Comment on "Magic Gap Ratio for Optimally Robust Fermionic Condensation and Its Implications for High-T_c Superconductivity"

Recently, Chan and Harrison (CH) [1] argued that the pseudogap in cuprates arises from incoherent bosonic pairs above the superconducting transition temperature T_c, and that these pairs Bose-Einstein condense (BEC) in the underdoped region, while they form a Bardeen-Cooper-Schrieffer (BCS) condensate in the more overdoped region. They propose a BEC to BCS crossover at a critical doping p^* by identifying a universal magic gap ratio $2\Delta = k_B T_c \approx 6.5$, where Δ is the antinodal (AN) gap, at which pair condensates become optimally robust. At this unitary point the jump $\Delta\gamma$ in specific heat coefficient should peak sharply. They draw extensively on the data of Loram *et al.* [2].

The analysis is interesting but there are many problems. The underdoped AN gap has been shown to be associated with the pseudogap while a distinct pairing gap opens on the residual Fermi arcs (or hole pockets) lying between the "pseudogapped" antinodes [3]. The pseudogap closes abruptly at $p^* = 0.19$, independent of temperature [4,5]. At lower doping this AN gap is often much larger than the pairing gap so the underdoped gap ratios used by CH are much larger than those relevant to pairing. Moreover, these gap ratios should be constructed not on the observed T_c but on their mean-field values which are significantly higher (up to 25 K higher for $(\mathrm{Y,Ca})\mathrm{Ba_2Cu_3O}_x$ and 45 K higher for $\mathrm{Bi_2Sr_2CaCu_2O}_{8+\delta}$) [6,7,4]. From our overdoped measurements we typically obtain $2\Delta = k_B T_{c,\mathrm{mf}} \approx 5$ [6] much closer to the weak-coupling BCS ratio. Further, the spectral pairing gap Δ is relevant in the absence of a competing gap but, when the pseudogap is present below p^*, one should use the order parameter Δ', which is the quadratic difference $\mathrm{SQRT}[\Delta(T,\mathbf{k})^2 - E^*(\mathbf{k})^2]$ [8] and thus smaller than Δ. This effectively establishes an unchanging gap ratio across the entire superconducting phase curve. It seems there is no tunable spectrum of pairing gap ratios as proposed by CH.

This competing pseudogap inevitably reduces $\Delta\gamma$ below p^*, because the temperature slope of $\mathrm{SQRT}[\Delta(T,\mathbf{k})^2\text{-}E^*(\mathbf{k})^2]$ at T_c decreases as E^* increases. However, contrary to CH, $\Delta\gamma$ does not peak at p^* for $\mathrm{Bi_2Sr_2CaCu_2O}_{8+\delta}$ or $\mathrm{La}_{2\text{-}x}\mathrm{Sr}_x\mathrm{CuO}_4$ [2], in fact the mean-field value, $\Delta\gamma^{\mathrm{mf}}/\gamma$ remains near the weak-coupling BCS value in overdoped $\mathrm{Bi_2Sr_2CaCu_2O}_{8+\delta}$. In $\mathrm{Y_{0.8}Ca_{0.2}Ba_2Cu_3O_x}$, $\Delta\gamma$ does fall for $p > p^*$ but only because the transitions are increasingly broadened as T_c falls, due to the distributed Ca doping.

The scaled condensation energy $U_0/\gamma T_c^{\mathrm{mf}\,2}$ at $T = 0$ is another important ratio. Under BCS and d-wave symmetry its value is 0.17 and that value is found across the overdoped region for $p > p^*$, essentially constant [9]. But for $p < p^*$ this ratio drops abruptly as the pseudogap opens and the order parameter Δ' falls below Δ. Crucially, the pseudogap is still present at $T = 0$ [9,4,5] where all pairs have condensed so the ground-state pseudogap cannot be associated with uncondensed pairs.

Finally, we note that CH draw heavily on the idea that γ itself maximizes at p^*, but this is not the case though it seems approximately so on the log scale used. For $\mathrm{Bi_2Sr_2CaCu_2O}_{8+\delta}$ and $\mathrm{La_{2\text{-}x}Sr_xCuO_4}$ it is true that γ passes through a maximum but this occurs at higher doping at the van Hove singularity crossing [10]. For $\mathrm{YBa_2Cu_3O}_{7-\delta}$ and $\mathrm{Y_{0.8}Ca_{0.2}Ba_2Cu_3O}_{7-\delta}$ there is no observed maximum in $\gamma(p)$ [2] because the van Hove singularity lies well below the Fermi surface.

In summary, we feel that our data do not support the idea of a unitary-point magic gap ratio in the cuprates. Rather they behave like near-weak coupling BCS superconductors with an independent competing gap below p^*.

Jeffery L. Tallon
Robinson Research Institute
Victoria University of Wellington
P.O. Box 33436, Lower Hutt 5046, New Zealand

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