Harrison and Chan Reply: Tallon posits [1] that there is a contribution E^* [black dashed line in Fig. 1(a) from Fig. 10(b) of Ref. [2]] to the antinodal gap Δ (purple line averaged over many measurements in Ref. [3]) whose origin is distinct from pairing and vanishes at $p^* = 0.19$. Were Tallon right, the pairing gap would be $\Delta' = \sqrt{\Delta^2 - E^{*2}}$ [4] and the gap ratio would therefore be $2\Delta'/k_B T_c$. However, because $E^* = 0$ at $p^* = 0.19$, we would still find $\delta\gamma(T_c)$ to peak at the same value of the gap ratio as in our Letter [3]. Tallon's main point [1] is therefore moot.

Tallon's vanishing E^* is widely considered as evidence for a quantum critical point at p^* in the cuprates [6]. We neglect it here because it is determined at $T > T_c$ (e.g., at $T = 110$ K in Ref. [7]), where it is nontrivial to ascertain whether or not Δ is due to pairing. Moreover, because the entropy $S(E^*)$ [renormalized by that $S(0)$ for $E^* = 0$] saturates for $E^* \lesssim 30$ meV [see Fig. 1(b)], Tallon's vanishing gap values at $p^* \approx 0.19$ are essentially an artifact of (i) $S(E^*)$ becoming insensitive to E^* once $E^* < 30$ meV and (ii) the introduction of additional assumptions, which are tailored to obtain the desired result [2,7–9].

By contrast, at $p = 0.205$, Vishik *et al.* [10] present direct spectroscopic evidence for gap in the normal state [labeled $\Delta_{T>T_c}$ in Fig. 1(a)]. Since $0.205 > p^*$, this clearly must be due to pairing. This gap persists for a range of temperatures above T_c before it is suppressed by line-broadening effects, which closely resembles the situation found in the unitary regime of a Fermi gas [11–14].

Without making any assumptions in our scaling analysis, we find there is only a single normal state energy scale [i.e., Δ in Fig. 1(a)] that is universal to *all* cuprates and that persists over a wide range of p and T [3]. Δ is an electronic excitation gap [15]. It is therefore expected to yield a maximum in γ at $T_\gamma \approx 0.3\Delta/k_B$ (roughly resembling a Schottky anomaly) and a broad maximum in χ at $T_\chi \approx 2\Delta/3k_B$. Such maxima are precisely what we find in the normal state experimental data for *all* cuprates [see Figs. 2(d), 3(a), and 3(b) of our Letter [3]].

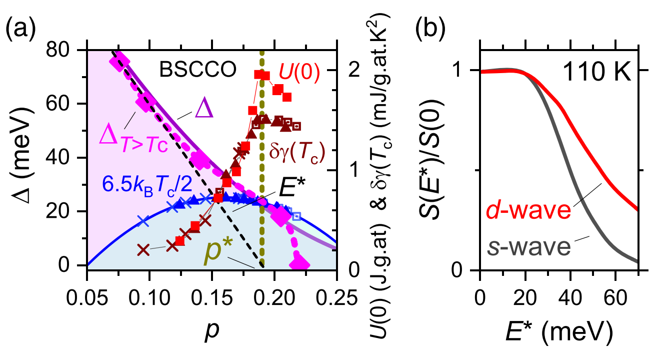

FIG. 1. (a) Experimental data for BSCCO from Refs. [3,5], including $U(0)$. (b) $S(E^*)/S(0)$ calculated for a gap E^*.

We respond to Tallon's other points in turn.

Second point: following standard usage, we use the condensation temperature (i.e., T_c) to determine the gap ratio [16–18].

Third point: since $\delta\gamma(T_c)$ often peaks at a different value of p from that $p \sim 0.16$ where T_c is maximum, a distribution of T_c values caused by disorder cannot be the sole origin of the suppression of $\delta\gamma(T_c)$ relative to its peak value. In fact, single crystalline samples of heavily overdoped TBCO with a T_c width of only ≈ 1 K [19] (therefore making them of arguably higher quality than those used for the $\delta\gamma(T_c)$ data in Fig. 1(c) of Ref. [3]) continue to exhibit an anomalously small $\delta\gamma(T_c) \approx 2.6$ mJ mol^{-1} K^{-2} [i.e., $\delta\gamma(T_c)/\gamma \approx 0.37$]. This points to the weakness of the pairing interactions in the limit $p > p^*$ as the cause.

Fourth point: there may be overdoped cuprates where $\gamma_{T\to 0}$ versus p peaks at a van Hove Singularity (VHS), but we make no such claim in our Letter [3]. Rather, we claim that the normal state γ exhibits a maximum as a function of T that coincides with T_c when $p = p^*$, which is a very different type of behavior and which becomes very clear upon rescaling T and T_c by 2Δ (plotted as reciprocals) in Fig. 3(a) of our Letter [3].

Final point: we note that while the VHS can in principle give rise to a maximum in γ [20,21], Δ easily gaps the VHS, causing it to become largely irrelevant at $p < p^*$.

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