond to UV-complete fundamental theories (red-shaded areas). In region I ($\delta\lambda_2 > 0$ and $0 < |\lambda_3^*| < \lambda_3^{\mathrm{crit}}$), they connect interacting UV conformal fixed points with the free theory in the IR. These theories remain strictly massless. Moreover, even though chiral symmetry is absent microscopically, it emerges in the IR. In region II ($\delta\lambda_2 < 0$ and $0 < |\lambda_3^*| < \lambda_3^{\mathrm{crit}}$), strong interactions lead to the dynamical generation of mass. Here, chiral symmetry is manifestly absent at all scales, and trajectories connect UV conformal fixed points with massive Gross-Neveu theories in the IR. Region III relates to all trajectories that do not start out at the UV line. These "swampland" theories are not UV-complete and must be seen as effective rather than fundamental. As soon as $\delta\lambda_1 \neq 0$, mass is also generated explicitly and trajectories starting from the UV line or in the swampland invariably lead to massive theories.

Bosonic phase diagram.—It is interesting to compare our results with $O(N)$ [or $U(N)$] symmetric sextic scalar theory of 3D real [or complex] bosons ϕ_a at large N [31,32,49] with action

$$S_b = \int_x \left\{ \frac{1}{2}(\partial\phi)^2 + \frac{M^2}{2}\phi^2 + \frac{G}{2}(\phi^2)^2 + \frac{H}{3!}(\phi^2)^3 \right\}. \quad (16)$$

The bosonic theory is super-renormalizable in the UV where it achieves a line of conformal fixed points owing to an exactly marginal sextic scalar self-coupling [49]. It also displays a strongly interacting Wilson-Fisher fixed point in the IR (Fig. 4). In turn, the fermionic theory, Eq. (1), is perturbatively nonrenormalizable, yet it develops a line of strongly interacting critical points in the UV owing to an exactly marginal six fermion coupling, alongside the isolated free fixed point in the IR (Fig. 3).

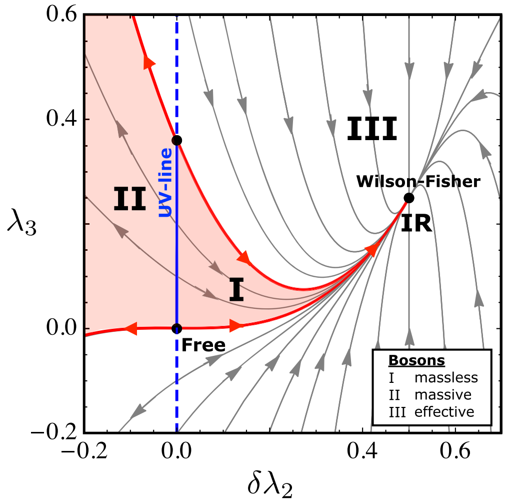

FIG. 4. Phase diagram of the bosonic theory, Eq. (16), in the $(\delta\lambda_2, \lambda_3)$ plane with $\delta\lambda_1 = 0$ (adapted from [31]).

Most notably, we observe that the sets of UV and IR scaling dimensions of the bosonic theory [31,32,48,49] are *identical* to those of the fermionic theory, Eqs. (13) and (14), respectively. We conclude that strongly interacting 3D fermions in the UV can be viewed as "quasibosons" in that they display the same conformal scaling dimensions as free bosons along the entire line of fixed points. By the same token, strongly interacting 3D scalars can be viewed as "quasifermions."

Comparing the large N phase diagrams and UV-IR connecting trajectories (Fig. 3 vs Fig. 4), either of which has been obtained using Eq. (3), we observe that both admit massless (region I) and massive theories (region II), and theories without UV completion (region III) [50]. Beyond large N, the line of fixed points collapses to a point. For the fermionic theory, this follows because the sextic becomes marginally irrelevant at $1/N$, which is mirrored in the scalar theory [31,32,49]. This is also in accord with the fact that the Townsend-Pisarski fixed point for scalars [51,52] and the Gat-Kovner-Rosenstein fixed point for fermions [53] at order $1/N$. We take these quantitative and structural similarities at large N as a hint for a deeper link between the theories, Eqs. (1) and (16), including away from critical points [47].

Discussion.—Using the renormalization group, we have studied large N Gross-Neveu models in three dimensions in the absence of chiral symmetry. We find that the theory remains nonperturbatively renormalizable, much like its chirally symmetric counterpart. The main novelty is that classically irrelevant 6F interactions of mass dimension −3 have become exactly marginal due to quantum fluctuations. This is also in agreement with earlier findings [53] based on a Gross-Neveu-Yukawa version of the theory. Hence, relaxing chiral symmetry adds a new fundamentally free parameter, and opens up an entire line of interacting UV fixed points. Fixed point solutions are well-defined globally, and limited by endpoints where scale symmetry is broken spontaneously [47]. Moreover, and even though fermion mass is no longer protected by symmetry, we find that massless theories prevail naturally, without any fine-tuning, and with chiral symmetry emerging in the IR. Further, the dynamical generation of mass, a combined effort of 4F and 6F interactions, proceeds without the breaking of a discrete symmetry.

An intriguing aspect of our results are the striking similarities with 3D scalar theories at large N. This includes equivalent conformal fixed points, phase diagrams, scaling dimensions, and UV-IR connecting trajectories. It will therefore be interesting to see whether explicit maps can be found relating RG flows in the theory of fermions, Eq. (1), to those in the theory of bosons, Eq. (16), in the spirit of a large N equivalence between *a priori* fundamentally different quantum field theories, e.g., [12]. While many large N equivalences or dualities in 3D involve Chern-Simons gauge fields, e.g., [13,14], the latter make no appearance in our setup. In the spirit of [53], it will also be interesting to cross-check the findings of this work within a Gross-Neveu-Yukawa formulation of the theory [39], given that their critical points are expected to be equivalent.

Finally, we look at our findings from the viewpoint of conformal field theory or higher spin gauge theories. The links between renormalization group fixed points and conformal field theories [54] can be exploited to extract further conformal data beyond scaling dimensions from our study. Moreover, certain versions of Vassiliev's higher spin theories on AdS_4 have been shown to be dual to free or interacting large N bosonic [55] or fermionic theories [21] on the boundary of AdS_4, with the parity-even Gross-Neveu UV fixed point ($\lambda_3 = 0$) relating to type-B Vassiliev theories [16]. Our study offers new large N critical fermions without parity symmetry ($\lambda_3 \neq 0$). It would also be interesting to understand whether critical endpoints ($\lambda_3 = \lambda_3^{\mathrm{crit}}$) continue to have higher spin duals [13], and whether the absence of parity or broken scale symmetry alters conformal field theory three-point functions of quasiboson or quasi-fermion theories [56,57].

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